International Institute of Information Technology Hyderabad

Modern Complexity Theory (CS1.405)

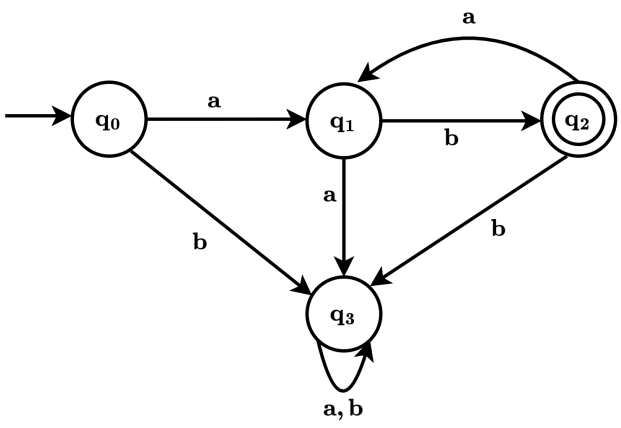
Assignment 1
Deadline: August 27, 2024 (Tuesday), 17:00 PM
Venue for Submission: CSTAR, A3-110, Vindhya Block, IIIT Hyderabad
Total Marks: 100

NOTE: It is strongly recommended that no student is allowed to copy from others.
No assignment will be taken after the deadline.
Write the following while submitting ONLY HARDCOPY:

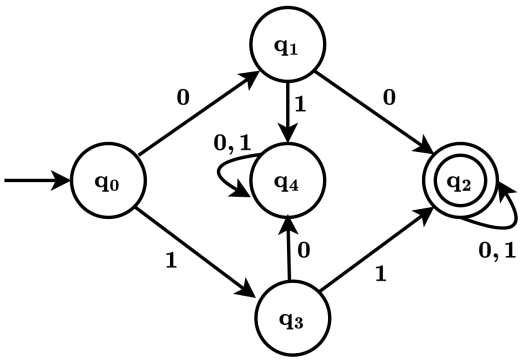
Modern Complexity Theory (CS1.405)
Assignment 1
Name:
Roll No.:

Questions

1. Determine the languages recognized by the following DFAs.



DFA M_1



DFA M_2

Figure 1:

2. Give state diagrams of DFAs recognizing the following languages. The alphabet is {0,1}.
   (a) $\{w | w$ is any string except 11 and 111$\}$
   (b) $\{w | w$ contains an even number of 0s, or contains exactly two 1s$\}$ **[(5+5)=10]**

3. (a) Consider the language L over $\{a, b\}$ which contains the strings whose lengths are from the arithmetic progression

$$P = \{2, 5, 8, 11, \cdots\} = \{2 + 3n \mid n \geq 0\}.$$

That is,

$$L = \{x \in \{a, b\}^* \mid |x| \in P\}.$$

Construct a DFA recognizing L.

(b) In general, for any arithmetic progression $P' = \{k' + kn \mid n \geq 0\}$ with $k, k' \in \mathbb{N}$ if we consider the language

$$L' = \{x \in \Sigma^* \mid |x| \in P'\}.$$

over an alphabet Σ, then design a generalized DFA that recognizes L'. **[(5+5)=10]**

4. (a) Prove that, if M is an NFA (Non-Deterministic Finite Automata) then $L(M) = \{w \mid w$ is accepted by $M\}$ is DFA-recognizable, where $L(M)$ is the language recognized by the NFA, M.

   (b) Prove that, FA-recognizable languages are closed under union. **[(5+5)=10]**

5. Show that the collection of decidable languages is closed under concatenation. **[10]**

6. Prove that, every nondeterministic Turing machine has an equivalent deterministic Turing machine. **[10]**

7. Let L be a language. Prove that, L is decidable if and only if both L and the complement $\bar{L}$ of L are Turing-recognizable. Give an example of a language which is not decidable but Turing-recognizable. **[10]**

8. Show that the collection of Turing-recognizable languages is closed under intersection. **[10]**

9. Prove that, every language accepted by a k-tape Turing machine is also accepted by a single-tape Turing machine. **[10]**

10. Show that any language decided by a Turing machine TM with work tape that is infinite in both directions in $T(n)$ steps can also be decided by a Turing machine TM with work tape infinite in only one direction in $O(T(n))$ steps. **[10]**

All the best!!!