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### **DISPLACED INFO:**

The general first-order differential equation is represented as:

$$\frac{\partial y(t)}{\partial t} = f(t, y), \quad t \in [a, b]$$

The general first-order differential equation is represented as:

$$y_t(t) = A + (t - a)z(t)$$

The error used to train the KAN model can be defined as such:

$$E(w) = \frac{1}{2} \left( \frac{\partial y_t(t)}{\partial t} - f(t, y) \right)^2$$

Finally, the new weights can be calculated using the error defined previously:

$$w_k = w_{k-1} - \eta \frac{\partial E(w)}{\partial w_k}$$

# Abstract WRITE THIS WHEN THE WORK IS DONE 1. Introduction

Ordinary differential equations (ODEs) serve as fundamental instruments in the mathematical modeling and analytical study of numerous scientific and engineering systems. They naturally emerge in diverse applications, including but not limited to fluid dynamics, chemical reaction kinetics, population dynamics, and structural analysis [1-5, 13]. The inherent complexity of ODEs presents significant challenges in their solution, as numerous cases do not yield closed-form solutions, necessitating the utilization of numerical or approximation techniques. Established numerical methodologies, including the Runge-Kutta technique, finite difference approach, and shooting method, have historically been employed to tackle these challenges. However, their drawbacks—such as considerable computational demands and the inability to produce closed-form





solutions—have motivated the investigation of alternative strategies for ODE resolution [1-3, 5, 12, 13].

The introduction of Artificial Neural Networks (ANNs) has facilitated novel methodologies for the numerical resolution of ODEs by recontextualizing the problem as an optimization framework. Preliminary investigations have demonstrated the efficacy of multilayer perceptrons (MLPs) in approximating solutions to both initial value problems (IVPs) and boundary value problems (BVPs).

These methods based on neural networks are much more efficient than the classical numerical techniques. In particular, ANNs are capable of formulating analytic solutions which eliminate the necessity for performing interpolation over discretized computational intervals, hence more flexibility in solving IVPs and BVPs [1, 3, 6, 7, 11, 13 -17]. On the other hand, the first generation of models based on ANNs had several challenges among which were the pronounced vulnerability to convergence at local minima and suboptimal rates of convergence [1].

As a solution to the shortcomings of the ANNs mentioned, new generation advanced architectures such as Radial Basis Function Neural Networks(RBFNNs) [2, 3, 12] and Wavelet Neural Networks (WNNs) [1] have emerged. These approaches have been recorded to have shorter convergence times and a higher accuracy compared to traditional techniques when applied to complex expressions of differential equations. Furthermore, WNNs have attracted considerable interest because their activation functions are concentrated so that the size of the network can be kept small which allows faster training while preserving the ability of any approximation that is said to be achieved by neural networks[1]. Furthermore, the implementation of sophisticated training methodologies, such as Extreme Learning Machines (ELM) and metaheuristic optimization techniques, including Particle Swarm Optimization (PSO), has substantially enhanced both the efficiency and accuracy of these neural network models [1, 2, 4, 5, 14].

In response to this assertion, the Kolmogorov-Arnold Network (KAN) architecture evolves a novel architecture which is robust and function approximation, which shows potential for solving ODEs. ODEs are appreciably accounted for in this





architecture. The KAN model is based on the Kolmogorov-Arnold representation theorem which states that every multivariate continuous function can be expressed as a finite sum of univariate functions [6-8, 13, 15-18]. This inbuilt KAN Thus, the KAN proves to be especially useful in terms of complex mathematical models that are developed using ODEs [6, 7, 13, 16-18]. This paper attempts to use KAN's systematic functioning on KAN as a function decomposition architecture on neural networks to overcome the shortcomings that are posed by the current neural network architectures on higher order differential equations.

The primary aim of the study is to make use of the KAN architecture for the approximation of the solutions of first order ODEs. This investigation is a major breakthrough in the fusion of sophisticated machine learning techniques with computational mathematics. The ANN-based approach is contrasted to the KAN structure, which is able to build a process-specific problem space and in this way improve the approximation of the results [17, 18]. It is also different from any other design in that the network can become more precise with a decrease in the number of parameters, which in turn makes it relatively faster while preserving the accuracy [6, 7,13,15,17,18].

The rationale behind the implementation of KAN in this framework is the existed capabilities to handle the critical ODEs solution. Mostly, first order ODEs are noted to be not serious with the many complicated boundary conditions. However, they may now and then show some anomalously typical of nonlinear dynamics which affects the conventional numeric technique [7, 8, 13, 17, 18]. The unique modification of KAN, characteristic of the agility of the system to the situation, along with an expressive mode of representation of the mentioned challenges, definitely leads to very good results in their solution. Furthermore, KAN can be easily upgraded by including advanced optimization algorithms [18], hence it is enhanced in solving ODEs with its robustness.

Recent investigations emphasize the efficacy of neural network architectures in the resolution of differential equations. Specifically, WNNs, when enhanced through sophisticated optimization techniques such as the butterfly optimization algorithm, exhibit superior capabilities in approximating solutions to ODEs. [1]. Moreover, RBFNNs trained via extreme learning methodologies demonstrate the high rates of convergence and high accuracy regarding fractional differential





equations [2, 3, 12]. These discoveries demonstrate the resurgence and importance of neural network models in computational mathematics.

Notwithstanding the advancements made in this field, significant deficiencies persist in the literature concerning the application of KAN to ODEs. Although the Kolmogorov-Arnold theorem (KAT) offers a theoretical framework for function approximation [15 - 18], its practical deployment for the resolution of ODEs remains insufficiently investigated. This research endeavors to fill this lacuna by executing a thorough assessment of KAN's effectiveness in solving both first- and second-order ODEs. Through methodical experimentation, this study aims to validate KAN as a robust and efficient methodology for function approximation specifically within the context of differential equations.

The implications of this research transcend the direct utilization of KAN in the context of ODEs. By establishing its efficacy as a versatile function approximator, this investigation enriches the field of computational mathematics and neural network-based modeling. The findings derived from this study are anticipated to guide the advancement of next-generation computational methodologies adept at solving intricate scientific and engineering challenges, consequently, the KAN constitutes a significant progression in the application of machine learning techniques for the resolution of differential equations. Its distinctive architectural framework and theoretical foundations establish it as a formidable alternative to prevailing ANN methodologies for function approximation. This research aims to enhance the current capabilities of neural network-based approaches in addressing first- and second-order ODEs by leveraging KAN, thereby facilitating advancements in computational mathematics and related fields.

Write the organization of the paper here when the content structure is clear.





# 2. KAN Model (General info of the architecture) We can use the initial paper here, or use the existing references for citing

The KAN model is optimally configured for function approximation tasks [15 - 18], including the resolution of ODEs, owing to its basis in the KAT. This theorem asserts that any continuous multivariate function can be expressed as a finite summation of univariate functions subjected to linear operations, thereby facilitating the ability of a KAN to approximate intricate functions with reduced network depth [6, 7, 13, 15 - 18]. By utilizing this property, KANs inherently diminish the computational complexity associated with multivariate functions while preserving accuracy, [6 - 8, 13, 17, 18] a critical factor for accurately modeling the complex dynamics of ODEs. KANs are architected to optimize the advantages of the KAT by structuring layers such that univariate basis functions are hierarchically composed [15, 17, 18], resulting in outputs that effectively approximate multivariate functions. In contrast to conventional MLPs, which depend on universal approximation via dense layers and nonlinear activation functions, KANs leverage the structural organization offered by KAT to attain efficient and precise function representations [6 - 8, 13, 15 - 18]. The hidden layers of the network typically utilize Gaussian radial basis functions (RBFs) as activation functions, selected for their smoothness properties and capacity for spatial localization of approximations [13, 17, 18]. These RBFs facilitate a concentration of response from each hidden layer neuron to distinct regions of the input space, which is essential for the accurate resolution of ODEs where localized dynamics predominantly influence system behavior. In contrast to WNNs, which employ wavelet transformations to achieve a compact topology and facilitate efficient training, KANs present an alternative framework founded on the theoretical assurances provided by the KAT [17, 18]. While both WNNs and RBF networks demonstrate proficiency in distinct application domains [1, 2, 3, 12], the hierarchical univariate decomposition characteristic of KANs is inherently more compatible with the requirements associated with ODE approximation [13]. This congruence enables KANs to deliver accurate gradient evaluations, a feature that is particularly beneficial for integration with differentiable ODE solvers. These





solvers exploit the structured outputs of KANs to simulate dynamical systems and extract latent physical phenomena while incurring minimal computational overhead.

A notable advantage of utilizing KANs is their ability to process high-dimensional input data effectively. The application of the superposition principle within KAT mitigates the complexity associated with high dimensionality by decomposing intricate functions into simpler, constituent components. This decomposition enhances model interpretability and streamlines the training process, as the optimization burden is reduced due to a smaller number of parameters relative to fully interconnected neural networks [6 - 8, 13, 15 - 18]. Additionally, the modular architecture of KANs supports their integration into hybrid systems [18], including Neural ODEs, where KANs function as gradient evaluators to iteratively optimize solutions to ODEs.

KANs utilize univariate function composition, which results in high convergence efficiency [6, 7, 8, 13, 17, 18]. The univariate basis functions are designed to capture distinct characteristics of the input, facilitating expedited learning and mitigating overfitting [17, 18]. This attribute is especially critical in addressing ODEs, where the solution landscape may present abrupt gradients or localized features. By integrating domain-specific insights into the selection of basis functions, such as Gaussian RBFs or B-splines, KANs demonstrate enhanced efficacy in approximating solutions to complex differential equations relative to alternative neural network frameworks, therefore, the KAN model serves as a powerful tool for tasks such as solving ODEs. Its theoretical foundation, coupled with its efficient architectural design and adaptability to high-dimensional parameter spaces, positions it as a superior alternative to conventional neural network paradigms. By decomposing multivariate functions into their univariate components, KANs enhance computational efficiency and exhibit strong approximation properties, thereby aligning optimally with the requirements of contemporary ODE-solving techniques.





# 3. KAN Model for differential equations (Later we can explain more in case we choose any specific subtype of KAN – FROM REFERENCES: 7,8, AND 9)

We can skip this part and immediately proceed with the numerical examples.

# 4. Numerical Examples

In this section, some example equations are evaluated by the KAN model. The results are then compared to other similar approaches.

#### 4.1.1. **Example 1**

Consider the following first-order differential equation:

$$\frac{\partial y(t)}{\partial dt} + \left(t + \frac{1 + 3t^2}{1 + t + t^3}\right) y(t) = 2t + t^3 + t^2 \cdot \frac{1 + 3t^2}{1 + t + t^3}, \quad t \in [0, 1]$$

With the initial condition:

$$y(0) = 1$$

Compared results to other references:

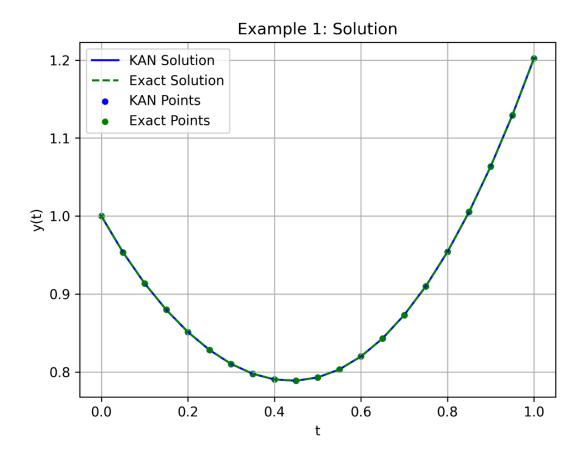
X	Exact solution	Euler	RungeKutta	RBFNet in Rizaner et al. 2018 (n=9)	RBFNN solution (n=9)	KAN
0.00	1.0000	1.000	1.0000	1.0000	1.0000	1.0000
0.05	0.9536	0.9500	0.9536	0.9536	0.9536	0.9535
0.10	0.9137	0.9072	0.9138	0.9137	0.9137	0.9137
0.15	0.8798	0.8707	0.8799	0.8798	0.8798	0.8798
0.20	0.8514	0.8401	0.8515	0.8514	0.8514	0.8514
0.25	0.8283	0.8150	0.8283	0.8283	0.8283	0.8283
0.30	0.8104	0.7953	0.8105	0.8104	0.8104	0.8104
0.35	0.7978	0.7810	0.7979	0.7978	0.7978	0.7977
0.40	0.7905	0.7721	0.7907	0.7905	0.7905	0.7905
0.45	0.7889	0.7689	0.7890	0.7889	0.7889	0.7888
0.50	0.7931	0.7717	0.7932	0.7930	0.7931	0.7930





x	Exact solution	Euler	RungeKutta	RBFNet in Rizaner et al. 2018 (n=9)	RBFNN solution (n=9)	KAN
0.55	0.8033	0.7805	0.8035	0.8033	0.8033	0.8033
0.60	0.8200	0.7958	0.8201	0.8199	0.8200	0.8199
0.65	0.8431	0.8178	0.8433	0.8431	0.8431	0.8431
0.70	0.8731	0.8467	0.8733	0.8731	0.8731	0.8731
0.75	0.9101	0.8826	0.9102	0.9100	0.9101	0.9100
0.80	0.9541	0.9258	0.9542	0.9540	0.9541	0.9540
0.85	1.0053	0.9763	1.0054	1.0052	1.0053	1.0052
0.90	1.0637	1.0342	1.0638	1.0637	1.0637	1.0637
0.95	1.1293	1.0995	1.1294	1.1293	1.1293	1.1293
1.00	1.2022	1.1721	1.2022	1.2021	1.2022	1.2021
Error	• 0	4.60e-10	1.24e-08	6.80e-10	7.56e-14	4.66e-11

Average MSE 4.6673 x 10<sup>-11</sup>, Average MAE 1.7985 x 10<sup>-6</sup>





#### 4.1.2. **Example 2**

Consider another first-order linear differential equation:

$$\frac{\partial y(t)}{\partial t} + 2y(t) = \cos(4t), \quad t \in [0,3]$$

With the initial condition:

$$y(0) = 3$$

Compared results to other references:

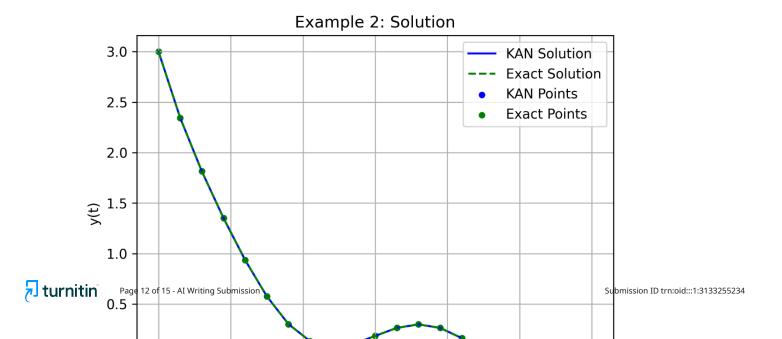
X	WNNIBOA	WNNBOA	WNNPSO	WNNPSOA	WNNMBP	WNNDEV	KAN
0.0000	0	0	0	0	0	0	0
0.1500	1.94e-07	3.83e-01	4.26e-03	3.96e-03	6.06e-03	2.06e-01	1.02e-04
0.3000	1.14e-07	4.91e-01	8.61e-03	8.60e-03	1.37e-02	2.78e-01	7.78e-05
0.4500	9.81e-08	4.55e-01	4.31e-03	4.78e-03	8.48e-03	9.93e-02	6.29e-05
0.6000	6.96e-08	3.72e-01	1.37e-03	7.14e-04	7.51e-04	3.56e-02	4.82e-05
0.7500	4.66e-08	3.07e-01	2.33e-03	2.00e-03	4.37e-03	3.90e-02	4.25e-05
0.9000	3.22e-08	2.89e-01	8.68e-04	4.93e-04	1.09e-03	2.33e-01	3.53e-05
1.0500	2.35e-08	3.18e-01	3.74e-03	2.90e-03	4.05e-03	3.49e-01	9.93e-06
1.2000	2.71e-08	3.74e-01	3.21e-03	2.65e-03	5.86e-03	2.80e-01	8.43e-06
1.3500	3.27e-08	4.27e-01	1.09e-04	4.75e-04	3.18e-03	8.31e-02	6.32e-05
1.5000	3.05e-08	4.48e-01	2.34e-03	1.22e-03	1.17e-03	8.79e-02	3.67e-05
1.6500	2.46e-08	4.20e-01	1.94e-03	1.06e-03	3.50e-03	1.25e-01	6.43e-06
1.8000	1.66e-08	3.48e-01	4.95e-04	2.19e-04	2.37e-03	4.67e-02	8.22e-06
1.9500	5.45e-09	2.54e-01	2.22e-03	1.01e-03	7.16e-04	3.59e-02	4.58e-06
2.1000	4.24e-10	1.74e-01	1.46e-03	6.39e-04	2.95e-03	2.95e-02	4.70e-06
2.2500	1.37e-09	1.42e-01	7.81e-04	1.47e-04	2.55e-03	5.14e-02	1.03e-05
2.4000	1.17e-08	1.87e-01	1.78e-03	3.94e-04	2.58e-05	1.04e-01	1.67e-05
2.5500	1.30e-08	3.23e-01	2.82e-04	1.18e-04	2.30e-03	5.27e-02	2.37e-05
2.7000	2.57e-09	5.54e-01	1.53e-03	8.06e-06	2.05e-03	4.40e-02	6.30e-06
2.8500	2.61e-08	8.80e-01	4.11e-04	3.69e-05	9.74e-04	5.01e-02	1.83e-06
3.0000	9.59e-08	1.31e+00	1.45e-03	1.24e-04	2.23e-03	2.43e-03	9.76e-05





X	Exact Solution	RBFNet Rizaner (n=21)	RBFNN (n=21)	RBFNN (n=90)	KAN
0.00	3.0000	3.0000	3.0000	3.0000	3.000
0.15	2.3438	2.2435	2.3438	2.3438	2.3437
0.30	1.8142	1.8138	1.8142	1.8142	1.8141
0.45	1.3511	1.3510	1.3511	1.3511	1.3511
0.60	0.9348	0.9344	0.9348	0.9348	0.9347
0.75	0.5763	0.5752	0.5763	0.5763	0.5762
0.90	0.3012	0.2998	0.3012	0.3012	0.3011
1.05	0.1318	0.1308	0.1318	0.1318	0.1317
1.20	0.0726	0.0720	0.0726	0.0726	0.0725
1.35	0.1038	0.1030	0.1038	0.1038	0.1038
1.50	0.1845	0.1834	0.1845	0.1845	0.1844
1.65	0.2643	0.2632	0.2643	0.2643	0.2642
1.80	0.2988	0.2982	0.2988	0.2988	0.2988
1.95	0.2638	0.2637	0.2638	0.2638	0.2638
2.10	0.1625	0.1622	0.1625	0.1625	0.1624
2.25	0.0235	0.02227	0.0235	0.0235	0.0235
2.40	-0.1095	-0.1107	-0.1095	-0.1095	-0.1094
2.55	-0.1937	-0.1952	-0.1937	-0.1937	-0.1936
2.70	-0.2025	-0.2043	-0.2025	-0.2025	-0.2025
2.85	-0.1348	-0.1373	-0.1348	-0.1348	-0.1348
3.00	-0.0157	-0.0186	-0.0157	-0.0157	-0.0156
Erro	r 0	1.44e-06	5.85e-12	9.95e-13	2.10e-10

Average MSE 2.1085 x 10<sup>-10</sup>, Average MAE 3.3426 x 10<sup>-6</sup>





### 4.1.3. Example 3

Consider another first-order linear differential equation:

$$\frac{\partial y(t)}{\partial t} = y(t) - t^2 + 1, \quad t \in [0,2]$$

With the initial condition:

$$y(0) = 0.5$$

Compared results to other references:

X	WNNIBOA	WNNBOA	WNNPSO	WNNPSOA	WNNMBP	WNNDEV	KAN
0.0000	0	0	0	0	0	0	0
0.2000	3.72e-09	1.54e-01	5.05e-05	1.64e-04	4.73e-04	1.01e-02	7.96e-05
0.4000	2.29e-09	2.94e-01	9.97e-05	2.15e-04	7.12e-04	3.50e-03	6.03e-05
0.6000	6.12e-09	4.22e-01	6.67e-06	9.45e-05	4.75e-04	7.78e-03	4.17e-05
0.8000	9.51e-09	5.28e-01	1.88e-04	2.53e-04	4.12e-04	1.36e-02	1.74e-05
1.0000	9.65e-09	6.06e-01	6.65e-05	6.29e-05	7.53e-04	8.46e-03	3.57e-06
1.2000	1.42e-08	6.68e-01	1.54e-04	4.41e-04	1.18e-03	8.34e-04	9.05e-06
1.4000	1.23e-08	7.37e-01	5.09e-05	2.93e-04	1.28e-03	5.92e-03	2.47e-05
1.6000	3.07e-08	8.32e-01	2.23e-04	1.78e-04	1.17e-03	1.06e-02	2.24e-05
1.8000	1.52e-08	9.65e-01	1.41e-04	1.18e-06	1.57e-03	1.20e-02	4.67e-05
2.0000	3.51e-07	1.14e+00	3.48e-05	6.41e-04	2.47e-03	1.51e-02	2.07e-04

Average MSE  $4.8837 \times 10^{-10}$ , Average MAE  $4.8980 \times 10^{-6}$ 

Example 3: Solution

2.5 - Exact Solution

Exact Points

Exact Points

1.5 - Loo

1.0 - Loo

1.0 - Loo

1.0 - Loo

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# 5. Conclusion WRITE THIS AND ABSTRACT IN THE END

# Acknowledgements (if there are any)

# References (APA for now unless we decide to change it.)

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