

$$2(a) \quad G(s) = \frac{K}{s^2(s+4)(s+12)}$$

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

$$1 + \frac{K}{s^2(s+4)(s+12)} = 0$$

$$s^4 + 16s^3 + 48s^2 + K = 0$$

Routh array

$$\begin{array}{ccc} s^4 & 1 & 48 & K \\ s^3 & 16 & 0 & \\ s^2 & 48 & K & \\ s^1 & -\frac{1}{3}K & & \\ s^0 & \frac{1}{K} & & \end{array}$$

range of K cannot be found since K cannot be greater than 0 and less than $-\frac{1}{3}$ at the same time.

\therefore The system cannot be stabilized by pure gain adjustment.

$$(b) \quad \text{damping ratio} = \zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}} = 0.4504$$

$$\text{The real part of dominant poles } \sigma_d = -\frac{4}{3}$$

$$\Rightarrow \omega_n = \frac{\sigma_d}{\zeta} = \frac{-\frac{4}{3}}{0.4504} = -2.9603$$

$$\therefore \text{dominant poles at } -(0.4504)(2.9603) \pm j(2.9603)\sqrt{1-(0.4504)^2}$$

$$= -1.333 \pm j2.643$$

The angle contributed by the added compensator =

$$-\theta - \left[180 - \tan^{-1} \left(\frac{2.643}{-1.333} \right) \right] - \tan^{-1} \left(\frac{2.643}{-4 + 1.333} \right) - \tan^{-1} \left(\frac{2.643}{-12.41 - 333} \right) - 1 = -180^\circ$$
$$+ \left[180 - \tan^{-1} \left(\frac{2.643}{-1.333 + 0.02} \right) \right]$$

$$\theta = 120.996^\circ$$

$$\frac{1.333}{-2.643 - 0.1} = \tan(120.996^\circ)$$

$$GA = -1.842$$