Shubhan Singh

SE-Comps B/Batch C

2022300118

DAA Experiment 7: Backtracking (Graph Coloring)

Aim – To implement an algorithm to generate graph colourings using backtracking

Problem statement:

Find all possible colourings of the given graph using the minimum number of colours i.e. chromatic number number of colours

Pseudocode :

Algorithm Graph-Coloring

Input: A graph represented by an adjacency matrix graph[1..v][1..v]

Output: A coloring of the vertices of the graph

1. chromatic = Find-Ch-Number(graph, v)
2. Initialize an array colors[1..v] with all elements as 0
3. Calculate(graph, v, colors, 0, chromatic)

Function Calculate(graph, v, colors, start, chromatic)

Input: The adjacency matrix graph, the number of vertices v, the colors array, the start index, and the chromatic number

Output: None

1. if start == v
2.   Print-Coloring(colors, v)
3. for color = 1 to chromatic

4. if Is-Safe(graph, v, colors, start, color)

5. colors[start] = color

6. Calculate(graph, v, colors, start+1, chromatic)

7. colors[start] = 0

Function Is-Safe(graph, v, colors, start, color)

Input: The adjacency matrix graph, the number of vertices v, the colors array, the start index, and the color

Output: True if it is safe to assign the color to the start vertex, False otherwise

1. for i = 1 to v

2. if graph[start][i] and color == colors[i]

3. return False

4. return True

The algorithm assigns colors to vertices of a graph such that no two adjacent vertices share the same color. The time complexity of this algorithm is O(v^chromatic), where v is the number of vertices in the graph and chromatic is the chromatic number of the graph.

Source code(C language):

```
#include<stdio.h>
#include<stdlib.h>

int count=0;

void printcoloring(int* colors,int n){
    printf("Coloring %d :\n",count);
    for(int i=0;i<n;i++){
        printf("%c -> %d\t",('a'+i),colors[i]);
        if(i%2==1){
            printf("\n");
        }
    }
    printf("\n");
}
```

```c
int find(int* arr, int n, int k){
    for(int i=0;i<n;i++){
        if(arr[i]==k){
            return 1;
        }
    }
    return 0;
}

int mex1(int* arr,int n){
    for(int i=1;i<=n;i++){
        if(find(arr,n,i)==0){
            return i;
        }
    }
}

int findchno(int** graph, int v){
    int colors[v];
    for(int i=0;i<v;i++){
        colors[i]=0;
    }
    colors[0]=1;
    int rowcolor[v];
    for(int i=1;i<v;i++){
        for(int j=0;j<v;j++){
            if(graph[i][j]==1){
                rowcolor[j]=colors[j];
            }
            else{
                rowcolor[j]=0;
            }
        }
        colors[i]=mex1(rowcolor,v);
    }
    int max=-1;
    for(int i=0;i<v;i++){
        if(colors[i]>max){
            max=colors[i];
        }
    }
    return max;
```

```
}

int chk_valid(int* row, int n, int* colors, int c){
    for(int i=0;i<n;i++){
        if(row[i]){
            if(colors[i]==c){
                return 0;
            }
        }
    }
    return 1;
}

void calculate(int** graph, int n, int* colors,int i, int
cno){
    for(int j=0;j<cno;j++){
        if(chk_valid(graph[i],n,colors,j+1)){
            colors[i]=j+1;
            if(i+1<n)
                calculate(graph,n,colors,i+1,cno);
            else{
                count++;
                printcoloring(colors,n);
            }
        }
    }
    colors[i]=0;
}

int main(){
    int V,E;
    char v1,v2;
    printf("Enter number of vertices: ");
    scanf("%d", &V);
    printf("Enter number of edges: ");
    scanf("%d", &E);
    int **graph = (int **)malloc(V * sizeof(int *));
    for(int i=0;i<V;i++){
        graph[i] = (int *)calloc(V,sizeof(int));
    }
```

```c
    printf("Enter the edges as vortex1 vortex2, where names
of vertices start from 'a': \n");
    for(int i=0;i<E;i++){
        while((getchar()) != '\n');
        scanf("%c %c", &v1, &v2);
        int v1i=v1-'a';
        int v2i=v2-'a';
        graph[v1i][v2i] = 1;
        graph[v2i][v1i] = 1;
    }


    int chromatic=findchno(graph,V);
    printf("\nThe chromatic number of the graph
is : %d\n",chromatic);
    printf("All the possible colorings of the graph using %d
colors are :\n",chromatic);
    int col[V];
    for(int i=0;i<V;i++){
        col[i]=0;
    }
    calculate(graph,V,col,0,chromatic);


    for(int i=0;i<V;i++){
        free(graph[i]);
    }
    free(graph);
return 0;
}
```

Output:

```
C:\Users\shubh\OneDrive - Bharatiya Vidya Bhavans Sardar Patel Institute Of Technology\DAA>.\graph_colorings
Enter number of vertices: 4
Enter number of edges: 4
Enter the edges as vortex1 vortex2, where names of vertices start from 'a':
a b
b c
c d
d a

The chromatic number of the graph is : 2
All the possible colorings of the graph using 2 colors are :
Coloring 1 :
a → 1  b → 2
c → 1  d → 2

Coloring 2 :
a → 2  b → 1
c → 2  d → 1

C:\Users\shubh\OneDrive - Bharatiya Vidya Bhavans Sardar Patel Institute Of Technology\DAA>
```

```
C:\Users\shubh\OneDrive - Bharatiya Vidya Bhavans Sardar Patel Institute Of Technology\DAA>.\graph_colorings
Enter number of vertices: 4
Enter number of edges: 6
Enter the edges as vortex1 vortex2, where names of vertices start from 'a':
a b
b c
c d
d a
a c
b d

The chromatic number of the graph is : 4
All the possible colorings of the graph using 4 colors are :
Coloring 1 :
a → 1  b → 2
c → 3  d → 4

Coloring 2 :
a → 1  b → 2
c → 4  d → 3

Coloring 3 :
a → 1  b → 3
c → 2  d → 4

Coloring 4 :
a → 1  b → 3
c → 4  d → 2

Coloring 5 :
a → 1  b → 4
c → 2  d → 3

Coloring 6 :
a → 1  b → 4
c → 3  d → 2
```

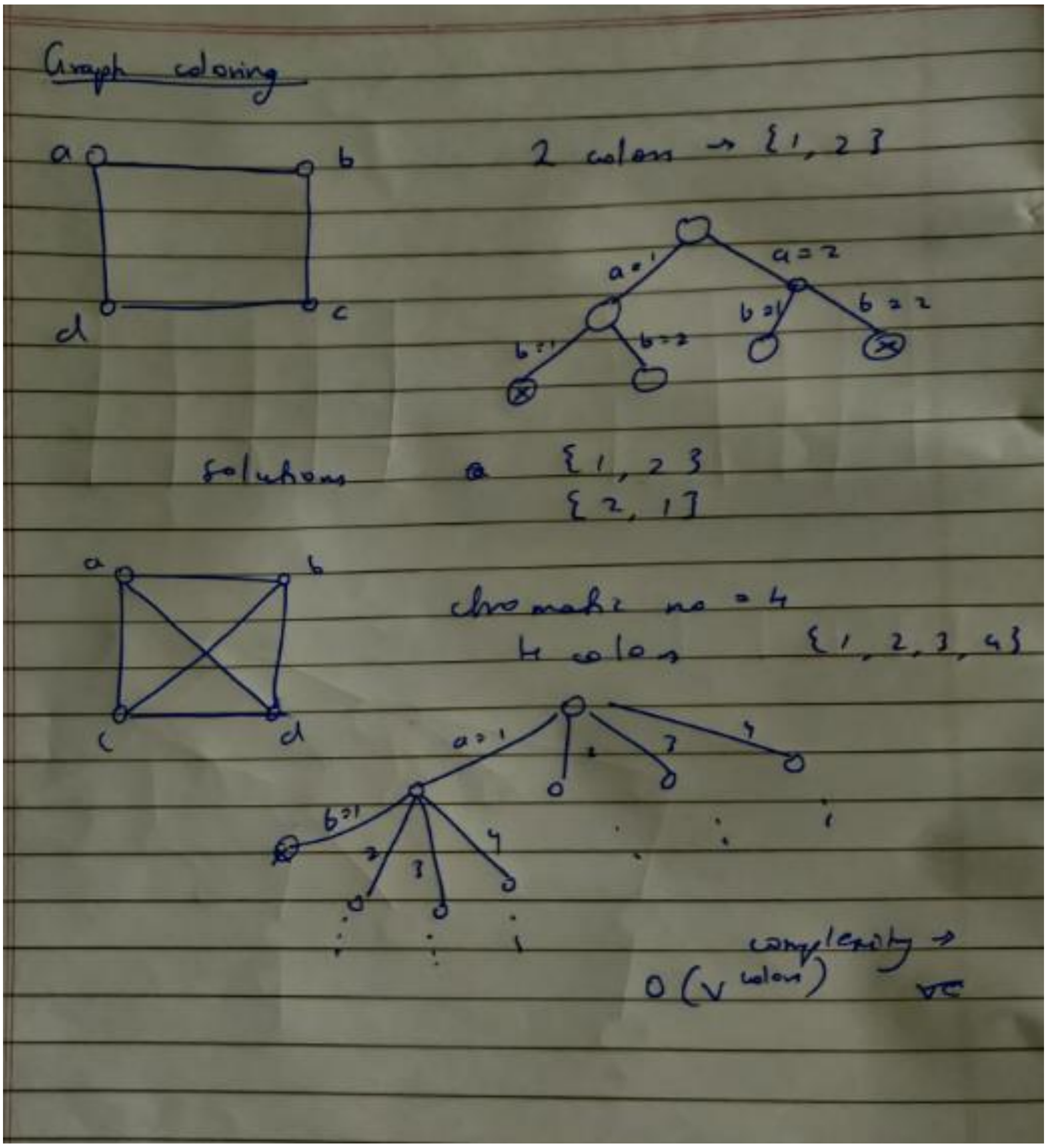
```
Coloring 17 :
a → 3  b → 4
c → 1  d → 2

Coloring 18 :
a → 3  b → 4
c → 2  d → 1

Coloring 19 :
a → 4  b → 1
c → 2  d → 3

Coloring 20 :
a → 4  b → 1
c → 3  d → 2

Coloring 21 :
a → 4  b → 2
c → 1  d → 3

Coloring 22 :
a → 4  b → 2
c → 3  d → 1

Coloring 23 :
a → 4  b → 3
c → 1  d → 2

Coloring 24 :
a → 4  b → 3
c → 2  d → 1


C:\Users\shubh\OneDrive - Bharatiya Vidya Bhavans Sardar Patel Institute Of Technology\DAA>
```

(All 24 colorings printed)

Conclusion:

- We wrote a program to find all the minimal colorings of a graph i.e. colorings using the minimum possible number of colors.
- The time complexity of this algorithm is O(v^no.of colors) where v is the number of vertices.
- We use backtracking to solve this problem, whenever we find that a certain color is not valid for the vertex in question, we backtrack and choose the next color.
- We find the chromatic number of the graph separately in O(v^2) time.

Rough Working :



Theory:

1. **Graph Coloring**: Graph coloring is a method of assigning colors to the vertices of a graph in such a way that no two adjacent vertices share the same color. This is a classic computer science problem and has many practical applications, such as in scheduling algorithms, register allocation, and solving puzzles like Sudoku.

2. **Chromatic Number**: The chromatic number of a graph is the smallest number of colors needed to color a graph. It is a fundamental concept in graph theory. For example, a graph is bipartite if and only if it is 2-colorable (i.e., its chromatic number is 2).

3. **Backtracking Algorithms for Graph Coloring**: Backtracking algorithms can be used to solve the graph coloring problem. The idea is to assign colors one by one to different vertices, starting from the first vertex. Before assigning a color, we check for safety by considering already assigned colors to the adjacent vertices. If we find a color assignment which is safe, we mark the color assignment as part of the solution. If we do not find a color due to clashes, we backtrack and return false.

4. **Efficiency**: Using backtracking, the graph coloring problem can be solved more efficiently than a naive approach that tries all possible combinations of colors. However, it is important to note that graph coloring remains a computationally challenging problem. In fact, determining if a graph can be colored with k colors for a given k is a well-known NP-complete problem.

5. **Applications**: Graph coloring algorithms have a wide range of applications in computer science and operations research, including register allocation in compilers, mobile radio frequency assignment, task scheduling, and many others.