Problem 1

Solve Sipser exercises (2nd edition) 1.7b, 1.7e, 1.16, 1.19, 1.21.

1.7b & 1.7e Give state diagrams of NFAs with the specified number of states recognizing each of the following languages. The alphabet is $\{0, 1\}$.

b. $\{w \mid \textit{contains the substring } 0101, \textit{ i.e., } w = x0101y \textit{ for some } x \textit{ and } y\}$ with fives states.

e. The language $0^*1^*0^+$ with three states.

1.16 Use the construction given in Theroem 1.39 to convert the following two nondeterministic finite automata to equivalent deterministic finite automata. Include diagrams here.
1.19 Use the procedure described in Lemma 1.55 to convert the following regular expressions to nondeterministic finite automata.

a. $((0 \cup 1)^*000(0 \cup 1)^*)$

b. $(((00)^*(11)) \cup 01)^*$

c. $\emptyset^*$

1.21 Use the procedure described in Lemma 1.60 to convert the following finite automata to regular expressions.
Include diagrams here.

Problem 2

Find an equivalent NFA for the following regular expression:

$R = (0(10)^* \cup (1(0 \cup 1)^*)$

Problem 3

For the following languages, give a corresponding regular expression. The languages are defined over the alphabet $\Sigma = \{a, b\}$

a. A_1: The set of all strings that contain ”a” as a substring

b. A_2: The set of all strings that do not contain ”bb” as a substring

c. A_3: The set of all string whose length is exactly three.

Problem 4

Using the pumping lemma, prove that:

$A_4 = \{w \in \Sigma^* |\ w\ contains\ more\ a's\ than\ b's\}$
with $\Sigma = \{a, b\}$, is not a regular language.

Problem 5

Are the following languages regular? Prove your answers.

- $C_1 = \{a^p b^q a^{p=q} \in \Sigma^* | \; p \geq 0, q \geq 0\}$
- $C_2 = \{a^{\binom{n}{s}} \in \Sigma^* | \; n \geq 2\}$