P1

Date: __/__/20

Conditional probability

formula

$$P(E/F) = \frac{P(E\cap F)}{P(F)} = \frac{n(E\cap F)}{n(F)}$$

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$.

let

E = At least 2 head Appears

F = last Coin shows tail.

Now talking from "S"

$E = \{HHH, HHT, HTH, THH\}$

$F = \{HHT, HTT, THT, TTT\}$. $\Rightarrow E\cap F = 1$ $\Rightarrow P(F) = 4$

$$\boxed{\cancel{P(E\cap F) = \frac{n(E\cap F)}{n(S)} = \frac{1}{8} = \frac{1}{8}} \quad \cancel{P(E/F) = \frac{P(E\cap F)}{P(F)} = \frac{\frac{1}{8}}{4}}}$$

$$P(E\cap F) = \frac{n(E\cap F)}{n(S)} = \frac{1}{8}$$

$$P(F) = \frac{n(F)}{n(S)} = \frac{\cancel{4}}{\cancel{8}_2}$$

$$P(E/F) = \frac{1/8}{1/2}$$

$$P(E/F) = \frac{\cancel{2}}{\cancel{8}4} = \frac{1}{4} \text{ Ans}$$

Slide 2

$S = \{ i = 1,2,3,4,5,6 ,\ j = 1,2,3,4,5,6 \}$.

let

E = dice on sid 3

F = Sum of 2 dice equal 8.

	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

$F = \{ 31, 32, 33, 34, 35, 36 \}$

$E = \{ 26, 35, 44, 53, 62 \}$

$$P(E \cap F) = \frac{n(E \cap F)}{n(S)} = \frac{1}{36}$$

$$P(F) = \frac{n(F)}{n(S)} = \frac{\cancel{6}\ 1}{\cancel{36}\ 6}$$

$$P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{36}}{\frac{5}{36}}$$

$$= \frac{1}{\cancel{36}} \times \frac{\cancel{36}}{5}$$

$$P(E/F) = \frac{1}{5}\ \text{Ans.}$$

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5 defective
10 partially
25 acceptable.

let

E: It is acceptable

F: partiall defective or acceptable

$E = \{25\}$

$F = \{25+10\} = \{35\}$

$$P(E/F) = \frac{P(E \cap F)}{P(F)}$$

$$P(E \cap F) = \frac{n(E \cap F)}{n(S)} = \frac{\cancel{25}\,5}{\cancel{40}\,8}$$

$$P(F) = \frac{n(F)}{n(S)} = \frac{\cancel{35}\,7}{\cancel{40}\,8}$$

$$P(E/F) = \frac{5}{8} \times \frac{8}{7}$$

$$\boxed{P(E/F) = \frac{5}{7}}$$

Slide 8

$S = \{(b,b), (b,g), (g,b), (g,g)\}$

E = Joh have both boy

F = Boy ~~and~~ / girl

$E = \{b, b\}$

$F = \{(b,g), (g,b), (b,g)\}$

$$P(E/F) = \frac{P(E \cap F)}{P(F)}$$

$$P(E \cap F) = \frac{n(E \cap F)}{n(S)} = \frac{1}{4}$$

$$P(F) = \frac{n(F)}{n(S)} = \frac{3}{4}$$

$$\boxed{P(E/F) = \frac{1}{4} \times \frac{4}{3} = \frac{1}{3} \text{ Ans}}$$

1/10

depart on time = 90%
Arrive on time = 80%
Arive and depart on time = 75%

P2

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(a)

E = Anna will arrive on time

F = John depart on time.

$P(E \cap F) = \frac{n(E \cap F)}{n(S)} = \left\{ \frac{0.75}{1} \right\}^{\text{for on time both}} = ~~0.833~~ \; 0.75$

$P(F) = \frac{n(F)}{n(S)} = \frac{0.90}{1} = 0.90$

$\boxed{P(E|F) = \frac{0.75}{0.90} = 0.833}$ Ans:-

(b)

E = Ana departed on time =

F = " Arrived on time :

$P(E \cap F) = \frac{n(E \cap F)}{n(S)} = \left\{ \frac{0.75}{1} \right\}^{\text{for on time both}} = 0.75$

$P(F) = \frac{n(F)}{n(S)} = \frac{0.80}{1} = 0.80$

$\boxed{P(E|F) = \frac{0.75}{0.80} = 0.9375}$ Ans:-

Slide12

Length test fail = 10%

Texture test fail = 5%

both fail 0.8%

E = texture defectiveness

F = ~~failing length test both~~ length test.

$P(E \cap F) = \frac{n(E \cap F)}{n(S)} = \frac{0.08}{0.1} = 0.05 \qquad \frac{0.008}{0.1} = 0.008$

$= 0.0080 = 0.5$

$P(F) = \frac{n(F)}{n(S)} = \frac{0.008}{0.1} = 0.05 \qquad \frac{0.10}{0.1}$

$= \frac{0.0008}{0.05} = 0. \qquad 0.1$

both fail

$P(E \cap F) = \frac{n(E \cap F)}{n(S)} = \frac{0.008}{1} = 0.008$

$P(F) = \frac{n(F)}{n(S)} = \frac{0.10}{10} = 0.1$

$$\boxed{P(E/F) = \frac{0.008}{0.1} = 0.08 \text{ Ans.}}$$

$P(AB) = P(A/B)\,P(B)$ —— ①

$P(AB) = P(B/A)\,P(A)$ —— ②

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Q

Slected 2 balls randomly r & b.

Let

E = 1st ball red & 2nd blue.

F = 2 balls Selected.

$$\boxed{\cancel{P(E/F) = \frac{P(E\cap F)}{n(S)} = \frac{r+b}{r+b}}}$$

⇒ Total probability theorem ← کوئی کام ہو بھی سکتا ہے یا نہیں اسکے لیے use کریں۔

$$P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + \cdots P(E_n)P(A/E_n)$$

$$\boxed{= \sum_{i=1}^{n} P(E_i)\,P(A/E_i)}$$

Q Question Screenshot prob "Strike"

Let

Event A = the Construction job will be Completed on time

B = there is Strike on Construction site

B′ = " " no " " " "

for B, B′ partition formula using theorem use ho ga.

$P(A) = ?$ $\quad P(B') = 1 - P(B) = 0.35$

$P(B) = 0.65$ $\quad P(A/B') = 0.8$ $\quad P(A/B) = 0.32$

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Now using theorem.

$$P(A) = P(B)\,P(A/B) + P(B')\,P(A/B')$$

$= 0.488$ Ans:-

Slide 27 $\quad$ Now $\quad B_1 = 30\%$, $B_2 = 45\%$ $\quad B_3 = 25\%$

Past $\rightarrow$ $B_1 = 2\%$, $B_2 = 3\%$, $B_3 = 2\%$ } defective

Let

Event A = The product is defected

B_1 = The product is made by machine B_1,

B_2 = " " " " " B_2,

B_3 = " " " " " B_3.

$$P(A) = P(B_1)\,P(A/B_1) + P(B_2)\,P(A/B_2) + P(B_3)\,P(A/B_3)$$

$= (0.3)(0.02) + (0.45)(0.03) + (0.25)(0.02)$

$= 0.006 + 0.0135 + 0.005$

$\boxed{P(A) = \cancel{0.006 + 0}\ 0.0245}$ Ans:-

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P3

Baye's theorem

these are dependent

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$$P(E_i/A) = \frac{P(E_i)\cdot P(A/E_i)}{\sum_{i=1}^{n} P(E_i)\cdot P(A/E_i)}$$

This will change

This will same for all

Screenshots

Q =

Let

Events

$\to E_1 =$ ~~Red~~ ball drawn from bag II.

$\to E_2 =$ ball draw from bag I.

$\to A =$ the ball is Red.

bag I $= 3R + 4B$, bag II $= 5R + 6B$

As Here we have to find prob of E_1 so,

$P(A/E_1) = P(\text{the red ball draw from bag I}) = \frac{3}{7}$

$P(A/E_2) = P(\text{" " " " " bag II}) = \frac{5}{11}$

$P(E_1) = P(E_2) = \frac{1}{2}$

By formula

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\frac{1}{2}\cdot\frac{3}{7}}{\frac{1}{2}\cdot\frac{3}{7}+\frac{1}{2}\cdot\frac{5}{11}} = \frac{308}{136} \times \frac{3}{14} = \frac{35}{68}\ \text{As.}$$

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Q Let

Even E_1 = Patient has kidney Cancer.

P_0 = 1 in every 1000 individual had kidney Cancer by test

E_1^c = patient have no kidney Cancer by test

P_0^c = patient have -ve test Cancer.

$P(P_0/C) = \frac{90}{100} = 0.9$

$P(P_0^c/C^c) = \frac{99.9}{100} = 0.999$

$P(P_0/C^c) = 1 - 0.999 = 0.001$

As 1 in every 10,000 have kidney Cancer

$P(C) = \frac{1}{10,000} = 0.0001$

$P(C^c) = 1 - 0.0001 = 0.9999.$

By Law of probability

$P(P_0) = P(P_0/C)P(C) + P(P_0/C^c)P(C^c)$

$= (0.9 * 0.0001) + (0.0001 * 0.9999)$

$= 0.00010899.$

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Bayes Law.

$$P(C/P_o) = \frac{P(P_o/C_1)\,P(C)}{P(P_o)} = 0.08 \text{ Ans.}$$

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~~E_1 = plant will be alive when you return~~

let A_1 = neighbour will water the plant.

A_2 = " " not " " "

B = The plant will die.

$P(A_1) = 0.9$, $P(A_2) = 0.1$; $P(B/A_1) = 0.15$, $P(B/A_2) = 0.8$

(a) Plant will alive?

~~Plant will die~~

$$P(B) = P(A_1)P(B/A_1) + P(A_2)P(B/A_2)$$

$$= (0.9)(0.15) + (0.1)(0.8)$$

$P(B) = 0.215$

as in Q plant will alive?

~~$P(B)$~~

$P(B') = 1 - 0.215 = 0.785$

$\boxed{P(B') = 0.785}$

(b) $$P(A_2/B) = \frac{P(A_2)P(B/A_2)}{P(B)}$$

$$= \frac{(0.10)(0.8)}{0.215} = 0.372 \text{ Ans.}$$

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= Q let

= $chol\ P(A) = 60\% = 0.6$

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$P(B/A^c) = 10\% = 0.1$

As $P(A \cap B) = 0$

So As one of them will occur due to which

$P(A \cup B) = P(A) + P(B)$

$= P(A) + P(B/A^c)P(A^c)$

$= 0.6 + (0.1)(1-0.6)$

$= 0.6 + 0.04$

$= 0.6$

Slide 37 $A=3$, B = Sum of dice 8, C = Sum of dice 7

(a) Are A and B Independent?

	1	2	3	4	5	6
1	11	12	13	14	15	16
2	21	22	23	24	25	26
3	31	32	33	34	35	36
4	41	42	43	44	45	46
5	51	52	53	54	55	56
6	61	62	63	64	65	66

$P(A \cap B) = P\{(3,5)\} = \frac{1}{36}$

1)

P4

$P(A) = P\{(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)\} = \frac{6}{36}$

$P(B) = P\{(2,6),(3,5),(4,4),(5,3),(6,2)\} = \frac{5}{36}$

$P(A \cap B) = P(A) \cdot P(B)$

$\frac{1}{36} = \frac{\cancel{6}}{36} \times \frac{5}{\cancel{36}\,6}$

$\frac{1}{36} \neq \frac{5}{36 \times 6} \Rightarrow P(A \cap B) \neq P(A) \cdot P(B)$

So A and B are independent.

(B)

$P(A \cap C) = \frac{1}{36}$

$P(A) = P\{(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)\} = \frac{6}{36}$

$P(B) = P\{\cancel{(2,6)},(2,5),(4,3),(5,3),(6,1),(3,4)\} = \frac{5}{36}$

$\frac{1}{36} \neq \frac{6}{36} \times \frac{5}{36}$ These are also not Independent.

Slide 39

A: Head on first Coin

B: Tail on 2nd Coin.

A and B are Indepent?

Sol $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{2} \Rightarrow P(A \cap B) = \frac{1}{4}$

$P(A) \cdot P(B) = \frac{1}{4} = P(A \cap B)$

So Events are Independent.

Slide 44

Let A_1 Event red on both side, A_2 black both side, A_3 black red

B be the event for one side

$P(A_1) = \frac{1}{3}$ | $P(B/A_1) = 1$

$P(A_2) = \frac{1}{3}$ | $P(B/A_2) = 0$

$P(A_3) = \frac{1}{3}$ | $P(B/A_3) = \frac{1}{2}$

By Bays Law.

$$P(A_1/B) = \frac{P(B/A_1)\,P(A_1)}{P(A_1)\times P(B/A_1) + P(A_2)\,(P(B/A_2)) + P(A_3)\,P(B/A_3)}$$

$$= \frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1 + \frac{1}{3}\times 0 + \frac{1}{3}\times\frac{1}{2}} = \frac{1}{1.5} = \frac{2}{3}$$

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$P(E_1)$ = 1st module error = 0.2

$P(E_2)$ = 2nd " " = 0.4

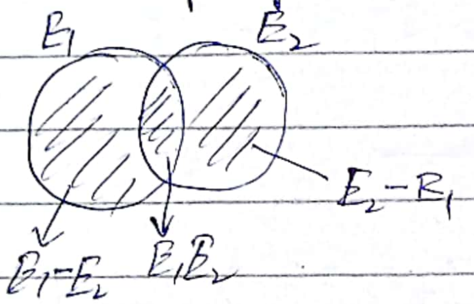
As in Q,

① E_1 & E_2 are independent so

$$P(E_1) \cap P(E_2) = P(E_1) \cdot P(E_2) = 0.2 \times 0.4 = 0.08$$

$$\boxed{P(E_1 E_2) = 0.08}$$

As the program crash due to some portion we will draw some sample space.



$P(C \mid E_1 - E_2) = 0.5$ → probability crash alone error in module 1

$P(C \mid E_2 - E_1) = 0.8$

$P(C \mid E_1 E_2) = 0.9$

$P(E_1 E_2 \mid C) = ?$ final. Error in $E_1 E_2$ but Crashed.

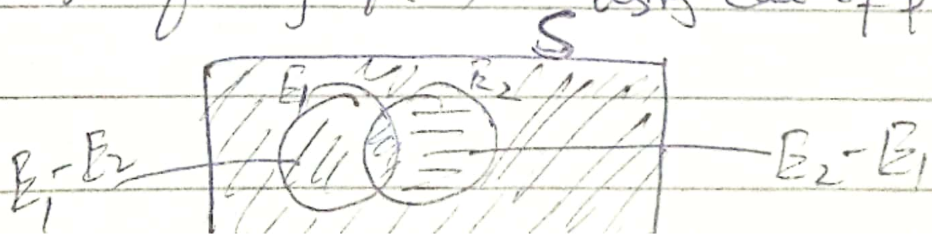
$$P(E_1 E_2 \mid C) = \frac{P(C/E_1 E_2)\, P(E_1 E_2)}{P(C)}$$

$P(C) \to ?$

Bay's rule

$$P(A/B) = \frac{P(B/A)\, P(A)}{P(B)}$$

Now finding $P(C) \Rightarrow ?$ using Law of total probability.



by picture possible. Crash are.

$E_1 - E_2$, $E_2 - E_1$, E_1E_2, $\overline{E_1 \cup E_2}$

using Law of total partition. using these

$$P(A) = P(B_i)(P(A/B_i))$$

~~$P(E) = P(C_i)P(E_i)$~~

$$P(C) = P(C|E_1 - E_2)P(E_1 - E_2) +$$
$$P(C|E_1E_2)P(E_1E_2) +$$
$$P(C|E_2 - E_1)P(E_2 - E_1) +$$
$$\underbrace{P(C|(E_1 \cup E_2)^c)P((E_1 \cup E_2)^c)}_{} \quad \text{— (A)}$$

this is for if no error as there is error so it is zero (0).

from figure

$$E_1 = (E_1 - E_2) \cup (E_1E_2)$$

$$P(E_1) = P(E_1 - E_2) \cup P(E_1E_2)$$

$$P(E_1 - E_2) = P(E_1) - P(E_1E_2) \quad \text{—— (a)}$$

put (a) in (A) then put values. $P(E_2 - E_1) = P(E_2) - P(E_1E_2)$ → (b)

$$= P[(0.5)(0.2) - (0.5)(0.08)] +$$

$$(0.9)(0.08) + [(0.8)(0.4) - (0.8)(0.08)] + 0$$

$$= (1 - \overset{0.96}{0.04}) + 0.072 + 0.256 = 1.28 \text{ Ans}$$

0.32

(P5)

Q50 $50\% \to S1$, $20\% \to S2$, $30\% \to S3$

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from $S1 \to 5\%$ deftive, from S_2 3% defective, from S_3 6% def

$P(S1) = 0.5$ | $P(D/S1) = 0.05$ | D = Random part is defective

$P(S_2) = 0.2$ | $P(D/S_2) = 0.03$

$P(S3) = 0.3$ | $P(D/S_3) = 0.06$

$P(D) = ?$

(a) $P(A_i) = P(A_i/B_i)\,P(B_i)$

$P(D) = P(D/S1)P(S1) + P(D/S_2)P(S_2) + P(D/S_3)P(S_3)$

$= (0.05)(0.5) + (0.03)(0.2) + (0.06)(0.3)$

$= 0.049$

(b) part is supplied by $S1 = ?$

$$P(S1/D) = \frac{P(D/S1)\,P(S1)}{P(D)}$$

$$= \frac{(0.05)(0.5)}{0.049} = \frac{25}{49}$$

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$P(j) = 10$ G = Grade of stu

$P(S) = 30$, $P(g) = 10$ $\boxed{\text{total} = 20A}$

$\boxed{P(G/j) = 5,\ P(G/S) = 10,\ P(G/g) = 5}$

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Probability those Students who got A grade.

$P(A) = \frac{2\cancel{0}}{5\cancel{0}}$

As All from 10 are senior students so

$P(S) = \frac{1\cancel{0}}{5\cancel{0}}$

$P(S/A) = \frac{\frac{2}{5}}{\frac{1}{5}}$

$P(S/A) = \frac{2}{\cancel{5}} \times \frac{\cancel{5}}{1}$

$\boxed{P(S/A) = 2}$

Slide 49

Let

Event A_1 = inspected parts $= 0.20$ ~~Fair inspection of parts~~

A_2 = not " " $= 0.80$

$P(D/A_1)$ = part defective and inspected $= 0.05$

$P(D/A_2)$ = part defective but not inspected $= 0.30$

$$P(D) = P(D/A_1)\,P(A_1) + P(D/A_2)\,P(A_2)$$

$= (0.05)(0.20) + (0.30)(0.80)$

$= 0.01 + 0.24 = 0.25$

...ng bays law.

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$$P(A_1/D) = \frac{P(D/A_1)\,P(A_1)}{P(D)}$$

$$= \frac{(0.05)(0.20)}{0.25}$$

$$P(A_1/D) = 0.04 \text{ Ans.}$$

Random variable Experiment

Slide 86

Ex $S = \{(d,d), (d,a), (a,d), (a,a)\}$

$X =$ number of acceptable parts.

from Sample space S.

$X(d,d) = 0$, $x(d,a) = x(a,d) = 1$

$X(a,a) = 2$

for acceptable

$p(x=0) = 0.09$

$p(x=1) = 0.21$

$p(x=2) = 0.49$

Let I is random variable where atleast on accepted.

$$I = \begin{cases} 1 & \text{if } X = 1 \text{ or } 2 \\ 0 & \text{if } X = 0 \end{cases}$$

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