МИНИСТЕРСТВО НАУКИ И ВЫСШЕГО ОБРАЗОВАНИЯ
РОССИЙСКОЙ ФЕДЕРАЦИИ

ФЕДЕРАЛЬНОЕ ГОСУДАРСТВЕННОЕ БЮДЖЕТНОЕ ОБРАЗОВАТЕЛЬНОЕ УЧРЕЖДЕНИЕ
ВЫСШЕГО ОБРАЗОВАНИЯ

**«БЕЛГОРОДСКИЙ ГОСУДАРСТВЕННЫЙ
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# Лабораторная работа №3.1

по дисциплине: Дискретная математика
тема: «Отношения и их свойства»

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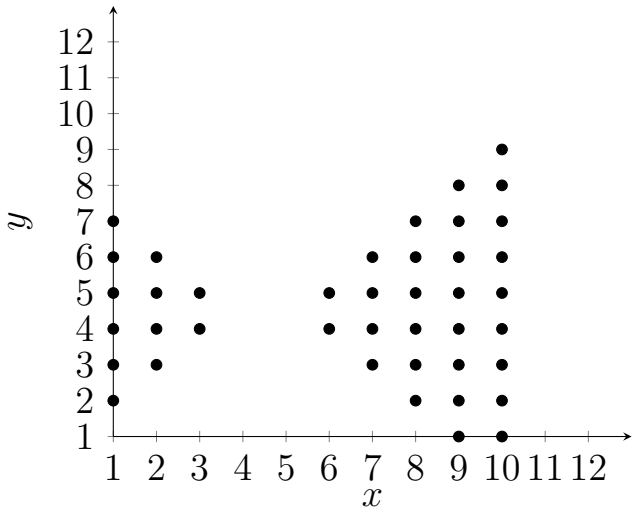
Белгород 2023 г.

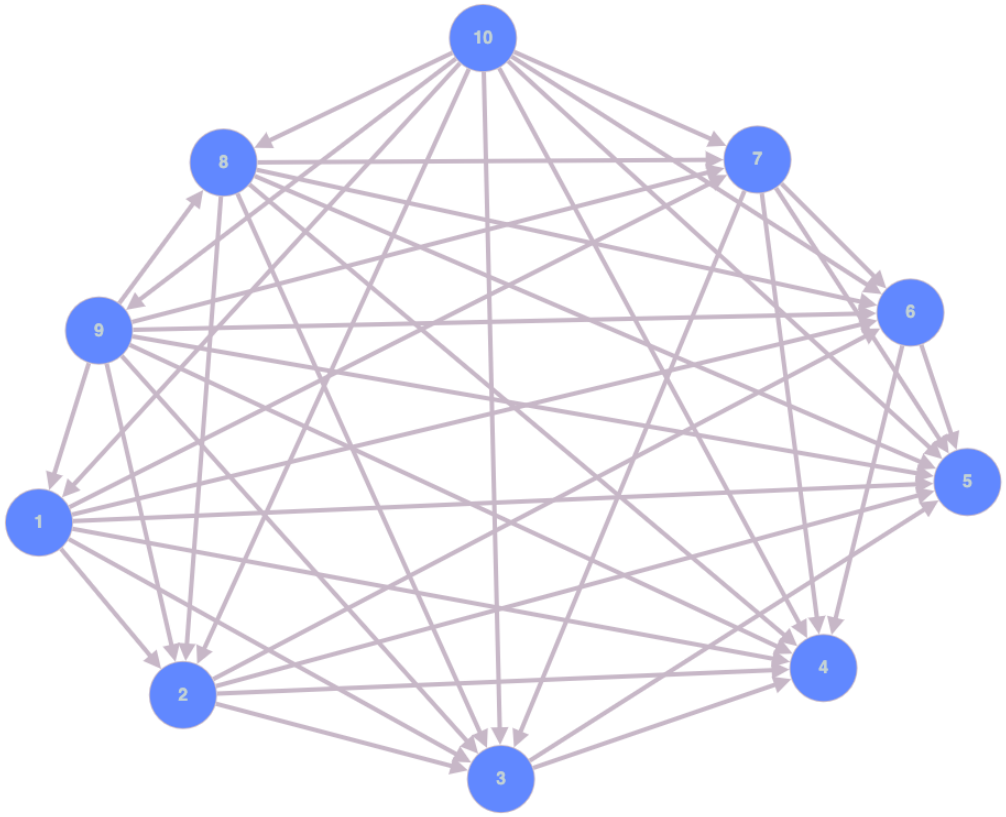
**Лабораторная работа №3.1**
Отношения и их свойства
Вариант 10

**Цель работы:** изучить способы задания отношений, операции над отношениями и свойства отношений, научиться программно реализовывать операции и определять свойства отношений.

## Часть 1. Операции над отношениями

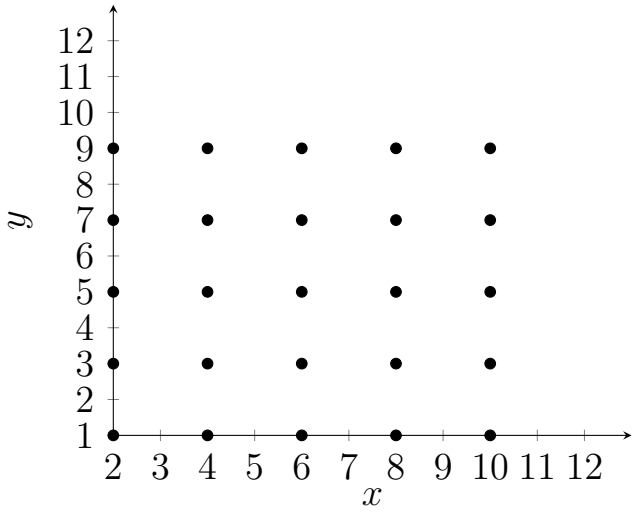
1.1. Представить отношения (см. ”Варианты заданий”, п.а) графиком, графом и матрицей.
$A = \{(x, y) | x \in N$ *и* $y \in N$ *и* $x < 11$ *и* $y < 11$ *и* $(x < y < (9 - x)$ *или* $(9 - x) < y < x)\}$

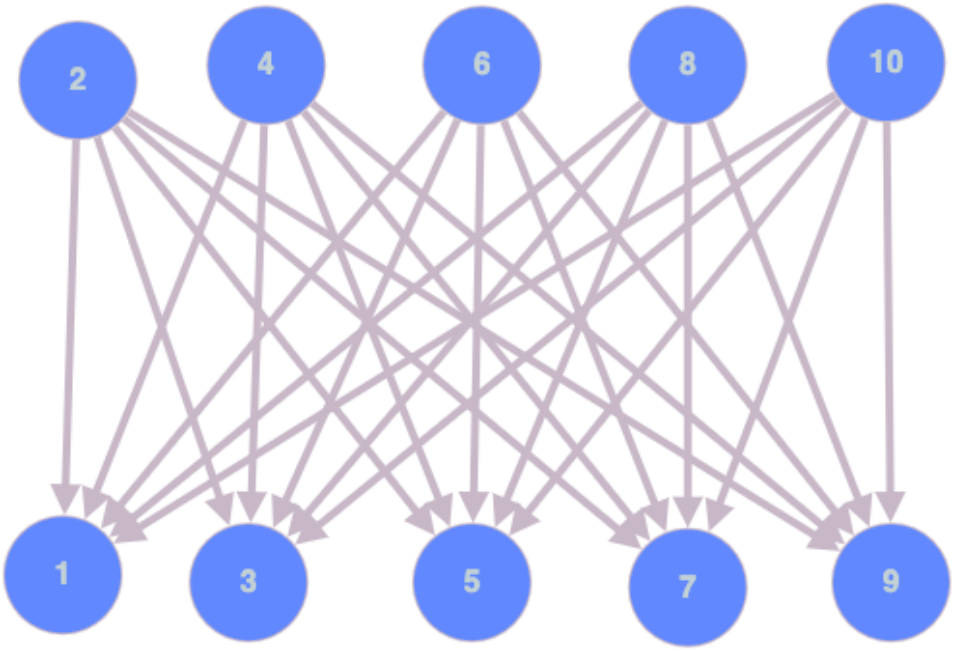


| | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| **2** | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| **3** | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| **4** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **5** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **6** | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| **7** | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| **8** | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| **9** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| **10** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

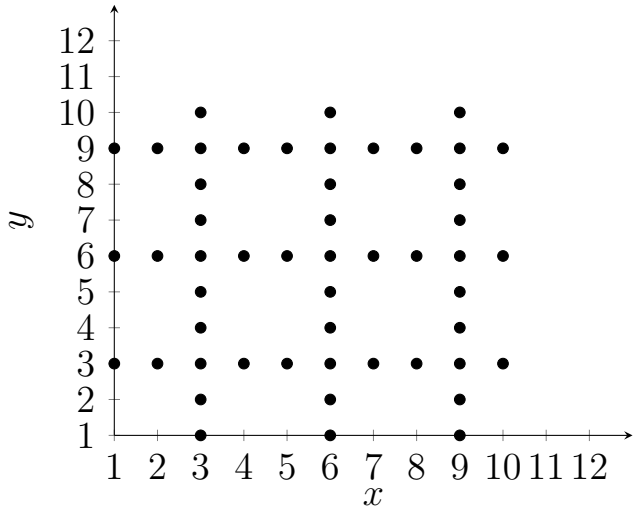
$B = \{(x, y) | x \in N$ *и* $y \in N$ *и* $x < 11$ *и* $y < 11$ *и* $x$ *- чётно и* $y$ *- нечётно*$\}$

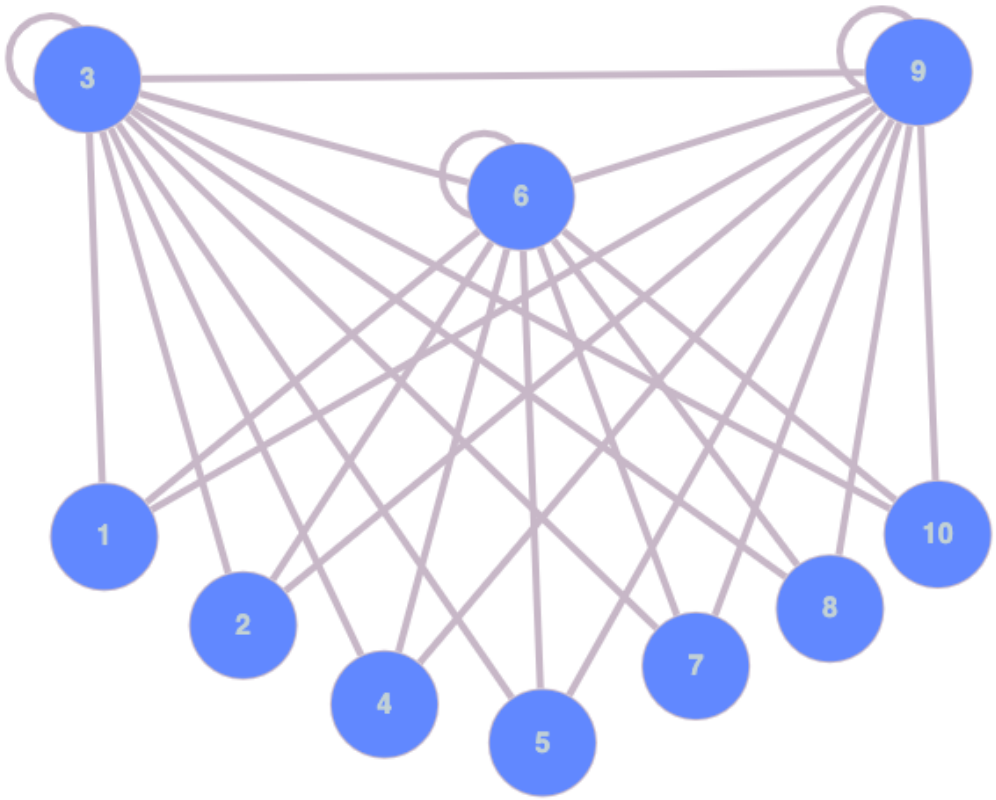


| | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **2** | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| **3** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **4** | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| **5** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **6** | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| **7** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **8** | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| **9** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **10** | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |

$C = \{(x, y) | x \in N$ *и* $y \in N$ *и* $x < 11$ *и* $y < 11$ *и* $x \cdot y$ кратно трём$\}$






| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| **2** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| **3** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **4** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| **5** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| **6** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **7** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| **8** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| **9** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **10** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |

1.2. Вычислить значение выражения (см. ”Варианты заданий”, п.б) при заданных отношениях (см. ”Варианты заданий”, п.а).

$D = A \circ B^2 - \overline{C} \cup C^{-1}$

$D = \overset{1}{A} \circ \overset{2}{B} \circ \overset{5}{B} - \overset{3}{\overline{C}} \cup \overset{4}{C^{-1}}$

1) $A \circ B =$

| | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| **2** | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| **3** | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| **4** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **5** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **6** | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| **7** | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| **8** | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| **9** | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| **10** | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |

2) $\_1 \circ B =$

| | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **2** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **3** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **4** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **5** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **6** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **7** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **8** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **9** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **10** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

3) $\overline{C} =$

| | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| **2** | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| **3** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **4** | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| **5** | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| **6** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **7** | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| **8** | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| **9** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **10** | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |

4) $C^{-1} =$

| | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| **2** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| **3** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **4** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| **5** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| **6** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **7** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| **8** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| **9** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **10** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |

5) $\_2 - \_3 =$

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **2** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **3** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **4** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **5** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **6** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **7** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **8** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **9** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **10** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

6) $\_5 \cup \_4 =$

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| **2** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| **3** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **4** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| **5** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| **6** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **7** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| **8** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| **9** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **10** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |

$D =$

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| **2** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| **3** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **4** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| **5** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| **6** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **7** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| **8** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| **9** | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| **10** | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |

1.3. Написать программы, формирующие матрицы заданных отношений (см. ”Варианты заданий”, п.а).

*main.cpp*

```
#include "../../libs/alg/alg.h"

bool predA(int x, int y) {
        return (x < y && y < (9 - x)) || ((9 - x) < y && y < x);
}

bool predB(int x, int y) {
        return x % 2 == 0 && y % 2 != 0;
```

```
}

bool predC(int x, int y) {
        return (x * y) % 3 == 0 ;
}

int main() {
        BoolMatrixRelation a(10, predA);
        std::cout << a << std::endl;

        BoolMatrixRelation b(10, predB);
        std::cout << b << std::endl;

        BoolMatrixRelation c(10, predC);
        std::cout << c << std::endl;
}
```

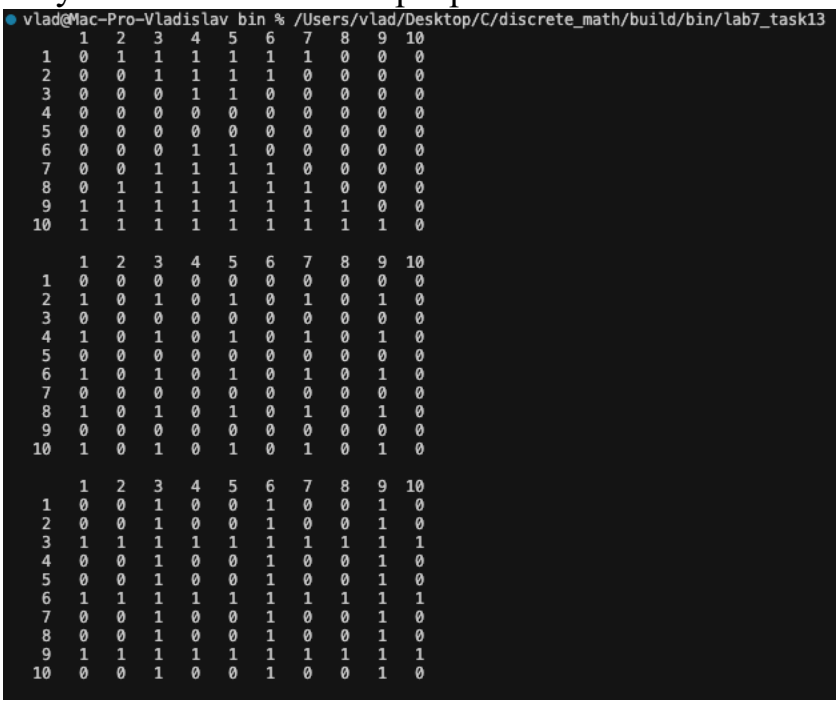
*alg.h* (объявление методов класса)

```
class BoolMatrixRelation
{
        private:
        std::vector<std::vector<bool>> data;
        int size;

        public:
        BoolMatrixRelation(const int size, bool (*pred)(int, int));
        ~BoolMatrixRelation();
        friend std::ostream& operator<<(std::ostream& out, BoolMatrixRelation &val) {
                out << std::setw(3) << "" << " ";
                for (int i = 1; i <= val.size; i++) {
                        out << std::setw(3) << i << " ";
                }
                out << "\n";

                for (int x = 0; x < val.size; x++) {
                        out << std::setw(3) << x + 1 << " ";
                        for (int y = 0; y < val.size; y++) {
                                out << std::setw(3) << val.data[x][y] << " ";
                        }

                        out << "\n";
                }

                return out;
        }
};
```

*task13.cpp* (реализация методов класса)

```cpp
#include "../alg.h"

BoolMatrixRelation::BoolMatrixRelation(const int size, bool (*pred)(int, int)) {
        this->size = size;

        for (int x = 1; x <= size; x++) {
                std::vector<bool> val;

                for (int y = 1; y <= size; y++) {
                        val.push_back(pred(x, y));
                }

                this->data.push_back(val);
        }
}

BoolMatrixRelation::~BoolMatrixRelation() {}
```

Результат выполнения программы:




1.4. Программно реализовать операции над отношениями.
Немного модифицируем класс BoolMatrixRelation. *alg.h* (объявление методов класса)

```cpp
class BoolMatrixRelation
{
        private:
        std::vector<std::vector<bool>> data;
        int size;

        static BoolMatrixRelation getDefault() {
                return BoolMatrixRelation();
        }
```

```
        public:
        BoolMatrixRelation(const int size, std::function<bool (int, int)> pred);
        BoolMatrixRelation() {
                this->size = 0;
        }
        ~BoolMatrixRelation();

        bool includes(BoolMatrixRelation b);
        bool equals(BoolMatrixRelation b);
        bool includesStrict(BoolMatrixRelation b);
        BoolMatrixRelation unite(BoolMatrixRelation b);
        BoolMatrixRelation intersect(BoolMatrixRelation b);
        BoolMatrixRelation diff(BoolMatrixRelation b);
        BoolMatrixRelation symDiff(BoolMatrixRelation b);
        BoolMatrixRelation non();
        BoolMatrixRelation transpose();
        BoolMatrixRelation compose(BoolMatrixRelation b);
        BoolMatrixRelation pow(int p);

        friend std::ostream& operator<<(std::ostream& out, BoolMatrixRelation &val) {
                out << std::setw(3) << "" << " ";
                for (int i = 1; i <= val.size; i++) {
                        out << std::setw(3) << i << " ";
                }
                out << "\n";

                for (int x = 0; x < val.size; x++) {
                        out << std::setw(3) << x + 1 << " ";
                        for (int y = 0; y < val.size; y++) {
                                out << std::setw(3) << val.data[x][y] << " ";
                        }

                        out << "\n";
                }

                return out;
        }
};
```

## *task13.cpp* (реализация методов класса)

```
#include "../alg.h"

BoolMatrixRelation::BoolMatrixRelation(const int size, std::function<bool (int, int)> pred)
{
        this->size = size;

        for (int x = 1; x <= size; x++)
        {
                std::vector<bool> val;

                for (int y = 1; y <= size; y++)
                {
```

```
                        val.push_back(pred(x, y));
                }

                this->data.push_back(val);
        }
}

BoolMatrixRelation::~BoolMatrixRelation() {}
```

---

## *task14.cpp* (реализация методов класса)

```
#include "../alg.h"

bool BoolMatrixRelation::includes(BoolMatrixRelation b)
{
        if (this->size != b.size) return false;

        for (int i = 0; i < size; i++) {
                for (int j = 0; j < size; j++) {
                        if (data[i][j] && !b.data[i][j])
                        return false;
                }
        }
        return true;
}
bool BoolMatrixRelation::equals(BoolMatrixRelation b)
{
        if (this->size != b.size) return false;

        for (int i = 0; i < size; i++) {
                for (int j = 0; j < size; j++) {
                        if (data[i][j] != b.data[i][j])
                        return false;
                }
        }
        return true;
}
bool BoolMatrixRelation::includesStrict(BoolMatrixRelation b)
{
        return (*this).includes(b) && !(*this).equals(b);
}
BoolMatrixRelation BoolMatrixRelation::unite(BoolMatrixRelation b)
{
        if (this->size != b.size) return BoolMatrixRelation::getDefault();

        return BoolMatrixRelation(size, [this, &b](int x, int y) {
                return data[x - 1][y - 1] || b.data[x - 1][y - 1];
        });
}
BoolMatrixRelation BoolMatrixRelation::intersect(BoolMatrixRelation b)
{
        if (this->size != b.size) return BoolMatrixRelation::getDefault();
```

```
	return BoolMatrixRelation(size, [this, &b](int x, int y) {
		return data[x - 1][y - 1] && b.data[x - 1][y - 1];
	});
}
BoolMatrixRelation BoolMatrixRelation::diff(BoolMatrixRelation b)
{
	if (this->size != b.size) return BoolMatrixRelation::getDefault();

	return BoolMatrixRelation(size, [this, &b](int x, int y) {
		return data[x - 1][y - 1] && !b.data[x - 1][y - 1];
	});
}
BoolMatrixRelation BoolMatrixRelation::symDiff(BoolMatrixRelation b)
{
	if (this->size != b.size) return BoolMatrixRelation::getDefault();

	return BoolMatrixRelation(size, [this, &b](int x, int y) {
		return data[x - 1][y - 1] ^ b.data[x - 1][y - 1];
	});
}
BoolMatrixRelation BoolMatrixRelation::non()
{
	return BoolMatrixRelation(size, [this](int x, int y) {
		return !data[x - 1][y - 1];
	});
}
BoolMatrixRelation BoolMatrixRelation::transpose()
{
	return BoolMatrixRelation(size, [this](int x, int y) {
		return data[y - 1][x - 1];
	});
}
BoolMatrixRelation BoolMatrixRelation::compose(BoolMatrixRelation b)
{
	if (this->size != b.size) return BoolMatrixRelation::getDefault();

	return BoolMatrixRelation(size, [this, &b](int x, int y) {
		for (int z = 0; z < size; z++) {
			if (data[x - 1][z] && b.data[z][y - 1])
			return true;
		}

		return false;
	});
}
BoolMatrixRelation BoolMatrixRelation::pow(int p)
{
	if (p < 0) return transpose();
	if (p == 0) return BoolMatrixRelation(size, [](int x, int y){return x == y;});
	if (p == 1) return *this;

	BoolMatrixRelation lowP = pow(p - 1);
	return compose(lowP);
```

```
}
```

1.5. Написать программу, вычисляющую значение выражения (см. “Варианты заданий”, п.б) и вычислить его при заданных отношениях (см.”Варианты заданий”, п.а).
*main.cpp*

```
#include "../../libs/alg/alg.h"

bool predA(int x, int y) {
        return (x < y && y < (9 - x)) || ((9 - x) < y && y < x);
}

bool predB(int x, int y) {
        return x % 2 == 0 && y % 2 != 0;
}

bool predC(int x, int y) {
        return (x * y) % 3 == 0 ;
}

int main() {
        BoolMatrixRelation a(10, predA);
        BoolMatrixRelation b(10, predB);
        BoolMatrixRelation c(10, predC);
        BoolMatrixRelation d = (((a.compose(b)).compose(b)).diff(c.non())).unite(c.pow(-1));

        std::cout << d << std::endl;
}
```

Результат выполнения программы:

```
vlad@Mac-Pro-Vladislav bin % /Users/vlad/Desktop/C/discrete_math/build/bin/lab7_task15
    1  2  3  4  5  6  7  8  9  10
 1  0  0  1  0  0  1  0  0  1  0
 2  0  0  1  0  0  1  0  0  1  0
 3  1  1  1  1  1  1  1  1  1  1
 4  0  0  1  0  0  1  0  0  1  0
 5  0  0  1  0  0  1  0  0  1  0
 6  1  1  1  1  1  1  1  1  1  1
 7  0  0  1  0  0  1  0  0  1  0
 8  0  0  1  0  0  1  0  0  1  0
 9  1  1  1  1  1  1  1  1  1  1
10  0  0  1  0  0  1  0  0  1  0
```

Значение формулы D, вычисленное вручную, и результат выполненния программы совпали.

## Часть 2. Свойства отношений

2.1. Определить основные свойства отношений (см. ”Варианты заданий”, п.а).

| | A | B | C |
|---|---|---|---|
| Рефлексивность | | | |
| Антирефлексивность | + | + | |
| Симметричность | | | + |
| Антисимметричность | + | + | |
| Транзитивность | + | + | |
| Антитранзитивность | | + | |
| Полнота | | | |

2.2. Определить, являются ли заданные отношения отношениями толерантности, эквивалентности и порядка.

| | A | B | C |
|---|---|---|---|
| Толерантно | | | |
| Эквивалентно | | | |
| Порядка | + | + | |
| Нестрогого порядка | | | |
| Строгого порядка | + | + | |
| Линейного порядка | | | |
| Нестрогого линейного порядка | | | |
| Строгого линейного порядка | | | |

2.3. Написать программу, определяющую свойства отношения, в том числе толерантности, эквивалентности и порядка, и определить свойства отношений (см. ”Варианты заданий”, п.а). *main.cpp*

```
#include "../../libs/alg/alg.h"

bool predA(int x, int y) {
    return (x < y && y < (9 - x)) || ((9 - x) < y && y < x);
}

bool predB(int x, int y) {
    return x % 2 == 0 && y % 2 != 0;
}

bool predC(int x, int y) {
    return (x * y) % 3 == 0 ;
}

void outputProperties(std::string name, BoolMatrixRelation a) {
    std::pair<int, int> failedAt;
    std::cout << "Properties for " << name << "\n";
    if (a.isReflexive(failedAt)) {
        std::cout << "Reflexive\n";
    } else {
        std::cout << "Non reflexive, failed pair: (" << failedAt.first << ", " << failedAt.second << ")\n";
    }
    if (a.isAntiReflexive(failedAt)) {
        std::cout << "Antireflexive\n";
    } else {
        std::cout << "Non antireflexive, failed pair: (" << failedAt.first << ", " << failedAt.second << ")\n";
```

```
    }

    if (a.isSymmetric(failedAt)) {
        std::cout << "Symmetric\n";
    } else {
        std::cout << "Non symmetric, failed pair: (" << failedAt.first << ", " << failedAt.second << ")\n";
    }

    if (a.isAntiSymmetric(failedAt)) {
        std::cout << "AntiSymmetric\n";
    } else {
        std::cout << "Non antisymmetric, failed pair: (" << failedAt.first << ", " << failedAt.second << ")\n";
    }

    if (a.isTransitive(failedAt)) {
        std::cout << "Transitive\n";
    } else {
        std::cout << "Non transitive, failed pair: (" << failedAt.first << ", " << failedAt.second << ")\n";
    }

    if (a.isAntiTransitive(failedAt)) {
        std::cout << "AntiTransitive\n";
    } else {
        std::cout << "Non antitransitive, failed pair: (" << failedAt.first << ", " << failedAt.second << ")\n";
    }

    if (a.isFull(failedAt)) {
        std::cout << "Full\n";
    } else {
        std::cout << "Non full, failed pair: (" << failedAt.first << ", " << failedAt.second << ")\n";
    }
    std::cout << (a.isTolerant() ? "Tolerant" : "Non tolerant") << "\n";
    std::cout << (a.isEquivalent() ? "Equivalent" : "Non equivalent") << "\n";
    std::cout << (a.isOrdered() ? "Ordered" : "Non ordered") << "\n";
    std::cout << (a.isOrderedNonStrict() ? "Ordered non strict" : "Non ordered non strict") << "\n";
    std::cout << (a.isOrderedStrict() ? "Ordered strict" : "Non ordered strict") << "\n";
    std::cout << (a.isOrderedLinear() ? "Ordered linear" : "Non ordered linear") << "\n";
    std::cout << (a.isOrderedLinearNonStrict() ? "Ordered linear non strict" : "Non ordered linear non strict")
    ↪  << "\n";
    std::cout << (a.isOrderedLinearStrict() ? "Ordered linear strict" : "Non ordered linear strict") << "\n" <<
    ↪  std::endl;
}

int main() {
    BoolMatrixRelation a(10, predA);
    BoolMatrixRelation b(10, predB);
    BoolMatrixRelation c(10, predC);

    outputProperties("A", a);
    outputProperties("B", b);
    outputProperties("C", c);
}
```

*alg.h*

```
class BoolMatrixRelation
{
private:
    std::vector<std::vector<bool>> data;
    int size;

    static BoolMatrixRelation getDefault() {
        return BoolMatrixRelation();
    }

public:
    BoolMatrixRelation(const int size, std::function<bool (int, int)> pred);
    BoolMatrixRelation() {
        this->size = 0;
    }
    ~BoolMatrixRelation();

    bool includes(BoolMatrixRelation b);
    bool equals(BoolMatrixRelation b);
    bool includesStrict(BoolMatrixRelation b);
    BoolMatrixRelation unite(BoolMatrixRelation b);
    BoolMatrixRelation intersect(BoolMatrixRelation b);
    BoolMatrixRelation diff(BoolMatrixRelation b);
    BoolMatrixRelation symDiff(BoolMatrixRelation b);
    BoolMatrixRelation non();
    BoolMatrixRelation transpose();
    BoolMatrixRelation compose(BoolMatrixRelation b);
    BoolMatrixRelation pow(int p);

    static BoolMatrixRelation getIdentity(int size);
    static BoolMatrixRelation getUniversum(int size);

    bool isEmpty();

    bool isReflexive(std::pair<int, int> &failed);
    bool isAntiReflexive(std::pair<int, int> &failed);
    bool isSymmetric(std::pair<int, int> &failed);
    bool isAntiSymmetric(std::pair<int, int> &failed);
    bool isTransitive(std::pair<int, int> &failed);
    bool isAntiTransitive(std::pair<int, int> &failed);
    bool isFull(std::pair<int, int> &failed);
    bool isTolerant();
    bool isEquivalent();
    bool isOrdered();
    bool isOrderedNonStrict();
    bool isOrderedStrict();
    bool isOrderedLinear();
    bool isOrderedLinearNonStrict();
    bool isOrderedLinearStrict();

    friend std::ostream& operator<<(std::ostream& out, BoolMatrixRelation &val) {
        out << std::setw(3) << "" << " ";
        for (int i = 1; i <= val.size; i++) {
            out << std::setw(3) << i << " ";
```

```
        }
        out << "\n";

        for (int x = 0; x < val.size; x++) {
            out << std::setw(3) << x + 1 << " ";
            for (int y = 0; y < val.size; y++) {
                out << std::setw(3) << val.data[x][y] << " ";
            }

            out << "\n";
        }

        return out;
    }
};
```

*task23.cpp*

```
#include "../alg.h"

bool BoolMatrixRelation::isEmpty() {
    for (int i = 0; i < size; i++) {
        for (int j = 0; j < size; j++) {
            if (data[i][j]) return false;
        }
    }

    return true;
}

BoolMatrixRelation BoolMatrixRelation::getIdentity(int size) {
    return BoolMatrixRelation(size, [](int x, int y) {
        return x == y;
    });
}
BoolMatrixRelation BoolMatrixRelation::getUniversum(int size) {
return BoolMatrixRelation(size, [](int x, int y) {
        return true;
    });
}

bool BoolMatrixRelation::isReflexive(std::pair<int, int> &failed)
{
    for (int i = 0; i < size; i++) {
        if (!data[i][i]) {
            failed = {i + 1, i + 1};
            return false;
        }
    }

    return true;
}
bool BoolMatrixRelation::isAntiReflexive(std::pair<int, int> &failed)
```

```
{
    for (int i = 0; i < size; i++) {
        if (data[i][i]) {
            failed = {i + 1, i + 1};
            return false;
        }
    }

    return true;
}
bool BoolMatrixRelation::isSymmetric(std::pair<int, int> &failed)
{
    for (int i = 0; i < size; i++) {
        for (int j = 0; j < size; j++) {
            if (data[i][j] && !data[j][i]) {
                failed = {i + 1, j + 1};
                return false;
            }
        }
    }

    return true;
}
bool BoolMatrixRelation::isAntiSymmetric(std::pair<int, int> &failed)
{
    for (int i = 0; i < size; i++) {
        for (int j = 0; j < size; j++) {
            if (data[i][j] && data[j][i]) {
                failed = {i + 1, j + 1};
                return false;
            }
        }
    }

    return true;
}
bool BoolMatrixRelation::isTransitive(std::pair<int, int> &failed)
{
    //return ((*this).pow(2)).includes((*this));
    for (int i = 0; i < size; i++) {
        for (int j = 0; j < size; j++) {
            for (int z = 0; z < size; z++) {
                if ((data[i][z] && data[z][j]) && !data[i][j]) {
                    failed = {i + 1, j + 1};
                    return false;
                }
            }
        }
    }

    return true;
}
bool BoolMatrixRelation::isAntiTransitive(std::pair<int, int> &failed)
{
```

```
    for (int i = 0; i < size; i++) {
        for (int j = 0; j < size; j++) {
            for (int z = 0; z < size; z++) {
                if ((data[i][z] && data[z][j]) && data[i][j]) {
                    failed = {i + 1, j + 1};
                    return false;
                }
            }
        }
    }

    return true;
}
bool BoolMatrixRelation::isFull(std::pair<int, int> &failed)
{
    for (int i = 0; i < size; i++) {
        for (int j = 0; j < size; j++) {
            if (!((i == j) || (data[i][j] || data[j][i]))) {
                failed = {i + 1, j + 1};
                return false;
            }
        }
    }

    return true;
}
bool BoolMatrixRelation::isTolerant()
{
    std::pair<int, int> ignored;
    return isReflexive(ignored) && isSymmetric(ignored);
}
bool BoolMatrixRelation::isEquivalent()
{
    std::pair<int, int> ignored;
    return isReflexive(ignored) && isSymmetric(ignored) && isTransitive(ignored);
}
bool BoolMatrixRelation::isOrdered()
{
    std::pair<int, int> ignored;
    return isAntiSymmetric(ignored) && isTransitive(ignored);
}
bool BoolMatrixRelation::isOrderedNonStrict()
{
    std::pair<int, int> ignored;
    return isOrdered() && isReflexive(ignored);
}
bool BoolMatrixRelation::isOrderedStrict()
{
    std::pair<int, int> ignored;
    return isOrdered() && isAntiReflexive(ignored);
}
bool BoolMatrixRelation::isOrderedLinear()
{
    std::pair<int, int> ignored;
```

```
    return isOrdered() && isFull(ignored);
}
bool BoolMatrixRelation::isOrderedLinearNonStrict()
{
    std::pair<int, int> ignored;
    return isOrderedNonStrict() && isFull(ignored);
}
bool BoolMatrixRelation::isOrderedLinearStrict()
{
    std::pair<int, int> ignored;
    return isOrderedStrict() && isFull(ignored);
}
```

Результат выполнения программы:

```
vlad@Mac-Pro-Vladislav bin % /Users/vlad/Desktop/C/discrete_math/build/bin/lab7_task23
Properties for A
Non reflexive, failed pair: (1, 1)
Antireflexive
Non symmetric, failed pair: (1, 2)
AntiSymmetric
Transitive
Non antitransitive, failed pair: (1, 3)
Non full, failed pair: (1, 8)
Non tolerant
Non equivalent
Ordered
Non ordered non strict
Ordered strict
Non ordered linear
Non ordered linear non strict
Non ordered linear strict

Properties for B
Non reflexive, failed pair: (1, 1)
Antireflexive
Non symmetric, failed pair: (1, 2)
AntiSymmetric
Transitive
AntiTransitive
Non full, failed pair: (1, 3)
Non tolerant
Non equivalent
Ordered
Non ordered non strict
Ordered strict
Non ordered linear
Non ordered linear non strict
Non ordered linear strict

Properties for C
Non reflexive, failed pair: (1, 1)
Non antireflexive, failed pair: (3, 3)
Symmetric
Non antisymmetric, failed pair: (1, 3)
Non transitive, failed pair: (1, 1)
Non antitransitive, failed pair: (1, 3)
Non full, failed pair: (1, 2)
Non tolerant
Non equivalent
Non ordered
Non ordered non strict
Non ordered strict
Non ordered linear
Non ordered linear non strict
Non ordered linear strict
```

**Вывод:** в ходе лабораторной работы изучили способы задания отношений, операции над отношениями и свойства отношений, научились программно реализовывать операции и определять свойства отношений.