

Small Substitution

$$10x_1 - x_2 = 9$$

$$-x_1 + 10x_2 - 2x_3 = 7$$

$$-2x_2 + 10x_3 = 6$$

$$Q = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} 9 \\ 7 \\ 6 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} - \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

k=0

it=1

$$x_1^{(1)} = \frac{9+0}{10} = 0,9$$

$$e = \left| \frac{0,9 - 0}{0,9} \right| \cdot 100 = 100\%$$

$$x_2 = \frac{7 + 0,9 + 0}{10} = 0,79$$

$$e = \left| \frac{0,79 - 0}{0,79} \right| \cdot 100 = 100\%$$

$$x_3 = \frac{6 + 2(0,79)}{10} = 0,758$$

$$e = \left| \frac{0,758 - 0}{0,758} \right| \cdot 100 = 100\%$$

it=2

$$x_1 = \frac{9 + 0,79}{10} = 0,979$$

$$e = \left| \frac{0,979 - 0,9}{0,979} \right| \cdot 100 = 8,069\%$$

$$x_2 = \frac{7 + 0,979 + 2(0,758)}{10} = 0,9495$$

$$e = \left| \frac{0,9495 - 0,79}{0,9495} \right| \cdot 100 = 16,738\%$$

$$x_3 = \frac{6 + 2(0,9495) + 0}{10} = 0,7899$$

$$e = \left| \frac{0,7899 - 0,758}{0,7899} \right| \cdot 100 = 4,038\%$$

it=3

$$x_1 = \frac{9 + 0,9495}{10} = 0,99495$$

$$e = \left| \frac{0,99495 - 0,979}{0,99495} \right| \cdot 100 = 1,603\%$$

$$x_2 = \frac{7 + 0,99495 + 2(0,7899)}{10} = 0,957475$$

$$e = \left| \frac{0,957475 - 0,9495}{0,957475} \right| \cdot 100 = 0,832\%$$

$$x_3 = \frac{6 + 2(0,95789)}{10} = 0,791495$$

$$e = \left| \frac{0,791495 - 0,7899}{0,791495} \right| \cdot 100 = 2,061\%$$

Método de Cramer

$$\begin{aligned} 10x_1 - x_2 &= 4 \\ -x_1 + 10x_2 - 2x_3 &= 7 \\ -2x_2 + 10x_3 &= 6 \end{aligned}$$

$$\det(A) = \begin{vmatrix} 10 & -1 & 0 \\ -1 & 10 & -2 \\ 0 & -2 & 10 \end{vmatrix} =$$

$$\det \begin{pmatrix} 10 & -2 \\ -2 & 10 \end{pmatrix} = 96 \quad \det \begin{pmatrix} -1 & -2 \\ 0 & 10 \end{pmatrix} = -10 \quad \det \begin{pmatrix} -1 & 10 \\ 0 & -2 \end{pmatrix} = 2$$

$$\det(A) = 10 \cdot 96 - (-1)(10) + 0 \cdot 2 = 950$$

$$\frac{\Delta x_1}{\Delta} = \frac{\begin{vmatrix} 4 & -1 & 0 \\ 7 & 10 & -2 \\ 6 & -2 & 10 \end{vmatrix}}{950} = \frac{4 \cdot 10 \cdot 10 + (-1)(-2) \cdot 6 + 0 \cdot 7 \cdot (-2) - 6 \cdot 10 \cdot 0 - (-2) \cdot 4 \cdot 10 \cdot 7 \cdot (-1)}{950}$$

$$\frac{\Delta x_1}{\Delta} = \frac{946}{950} = 0,995789$$

$$\frac{\Delta x_2}{\Delta} = \frac{\begin{vmatrix} 10 & 4 & 0 \\ -1 & 7 & -2 \\ 0 & 6 & 10 \end{vmatrix}}{950} = \frac{10 \cdot 7 \cdot 10 + 4(-2)(6) + 0(-1)(6) - 0(7)(0) - 6(-2)(10) - 10(-1)4}{950}$$

$$\frac{\Delta x_2}{\Delta} = \frac{940}{950} = 0,989474$$

$$D_{x_3} = \frac{\begin{vmatrix} 10 & -1 & 9 \\ -1 & 10 & 2 \\ 0 & -2 & 6 \end{vmatrix}}{950} = \frac{10 \cdot 10 \cdot 6 + (-1) \cdot 12 \cdot 0 + 9 \cdot (-1) \cdot (-2) - 0 \cdot 10 \cdot 9 - (-2) \cdot 12 \cdot 10 - 6 \cdot (-1) \cdot (-1)}{950} = \frac{782}{950}$$

$$\frac{D_{x_3}}{D} = \frac{782}{950} = 0,791578 //$$

$$e_1 = \left| \frac{0,995789 - 0,995789}{0,995789} \right| \cdot 100 = 0 \% //$$

$$e_2 = \left| \frac{0,957584 - 0,957584}{0,957584} \right| \cdot 100 = 0 \% //$$

$$e_3 = \left| \frac{0,791578 - 0,791578}{0,791578} \right| \cdot 100 = 0 \% //$$