Assignment8-EE2703

Nihal Gajjala

April 2021

Introduction

The objective of the assignment is to solve the low pass filters and high pass filters using Sympy in Python. We would be analysing circuits using the Laplace Transform of the impulse response, using symbolic algebra capabilities of Python.

Circuits

Low Pass Filter

An ideal low pass filter is a circuit that allows only the lower frequency components (i.e, input frequencies below a cut off frequency) to pass through and blocks all the higher frequency components. In reality, a low pass filter also allows high frequency components to pass through but their amplitude is highly attenuated.

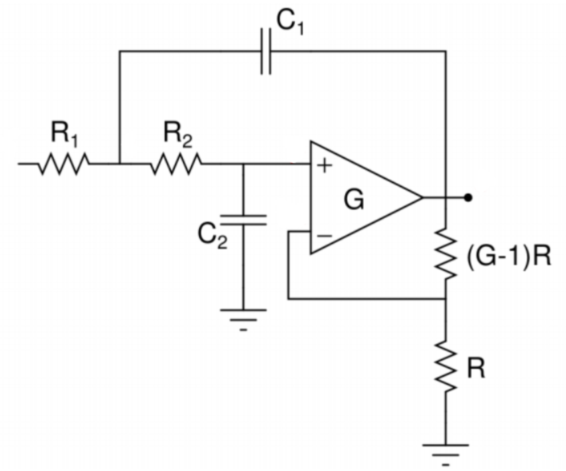

Figure 1: Low Pass Filter

$$V_m = \frac{V_o}{G} \tag{1}$$

$$V_o = G(V_p - Vm) \tag{2}$$

$$V_p = \frac{V_1}{1 + sC_2R_2} \tag{3}$$

$$\frac{V_i - V_1}{R_1} + \frac{V_p - V_1}{R_2} + sC_1(V_o - V_1) = 0 \tag{4}$$

We give an input signal at node 'V_i' and obtain the output at node 'V_o'. The above equations are obtained by applying nodal analysis.

High Pass Filter

An ideal high pass filter is a circuit that allows only the higher frequency components (i.e, input frequencies above a cut off frequency) to pass through and blocks all the lower frequency components. In reality, a high pass filter also allows low frequency components to pass through but their amplitude is highly attenuated.

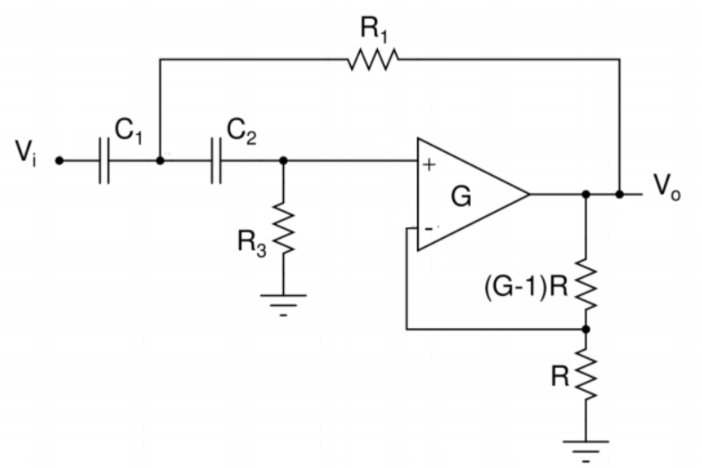

Figure 2: High Pass Filter

$$V_m = \frac{V_o}{G} \tag{5}$$

$$V_o = G(V_p - Vm) \tag{6}$$

$$V_p = \frac{V_1 s R_3 C_2}{1 + sC_2R_2} \tag{7}$$

$$sC_1(V_i - V_1) + sC_2(V_p - V_1) + \frac{V_o - V_1}{R_1} = 0 \tag{8}$$

We give an input signal at node 'V_i' and obtain the output at node 'V_o'. The above equations are obtained by applying nodal analysis.

Magnitude Response And Step Response

Low Pass Filter

The magnitude response of an ideal low pass filter should be equal to 1 for lower frequencies and 0 for higher frequencies.

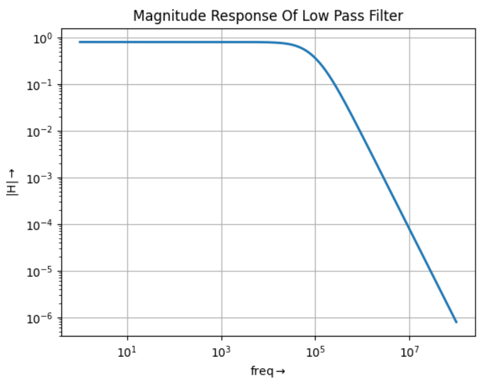

Figure 3: Magnitude Response Of LPF

In the above figure, we observe that before $\omega = 10^4$, the magnitude of $H(j\omega)$ is 1 i.e. the output will be equal to the input for frequencies below 10^4. For $\omega > 10^4$, the magnitude of $H(j\omega)$ falls by 40dB per decade (if ω increases by a factor of 10, then the magnitude of $H(j\omega)$ decreases by a factor of 100).

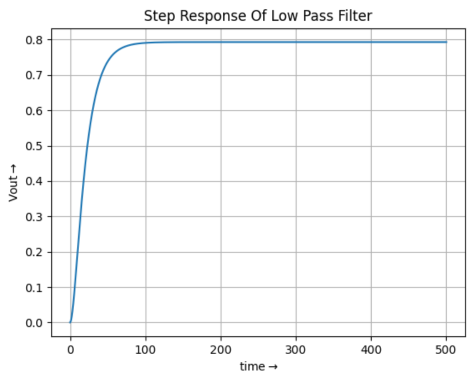

Figure 4: Step Response Of LPF

High Pass Filter

The magnitude response of an ideal low pass filter should be equal to 1 for higher frequencies and 0 for lower frequencies.

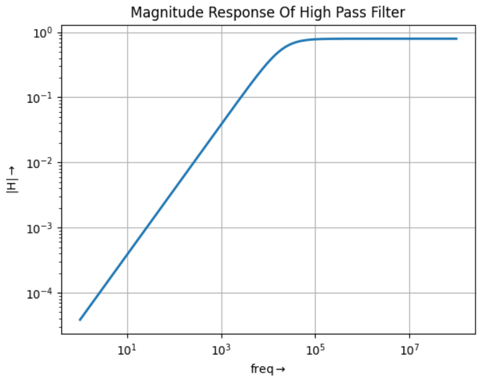

Figure 5: Magnitude Response Of HPF

In the above figure, we observe that after $\omega = 10^5$, the magnitude of $H(j\omega)$ is 1 i.e. the output will be equal to the input for frequencies above 10^5. For $\omega < 10^4$, the magnitude of $H(j\omega)$ increases by 20dB per decade (if ω increases by a factor of 10, then the magnitude of $H(j\omega)$ increases by a factor of 10).

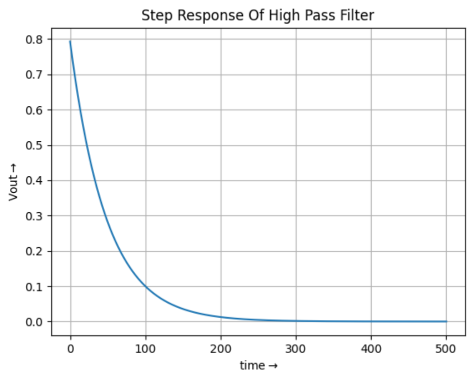

Figure 6: Step Response Of HPF

Step Analysis

For circuits where we connect a dc input at t=0, the capacitor behaves differently at t=0 and $t \to \infty$.

- At t=0, capacitor behaves like a short circuit.
- At $t \to \infty$, capacitor behaves like an open circuit.

For the low pass filter, we see that at t = 0, C_1 and C_2 behaves as a short circuit and $V_p = 0 \implies V_o = 0$. At $t \to \infty$, C_1 and C_2 behaves as an open circuit. This means that $V_p = V_i \implies V_o = GV_i$. Using the above argument, we can say that the step response of low pass filter should go from 0 to GV_i.

For the high pass filter, we see that at t = 0, C_1 and C_2 behaves as a short circuit and $V_p = V_i \implies V_o = GV_i$. At $t \to \infty$, C_1 and C_2 behaves as an open circuit. This means that $V_p = 0 \implies V_o = 0$. Using the above argument, we can say that the step response of low pass filter should go from GV_i to 0.

Inputs

Input 1: Sum of two sinusoidal of different frequencies

$V_i(t) = (\sin 2 * 10^3 \pi t + \cos 2 * 10^6 \pi t)u(t)$

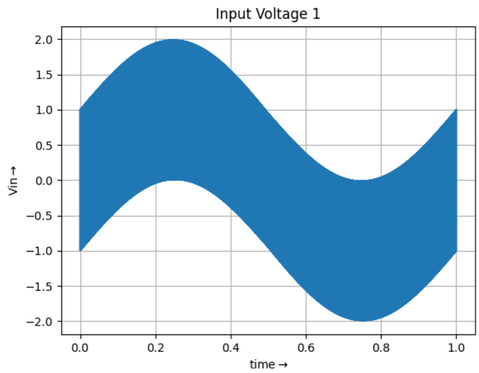

Figure 7: Input Voltage 1

Input 1 is a sum of 2 sinusoidal of different frequencies:

1. High Frequency Component:$V_i(t) = (\cos 2 * 10^6 \pi t)u(t)$
2. Low Frequency Component:$V_i(t) = (\sin 2 * 10^3 \pi t)u(t)$

Ideally the low pass filter should eliminate the high frequency 'cos' component and the high pass filter should eliminate the low frequency 'sin' component.

High Frequency Component:$V_i(t) = (\cos 2 * 10^6 \pi t)u(t)$

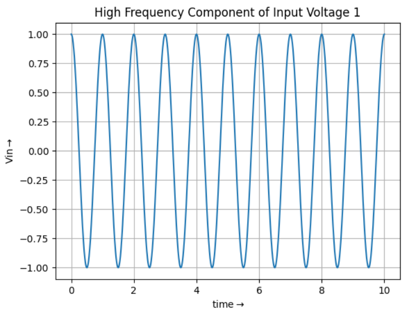

Figure 8: Input Voltage 1 High Frequency Component

Low Frequency Component:$V_i(t) = (\sin 2 * 10^3 \pi t)u(t)$

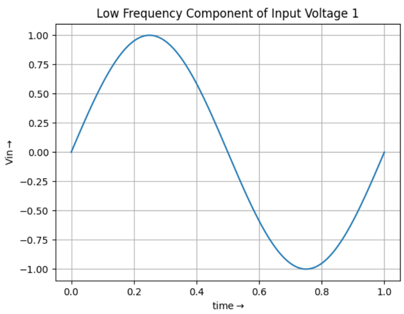

Figure 9: Input Voltage 1 Low Frequency Component

Input 2: Damped sinusoidal (sin)

$V_i(t) = \sin((freq)\pi t)e^{-decay}(t)u(t)$

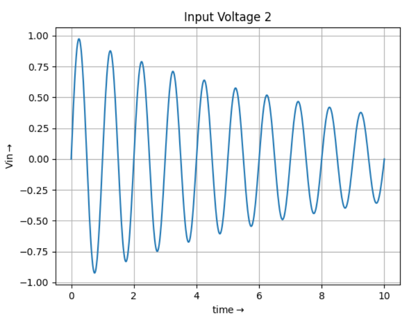

Figure 10: Input Voltage 2

Input 3: Damped sinusoidal (cos)

$$V_i(t) = \cos((freq)\pi t)e^{-decay}(t)u(t)$$

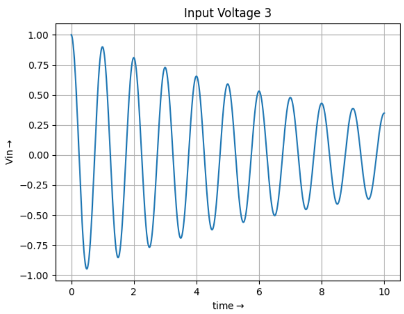

Figure 11: Input Voltage 3

Outputs

Low Pass Filter

The low pass filter has eliminated the high frequency component (cos). The output voltage consists of only the low frequency component (sin).

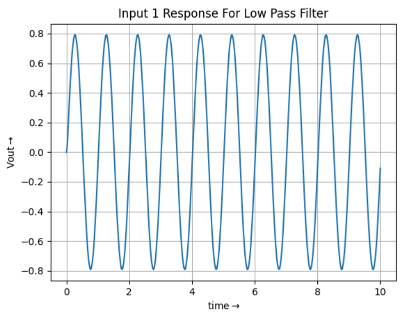

Figure 12: Input 1 Response of LPF

The output voltage for the damped sinusoidal input is similar to the input voltage with slightly reduced amplitude. This reduction in amplitude is due to the non-ideality of the low pass filter.

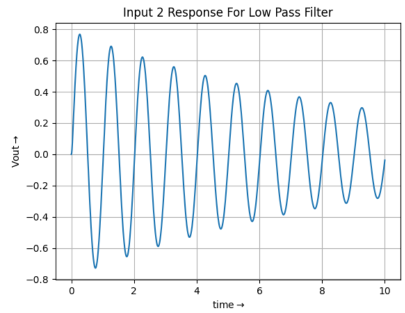

Figure 13: Input 2 Response of LPF

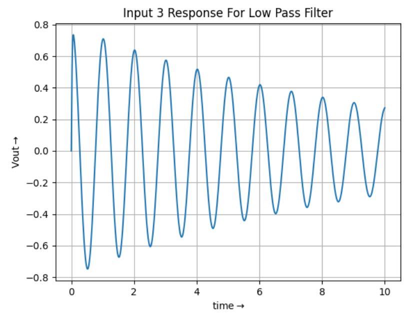

Figure 14: Input 3 Response of LPF

High Pass Filter

The high pass filter has partially eliminated the low frequency component (sin). The output voltage consists of only the high frequency component (cos) with distortions due to low frequency component. The distortions are due to the non-ideality of the high pass filter.

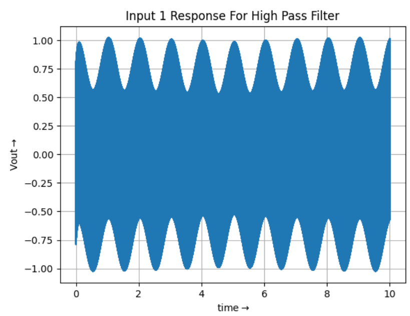

Figure 15: Input 1 Response of HPF

The output voltage is an highly attenuated damped sinusoidal input. Most of the low frequency components are eliminated or highly attenuated whereas the high frequency components pass freely through the high pass filter.

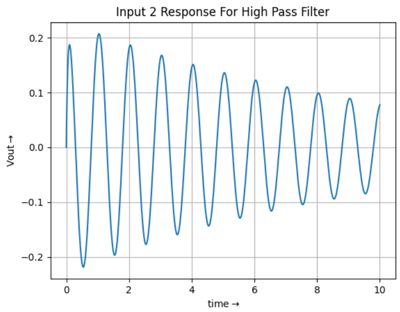

Figure 16: Input 2 Response of HPF

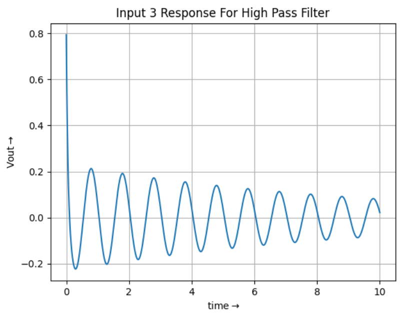

Figure 17: Input 3 Response of HPF

Conclusion

Sympy provides a convenient way to analyse LTI systems using their Laplace transforms. The toolbox was used to study the behaviour of a low pass filter, implemented using an op-amp of gain G. For a mixed frequency sinusoidal as input, it was found that the filter suppressed the high frequencies while allowing the low frequency components. Similarly, a high pass filter was implemented using an op-amp with the same gain. The magnitude response of the filter was plotted and its output was analysed. The output was analysed for a damped sinusoidal input. The step response of the filter was found to have a non-zero peak at t = 0, due to the sudden change in the input voltage.