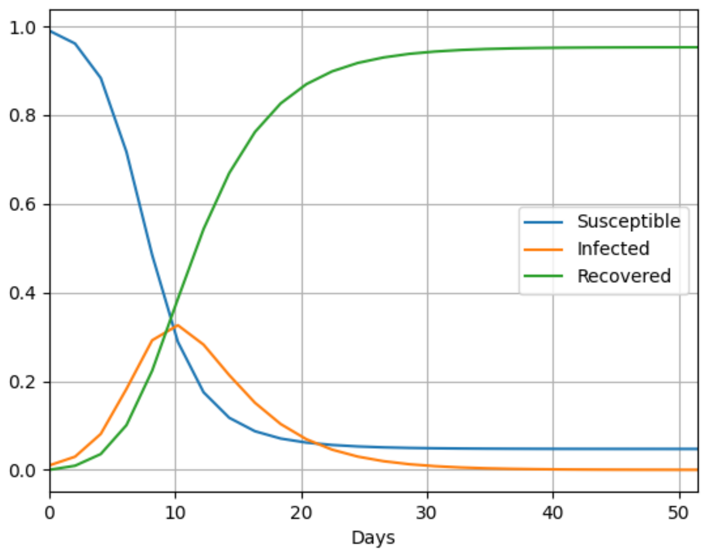
<u>Question 1.1</u>

I am not entirely sure how to mathematically get the fixed points in this scenario. Logically, the point (0, 0, 0), which expresses extinction, is an unstable fixed point. In addition, since the changes in S(t) and I(t) depend on the infected, (S=1, 0, 0) will be an unstable fixed point (as the lines will move away from that point if an infected is added). Finally, based on the graphs, when the number of infected reaches 0 (after infection), (S, 0, R) is a stable fixed point (where S + R = 1).

<u>Question 1.2</u>

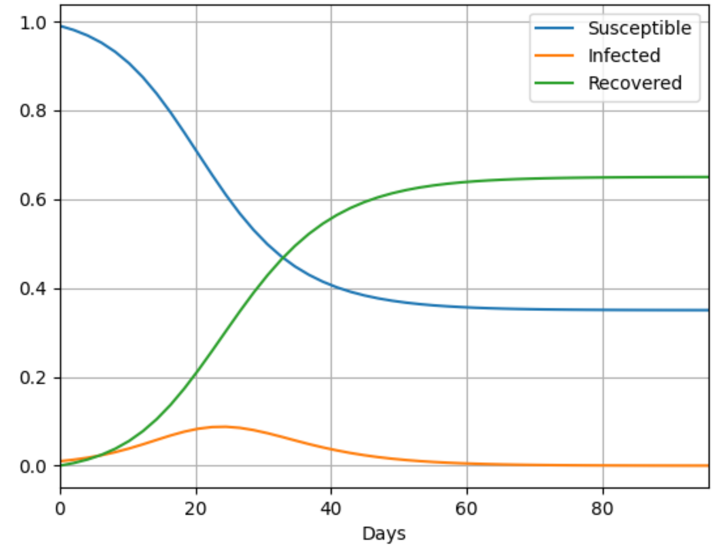
All simulations have initial conditions I(0) = 0.01, S(0) = 0.99, R(0) = 0 and a stopping condition of I(t) < 0.0001. The time intervals are assumed to be days (for labeling purposes).

Simulation with $\tau = 0.8$, $\kappa = 4$:



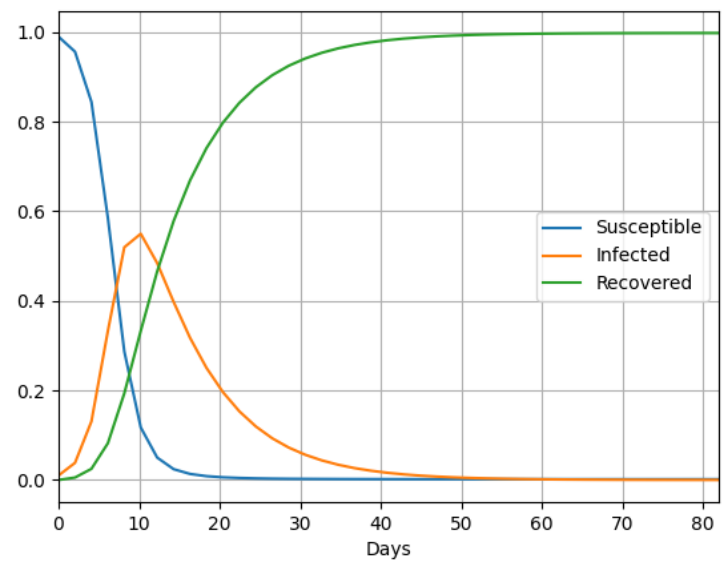

```
The stopping condition occurs at:
[51.46639496]
```

Simulation for τ = 0.4, κ = 4:




```
The stopping condition occurs at:
[95.6855907]
```

Simulation with τ = 0.8, κ = 8:




```
The stopping condition occurs at:
[81.94656937]
```

Analysis:

Overall, each plot has the same shape: the susceptible line simply drops down and then levels out, the recovered line grows and then levels out, and the infected line grows, reaches a maximum, then levels out getting close to 0. However, the different τ and κ cause the lines to have different growth/ decline rates resulting in them leveling out at different values.

Since τ measures how fast the disease can spread, a higher τ causes the infected line to reach its maximum faster ($\tau = 0.8$, in the 1st and 3rd scenario, has a spike near 10 days while $\tau = 0.4$, the 2nd scenario, has one at about 25 days). Since the number of recovered depends on the number infected, a higher τ also drives the recovered line up faster.

Since κ measures how long it takes an infected person to recover, a higher κ causes the recovered line to grow slower (the 1st scenario, $\kappa = 4$, has the recovered line level out at 30 days while the 3rd scenario, $\kappa = 8$, has it level out at 50 days). However, how this truly affects the recovered line also depends on τ since τ would cause changes in the number of infected people which means a lot of people or a small amount could be recovering at one time (which is why the recovered line in the 1st scenario grows faster than the one in the 2nd).

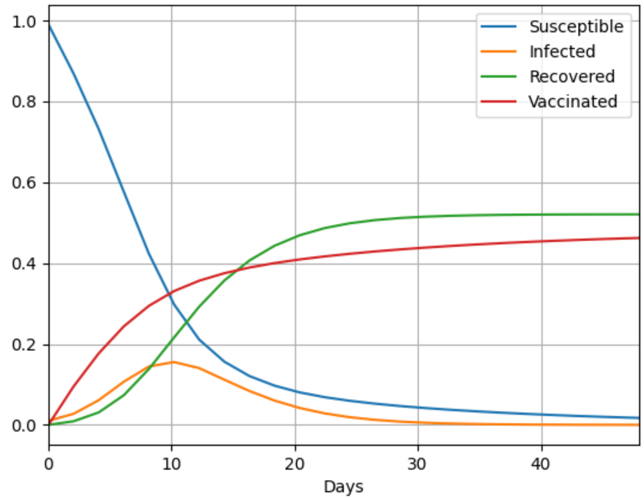
Since the stopping condition is based on the number of infected people, the 2nd scenario (slowest infection rate, $\tau = 0.4$) took the longest to reach the stopping condition as it had to slowly grow (and the recovery process had a smaller amount of infected people to work with at a time). Then, when the infection rate was the same, the third scenario took longer than the first because the recovery of infected people took longer ($I(t)/8$ instead of $I(t)/4$).

Question 1.4

In addition to the previous listed initial conditions and stopping condition ($I(0) = 0.01$, $S(0) = 0.99$, $R(0) = 0$, and $I(t) < 0.000$), another initial condition ($V(0) = 0$) was added to denote the starting amount of vaccinated people.

Since when people are vaccinated, $S(t)$ should decrease, and $V(t)$ should increase by the same amount. Based on the relationship between the infected people and the recovered people, which is $I(t)/\kappa$, the rate at which $S(t)$ and $V(t)$ changed was implemented to be $S(t)*R$ (where R was chosen to be 0.05).

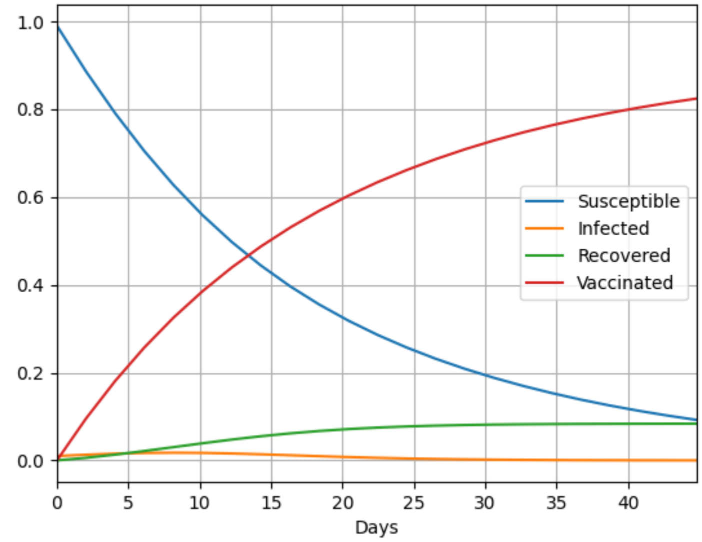
Simulation with τ = 0.8, κ = 4:




```
The stopping condition occurs at:
[47.97551469]
```

Comparing this graph to the first one in Question 1.2, the stopping condition is met earlier. However, the change in time isn't that much (probably due to the change in the number of infected people decreasing the closer the number approaches 0). In addition to the earlier stopping time, the height of the infected and recovered graphs got a lot shorter (recovered stopped at about 0.55 vs. Q1.2's 0.95, and the peak in the infected number went from Q1.2's 0.33 to 0.15).

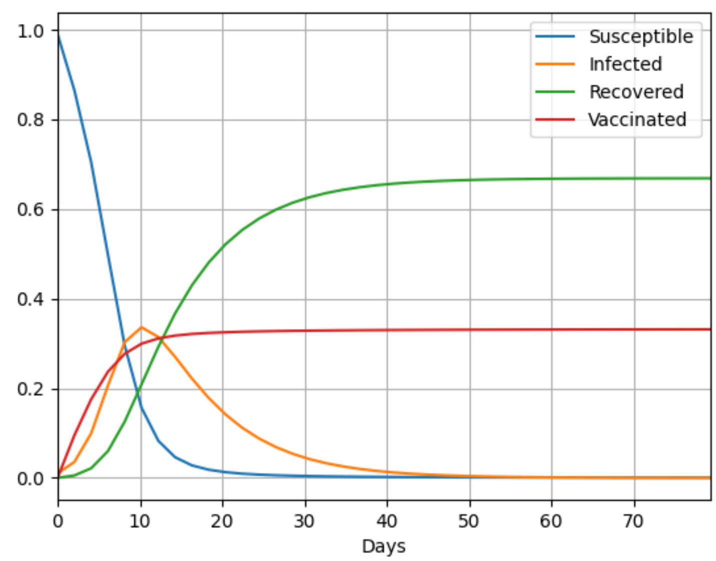
Simulation with τ = 0.4, κ = 4:




```
The stopping condition occurs at:
[44.80682568]
```

This stopping condition occurred a lot faster than Q1.2 2nd scenario's stopping condition. This may have occurred due to the low infection rate, which meant that an insanely large number of susceptible people had time to vaccinate themselves and not become infected.

Simulation with τ = 0.8, κ = 8:




```
The stopping condition occurs at:
[79.34406218]
```

As in Q1.4's first scenario, the high infection rate most likely caused the stopping condition to not decrease as much as in the second scenario.