Problem 1

8.6 Which of the following are valid (necessarily true) sentences?

a. $(\exists x\ x = x) \Rightarrow (\forall y\ \exists z\ y = z)$

b. $\forall x\ P(x) \lor \neg P(x)$

c. $\forall x\ Smart(x) \lor (x = x)$

a. Valid.

b. Valid. Either $P(x)$ is true or it is not.

c. Valid. The right hand side has $x = x$ which will always be true, thus the sentence will always be true.

Problem 2

8.10 Consider a vocabulary with the following symbols:

- $Occupation(p, o)$: Predicate. Person p has occupation o.
- $Customer(p1, p2$: Predicate. Person $p1$ is a customer of person $p2$.
- $Boss(p1, p2)$: Predicate. Person $p1$ is a boss of person $p2$.
- $Doctor, Surgeon, Lawyer, Actor$: Constants denoting occupations.
- $Emily, Joe$: Constants denoting people.

Use these symbols to write the following assertion in first-order logic:

a. Emily is either a surgeon or a lawyer.

b. Joe is an actor, but he also holds another job.

c. All surgeons are doctors.

d. Joe does not have a lawyer (i.e., is not a customer of any lawyer).

e. Emily has a boss who is a lawyer.

f. There exists a lawyer all of whose customers are doctors.

g. Every surgeon has a lawyer.

a. $Occupation(Emily, Surgeon) \lor Occupation(Emily, Lawyer)$

b. $Occupation(John, Actor) \land \exists o \neq Actor \land Occupation(John, o)$

c. $\forall p\ Occupation(p, Surgeon) \Rightarrow Occupation(p, Doctor)$

d. $\neg\exists p\ Customer(John, p) \land Occupation(p, Lawyer)$

e. $\exists p\ Boss(p, Emily) \land Occupation(p, Lawyer)$

f. $\exists p\ Occupation(p, Lawyer) \land \forall q\ Customer(q, p) \Rightarrow Occupation(q, Doctor)$

g. $\forall p\ Occupation(p, Surgeon) \Rightarrow \exists q\ Occupation(q, Lawyer) \land Customer(p, q)$

Problem 3

8.23 For each of the following sentence in English, decide if the accompanying first-order logic sentence is a good translation. If not, explain why not and correct it. (Some sentences have more than one error!)

a. No two people have the same social security number.
$\neg\exists x, y, n\ Person(x) \land Person(y) \Rightarrow [HasSS\#(x, n) \land HasSS\#(y, n)]$

b. John's social security number is the same as Mary's.
$\exists n\ HasSS\#(John, n) \land HasSS\#(Mary, x)$

c. Everyone's social security number has nine digits.
$\forall x, n\ Person(x) \Rightarrow [HasSS\#(x, n) \land Digits(n, 9)]$

d. Rewrite each of the above (uncorrected) sentence using a function symbol $SS\#$ instead of the predicate $HasSS\#$.

a. Incorrect.
$\neg\exists x, y, n\ Person(x) \land Person(y) \Rightarrow [HasSS\#(x, n) \land HasSS\#(y, n)]$

b. Correct.

c. Incorrect.
$\forall x, n\ Person(x) \land HasSS\#(x, n) \Rightarrow Digits(n, 9)$

d. $\neg\exists x, y\ Person(x) \land Person(y) \Rightarrow [SS\#(x) = SS\#(y)]$
$SS\#(John) = SS\#(Mary)$
$\forall x\ Person(x) \Rightarrow Digits(SS\#(x), 9)$

Problem 4

9.4 For each pair of atomic sentences, give the most general unifier if it exists:

a. $P(A, B, B), P(x, y, z)$

b. $Q(y, G(A, B)), Q(G(x, x), y)$

c. $Older(Father(y), y), Older(Father(x), John)$

d. $Knows(Father(y), y), Knows(x, x)$

a. x is A, y is B, and z is B.

b. Not possible.

c. x and y are John

d. Not possible.

Problem 5

9.6 Write down logical representations for the following sentences, suitable for use with Generalized Modus Ponens:

a. Horses, cows, and pigs are mammals.

b. An offspring of a horse is a horse.

c. Bluebeard is a horse.

d. Bluebeard is Charlie's parent.

e. Offspring and parent are inverse relations.

f. Every mammal has a parent.

a. $Horse(x) \Rightarrow Mammal(x)$
$Cow(x) \Rightarrow Mammal(s)$
$Pig(x) \Rightarrow Mammal(s)$

b. $Offspring(a, b) \wedge Horse(b) \Rightarrow Horse(a)$

c. $Horse(Bluebeard)$

d. $Parent(Bluebeard, Charlie)$

e. $Offspring(a, b) \Leftrightarrow Parent(b, a)$