Assignment7-EE2703

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Introduction

The objective of this assignment is to solve Laplace equations, obtain the time response after solving for the system and plotting the bode plots. We calculate the initial conditions and use them in solving inverse unilateral Laplace transform.

In this assignment, we will look at how to analyse “Linear Time-invariant Systems” with numerical tools in Python.

Spring System

The spring equation is given by:

$$\ddot{x} + 2.25x = f(t) \tag{1}$$

where:

- x is the displacement of the spring
- $\ddot{x}$ is the acceleration of the spring (second derivative of x)
- f(t) is the driving force

For this equation we will have 2 initial conditions:

- x=0
- $\ddot{x}$=0

The driving force is f(t) is given by:

$$f(t) = cos(\omega dt)e^{-(decay)t}u(t) \tag{2}$$

The Laplace transform of f(t) is given by:

$$F(s) = \frac{s + decay}{(s + decay)^2 + \omega_d^2} \tag{3}$$

We will solve the above Laplace equation for decay equal to 0.5 and 0.05 and ω_d varying between 1.4 and 1.6.

We will be plotting the values of x for time between 0 to 50.

Time Response Of Spring System

We will observe the dependence of the time response of the spring system on decay and frequency of operation (ω_d).

For $\omega_d = 1.5$ and decay=0.5 we get the following graph:

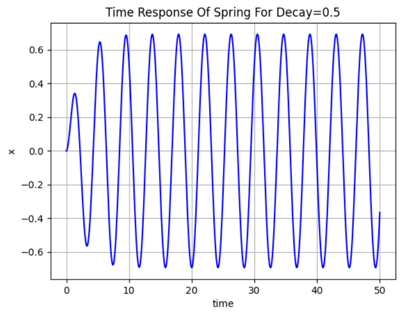


Figure 1: Time Response Of Spring For Decay=0.5

For $\omega_d = 1.5$ and decay=0.05 we get the following graph:

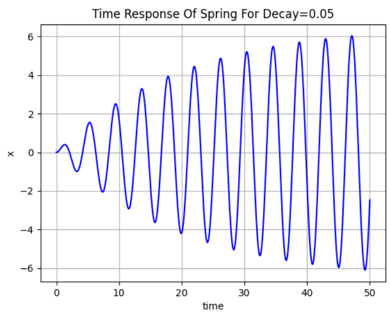


Figure 2: Time Response Of Spring For Decay=0.05

For varying ω_d between 1.4 to 1.6 and decay=0.5 we get the following graph:

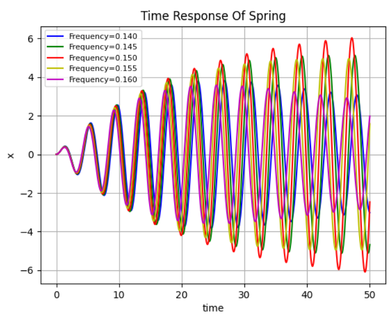


Figure 3: Time Response Of Spring

From the graphs we observe that:

- The amplitude increases as frequency increases from 1.4 to 1.5
- The amplitude is maximum for frequency equal to 1.5
- The amplitude decreases as the frequency increases from 1.5 to 1.6

We observe that amplitude is maximum when frequency of driving force is equal to the natural frequency of the spring. The natural frequency of the spring in the above case is 1.5. So the amplitude of the spring is maximum at frequency equal to 1.5.

Coupled Spring System

The coupled spring equations are given by:

$$\ddot{x} + (x - y) = 0 \tag{4}$$

$$\ddot{y} + 2(y - x) = 0 \tag{5}$$

For these equations we will have 4 initial conditions:

- x=1
- y=0
- $\ddot{x}$=0
- $\ddot{y}$=0

The Laplace transform of the above equations are given by:

$$X(s) = \frac{s^2 + 2}{s^3 + 3s} \tag{6}$$

$$Y(s) = \frac{2}{s^3 + 3s} \tag{7}$$

The time response of the coupled spring system is:

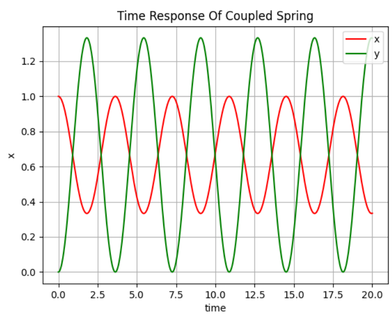


Figure 4: Time Response Of Coupled Spring

The above graph shows the variation of x and y as a function of time in the range 0 to 20.

We observe that the amplitude of spring y is greater than the amplitude of spring x.

Two Port Network

The steady state transfer function of the RLC two port network is given by:

$$H(s) = \frac{1}{s^2LC + sRC + 1} \tag{8}$$

here:

- R=100Ω
- L=1μH
- C=1μF

The Bode plot represents the change in magnitude or phase of the system for varying frequency on a logarithmic scale. The Bode plot of the above transfer function is as follows:

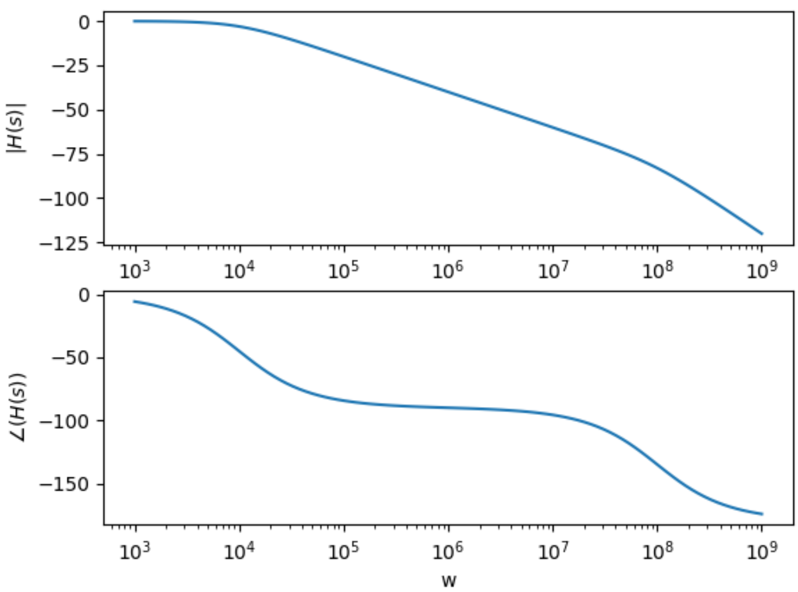


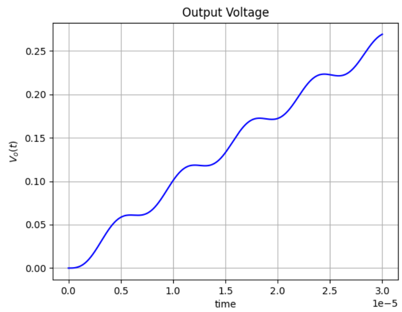
Figure 5: Bode Plot

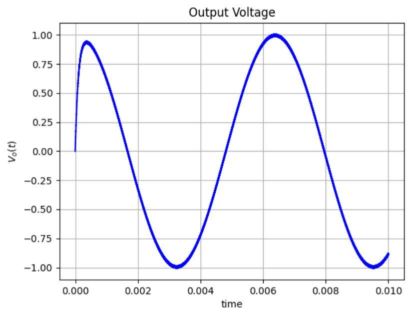
The poles of the RLC transfer function are at s=10^8 and s=10^4.

Output Voltage

Input Voltage

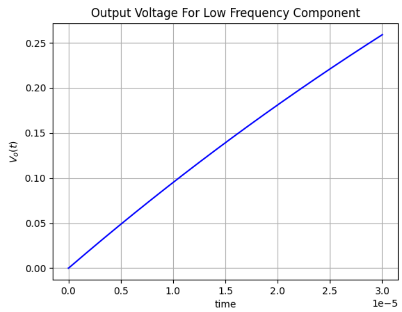
Output voltage V_o for Input Voltage $V_i = cos(10^3 t)u(t) - cos(10^6 t)u(t)$

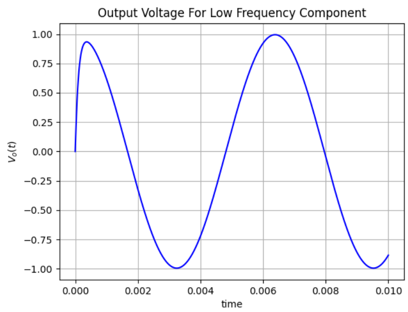




Low Frequency

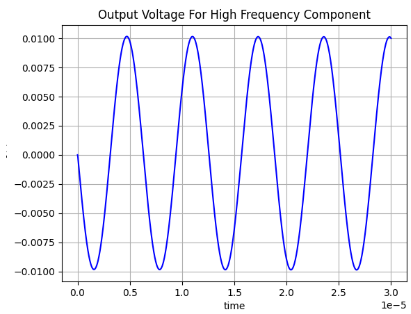
Output voltage V_o for Input Voltage $V_i = cos(10^3 t)u(t)$

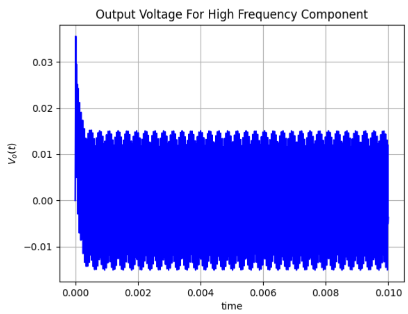




High Frequency

Output voltage V_o for Input Voltage $V_i = -cos(10^6 t)u(t)$





From the above graphs we can infer that:

1. The output voltage for the high frequency component has a very low amplitude of almost 0.01 V.

2. The output voltage for the low frequency component has a amplitude of almost 1 V. This is 100 times the amplitude obtained due to high voltage component.

The output voltage for the complete input signal and the output voltage for the low frequency input signal is almost the same in the time scale of 0 to 10 msec. However during the first 30µsec, the output voltage increases linearly till 0.25 V. This is comparable to the output voltage for the high frequency component. So, we see a distorted graph, where the output looks like the sum of a linear signal and a sinusoidal signal.

Conclusion

In this assignment, we have learnt how to handle polynomials in python. We also learnt how to handle Laplace equations and plot bode plots. Using this knowledge, we solved the equations of a single spring, a coupled spring, and an RLC circuit.