Problem 1

Solve Sipser exercises 0.3, 0.4, 0.5, and problem 0.10.

0.3

a. No.

b. Yes.

c. x, y, z

d. x, y

e. $xx, xy, yx, yy, zx, zy, xz, yz$

f. x, y, x, y

0.4 The cartesian product has $a * b$ elements. Since the cartesian product is b ordered pairs for each of the m elements.
0.5 There are 2^c elements in the power set of C. Since the subsets in the power set either include an element of C or they do not then it holds that there are $2^{|}C|$ elements in the power set.
0.10 In the step where we divide both sides by $(a - b)$ if $a = b$ then $(a - b) = 0$. This leads to a divide by zero error.

Problem 2

For a DFA, $M = (Q, \sum, \delta, q_0, F)$ in which the set of states is $Q = \{q_1, q_2, q_3, q_4, q_5\}$, $\sum = \{a, b\}$, $q_0 = q_1$, $F = \{q_2, q_3, q_4, q_5\}$, and δ is specified by the table:

δ	q_1	q_2	q_3	q_4	q_5
a	q_1	q_3	q_5	q_2	q_4
b	q_2	q_4	q_1	q_3	q_5

Do the following:

(a) Draw the state diagram of the DFA.

(b) For the strings below, give the corresponding computation of the automaton and say whether it accepts or rejects them. The definition of computation is given in page 40.

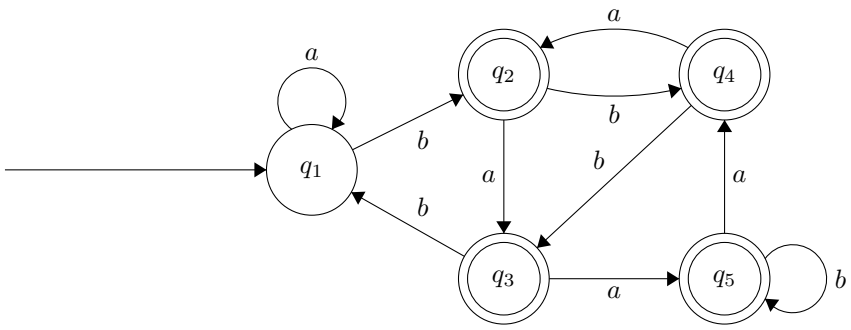
(a) baab

(b) abbb

(c) bbba

(c) Give a succinct English description of the string accepted by M.

a.



b. *baab* is accepted. *abbb* is accepted. *bbba* is accepted.

c. It accepted strings that do not have four consecutive b's.

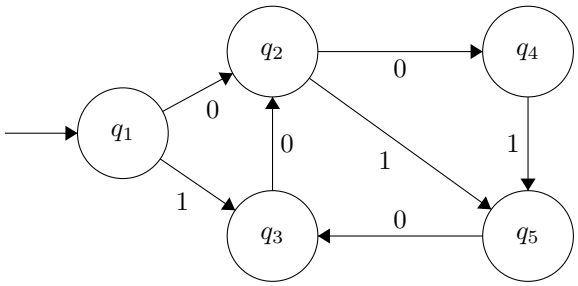
Problem 3

For each of the following languages give a state diagram of a DFA that recognize it. The alphabet is $\sum = \{0, 1\}$

(a) $\{w|w$ does not contain 000 or 11 as a substring$\}$

(b) $\{w|w$ contain at least two 0's and at least two 1's$\}$. The 0's and 1's do not need to be consecutive.

(a)

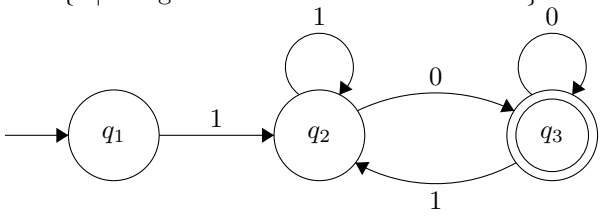


(b) Draw Tikz here.

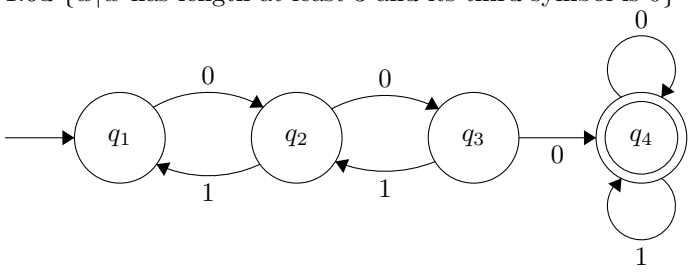
Problem 4

Solve Sipser exercises 1.6b, 1.6d, 1.5c, 1.4c.

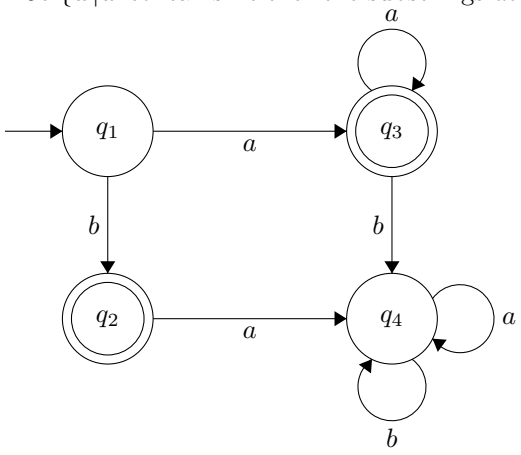
- 1.6b $\{w|w$ begins with a 1 and ends with a 0$\}$



- 1.6d $\{w|w$ has length at least 3 and its third symbol is 0$\}$



- 1.5c $\{w|w$ contains neither the substrings ab nor $ba\}$



- 1.4c $\{w|w$ has an even number of a's and one or two b's$\}$