COT 5407 Analysis of Algorithms

Homework 1

DUE: **Friday** January 30 at 11:55 PM

- *Please remember that all submissions must be typeset. Handwritten submissions will NOT be accepted. These must be uploaded to SCIS moodle in PDF format only.*
- *Please remember to type your name on top of your submission.*

1. (20 points) Order the following functions by growth rate. $n!$, $n^2 + \sqrt{n}\log^{10} n$, $n^{1/3}$, $\log^{100} n$, n^3, 2^n, $10^{\sqrt{n}}$, $2^{\log n}$, $2^{2\log n}$, $2^{\sqrt{\log n}}$, 128, $128n$. Indicate which functions grow at the same rate (all logarithms are base 2). For example, if you are asked to order n, $2n$ and $2n^2$, then you answer should be "$n = \Theta(2n)$, $2n = o(2n^2)$".

2. (20 points) Solve the following recurrence equations, expressing the answer in Big-Oh notation. Assume that $T(n)$ is constant for sufficiently small n.

   (a) $T(n) = T(n/2) + 100$
   (b) $T(n) = 8T(n/2) + n^2$
   (c) $T(n) = 8T(n/2) + n^3$
   (d) $T(n) = 8T(n/2) + n^4$
   (e) $T(n) = T(n-1) + \log n$
   (f) $T(n) = T(n-3) + n$

3. (20 points) You implemented a quadratic time algorithm for a problem P. On a test run, your algorithm takes 50 seconds for inputs of size 1000. Your classmate found a clever algorithm solving the same problem with running time $O(n^{3/2})$. However, the faster algorithm takes 150 seconds for inputs of size 1000. Explain how can this happen. If you need to solve a problem of size 4000, which algorithm you should use? What about inputs of size 10,000? Explain your answers (assume low-order terms are negligible).

4. Recall that in the *testing safe height to drop a cellphone* problem we discussed in the class, the goal is to find out the maximum safe height to drop a cellphone without breaking it. In the class we saw that if the maximum safe height is n, then in the worst case we can perform n tests if there is only one cellphone and $2\sqrt{n}$ tests if there are two cellphones. Give an algorithm to minimize the number of tests if there are k cellphones available (assume k is a constant). How many tests do you need to perform?

5. (20 points) You are given a set of n numbers. Give an $O(n^2)$ algorithm to decide if there exist three numbers a, b and c in the set such that $a + b = c$ (Hint: sort the numbers first).