GEOMETRIA

ESERCIZIO 1 Compito 7-6-07

$U = \{(x,y,z,t) \in \mathbb{R}^4 / x+y+z=0\}$

$V = \{(x,y,z,t) \in \mathbb{R}^4 / x-2z=0 \ , \ y+t=0\}$

$x+y+z=0 \Rightarrow z=-x-y$

$U = \{(x,y,-x-y,t) \ x,y,t \in \mathbb{R}\} = <(1,0,-1,0),(0,1,-1,0),$
$(0,0,0,1)> \qquad \dim U = 3$

$B_U = \{(1,0,-1,0),(0,1,-1,0),(0,0,0,1)\}$

$x-2z=0 \Rightarrow x=2z \qquad y+t=0 \Rightarrow t=-y$

$V = \{(2z,y,z,-y) \ y,z \in \mathbb{R}\} = <(2,0,1,0),(0,1,0,-1)>$

$\dim V = 2 \qquad B_V = \{(2,0,1,0),(0,1,0,-1)\}$

$U \cap V = \{(x,y,-x-y,t) / x-2(-x-y)=0 \ , \ y+t=0\}$

$3x+2y=0 \qquad t+y=0 \qquad 2z = -\frac{2}{3}y$

$x = -\frac{2}{3}y \qquad t=-y \qquad \frac{\cancel{2}z}{\cancel{2}} = \frac{1}{\cancel{2}} \cdot -\frac{\cancel{2}}{3}y$

$z = -\frac{1}{3}y$

$U \cap V = \{(-\frac{2}{3}y, y, -\frac{1}{3}y, -y), y \in \mathbb{R}\}$

$U \cap V = <(-2,3,-1,-3)>$

$U+V = \mathbb{R}^4$

ESERCIZIO 2 Compito del 7-06-07

Discutere il seguente sistema lineare

$$\begin{cases} x+y+z=1 \\ kx-ky+z=0 \\ x-y+z=1 \end{cases}$$

al variare del parametro reale k e darne le eventuali soluzioni

$$\det A = \begin{vmatrix} 1 & 1 & 1 \\ k & -k & 1 \\ 1 & -1 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 1 \\ k-1 & -k & 1 \\ 0 & -1 & 1 \end{vmatrix} = 2(1-k)=0 \implies k=1$$

$\forall x \in \mathbb{R} - \{1\}$ sistema completo $\exists !$ soluzione

$$\begin{cases} x+y+z=1 \\ x-y+z=0 \\ x-y+z=1 \end{cases} \text{IMPOSSIBILE}$$

$$x = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 0 & -k & 1 \\ 1 & -1 & 1 \end{vmatrix}}{\det A} = \frac{\begin{matrix} 1 & 1 & \cancel{0} \\ \cancel{0} & \cancel{k} & \cancel{1} \\ 1 & -1 & \cancel{0} \end{matrix}}{2(1-k)} = \frac{2}{2(1-k)} = \frac{1}{1-k}$$

$$y = \frac{\begin{vmatrix} 1 & 1 & 1 \\ k & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}}{2(1-k)} = 0 \quad ;$$

$$z = \frac{\begin{vmatrix} 1 & 1 & 1 \\ k & -k & 0 \\ 1 & -1 & 1 \end{vmatrix}}{2(1-k)} = \frac{\begin{vmatrix} 1 & 2 & 1 \\ k & 0 & 0 \\ 1 & 0 & 1 \end{vmatrix}}{2(1-k)} = \frac{-2k}{2(1-k)} = \frac{k}{k-1}$$

$$\text{Solutione} = \left(\frac{1}{1-k}, 0, \frac{k}{k-1} \right)$$

$\boxed{\text{ESERCIZIO 3}}$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & -1 & 1 \end{pmatrix}$$

$\det A = 0$ poichè 2 colonne sono uguali

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 2-\lambda & 1 \\ 1 & -1 & 1-\lambda \end{vmatrix} = (2-\lambda)(1-\lambda)^2 \cancel{-1} \cancel{+1} - (2-1) \cancel{+1} \cancel{-\lambda} - \cancel{(1-\lambda)} = (2-\lambda)\left[(1-\lambda)^2 - 1\right] =$$

$$= (2-\lambda)\left[1-\lambda+1\right]\left[\cancel{1}-\lambda\cancel{-1}\right] = -\lambda(2-\lambda)^2 = 0$$

$\boxed{\lambda = 0}$ $m_a(0) = 1$

$\boxed{\lambda = 2}$ $m_a(2) = 2$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & -1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \overline{0}$$

$$\begin{cases} x + y + z = 0 \\ x - y + z = 0 \end{cases} \qquad 2(x+y) = 0 \qquad \begin{cases} x + z = 0 \\ y = 0 \end{cases} \qquad z = -x$$

$$V_{(0)} = \{(x, 0, -x),\ x \in \mathbb{R}\} = \langle (1, 0, -1) \rangle$$

$$A - 2I = \begin{pmatrix} -1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \overline{0}$$

$$\mathrm{Ker}(A - 2I) = \left\{(x, y, z) \in \mathbb{R}^3 \,/\, \begin{matrix} x + z = 0 \\ x - \cancel{y} - z = 0,\ x - y + \cancel{x} = 0 \end{matrix}\right\} =$$

$y = 2x$

$$= \{(x, 2x, -x),\ x \in \mathbb{R}\} \qquad m_g(2) = 1$$

$$V_{(2)} = \langle (1, 2, -1) \rangle$$

A non è diagonalizzabile

JORDAN

$$\begin{pmatrix} -1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & 1 \end{pmatrix}\begin{pmatrix} -1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & -1 \end{pmatrix} = \begin{pmatrix} 3 & -2 & -1 \\ 0 & 0 & 0 \\ -3 & 2 & 1 \end{pmatrix}$$

RIGHE × COLONNE →

Il rango di questa matrice è 1

$$\ker (A-2I)^2 = \{(x,y,z) \in \mathbb{R}^3 / 3x-2y-z=0\} =$$

$$= \{(x,y,3x-2y)\,;\, x,y \in \mathbb{R}\} = \underset{\substack{\uparrow \\ \text{è generato}}}{<}(1,0,3),(0,1,-2)>$$

$$\dim \ker (A-2I)^2 = 2$$

$V_1 = (1,0,-1)$ $\qquad\qquad$ $AV_1 = \overline{0}$

$(1,0,3)$

↑
VETTORE
COLONNA

$$(A-2I)\overset{V_3}{\begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}} = \overset{V_2}{\begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix}}$$

$$\underset{\substack{\| \\ (A-2I)^2\begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}}}{(A-2I)\begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix}} = \overline{0} = AV_2 - 2\begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix} =$$

$$AV_2 = 2V_2$$

$$(A-2I)V_3 = V_2 \qquad AV_3 = V_2 + 2V_3$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$$

ESERCIZIO 4

$P(1,0\,1)$

$r = \begin{cases} x = z+1 \\ y = z \end{cases}$ $\quad \vec{r}\,(1,1,1)$

$\cancel{\quad}$ $x-1+y+z-1=0$

$r = \begin{cases} x+y+z=2 \\ x = z+1 \\ y = z \end{cases}$ $\quad \left(\frac{4}{3}, \frac{1}{3}, \frac{1}{3}\right)$

$3z=1 \quad z=\frac{1}{3}$

$\mathrm{dist}(P,r) = \sqrt{\frac{1}{9}+\frac{1}{9}+\frac{4}{9}} = \frac{1}{3}\sqrt{6}$

ESERCIZIO 5

$r: \begin{cases} x=-y \\ z=2y+1 \end{cases}$ $\quad s: \begin{cases} x=1 \\ z=y+1 \end{cases}$

r e s sono sghembe

non esiste nessun punto in comune

non sono parallele

$\vec{r}(-1,1,2)$ $\quad \vec{s}(0,1,1)$

$R(-k,k,2k+1)$ $\quad y-k+(z-2k-1)=0$

$\begin{cases} y-z=3k+1 \\ (x-1)^2+y^2+(z-1)^2 = \\ =(1+k)^2+k^2+4k^2 \end{cases}$

$y-z=3k+1$ ← piano della circonferenza

$(x-1)^2+y^2+(z-1)^2=$ ← equazione sfera

$K=\frac{1}{3}(y+z-1)$ e si sostituisce alla equazione della sfera