Лабораторная работа №5

Решение интегральных уравнений Фредгольма

Необходимо решить интегральное уравнение Фредгольма
$x(t) +$
$\int_1^3 \left(\frac{t}{s^2} - 2\right) x$
$(s) ds = t^2 + \frac{t}{3}$
$- \frac{1}{3}$
квадратурным методом с тремя и десятью узлами, пользуясь:
●формулой трапеций
●формулой Гаусса

Используемые библиотеки

In [1]:

```
import matplotlib.pyplot as plt
from typing import Callable
import numpy as np
import math
```

Условия

In [2]:

```
low_lim, up_lim = 1, 3


def K(t: float, s: float) -> float:
    return t / s**2 - 2


def func(t: float) -> float:
    return t**2 + t/3 - 1/3
```

Функция отрисовки

In [3]:

```
def draw(t: list, u: list):
    fig = plt.figure()
    plt.title('Fredholm solution')
    plt.ylabel('x(t)')
    plt.xlabel('t')
    l1 = plt.plot(t, u)
    fig.legend((l1), ('x'))
    plt.grid(True)
    plt.show()
```

Метод трапеций

$\int_a^b f(x) dx$
$= h \left(\frac{f_0 + f_n}{2} +\right.$
$\left.\sum_{i=1}^{n-1} f_i\right)$

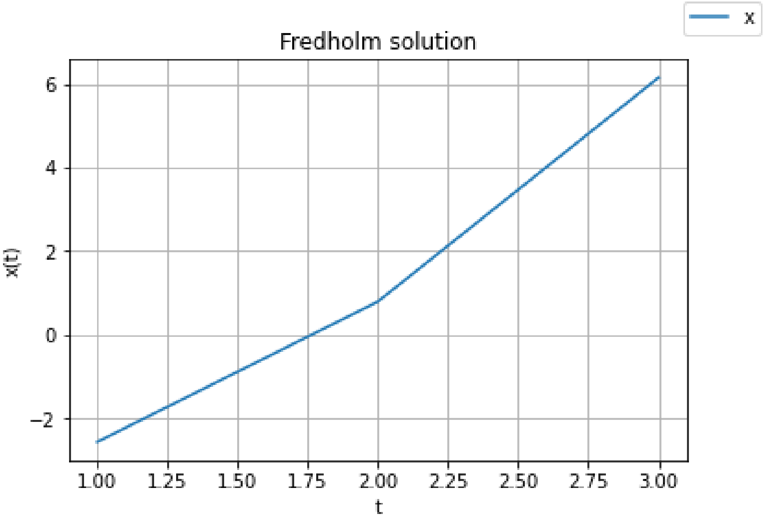
In [4]:

```
def trapez_method(K: Callable, func: Callable, node: int) -> list and list:
    x = np.linspace(low_lim, up_lim, node)
    size = len(x)
    wt, wj = 0.5, 1
    h = (up_lim - low_lim) / (node - 1)
    A = np.zeros((size, size))

    for i in range(0, size):
        A[i][0] = -h * wt * K(x[i], x[1])
        for j in range(2, size - 1):
            A[i][j] = -h * wj * K(x[i], x[j])
        A[i][size - 1] = -h * wt * K(x[i], x[size - 1])
        A[i][i] += 1

    B = np.zeros((size, 1))
    for j in range(0, size):
        B[j][0] = func(x[j])
    return np.linalg.solve(A, B), x
```

Результаты

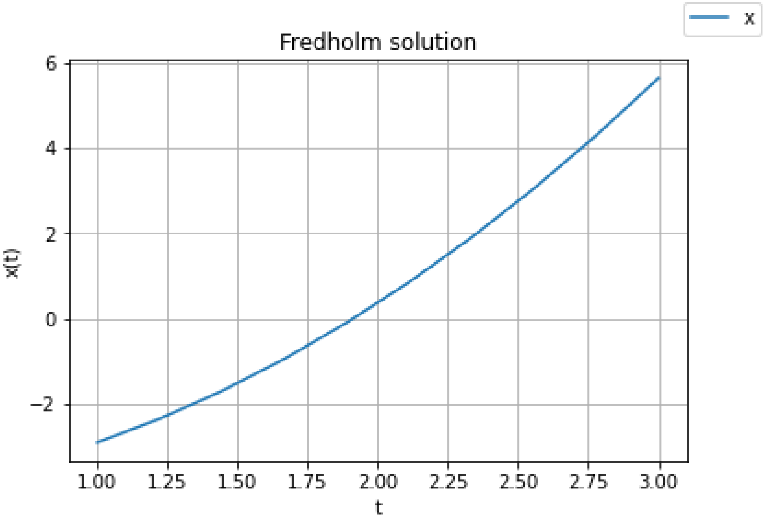
In [5]:

```
x, t = trapez_method(K=K, func=func, node=3) # для 3х узлов
draw(t, x)
```



In [6]:

```
x, t = trapez_method(K=K, func=func, node=10)  # для 10 узлов
draw(t, x)
```



Формула Гаусса

$\int_a^b f(x)dx$

$= \frac{b-a}{2}$

$\sum_{i=1}^n c_i f(s_i)$,

где s_i

$= \frac{a+b+(b-a)x_i}{2}$

,

где x_i — корни уравнения Лежандра

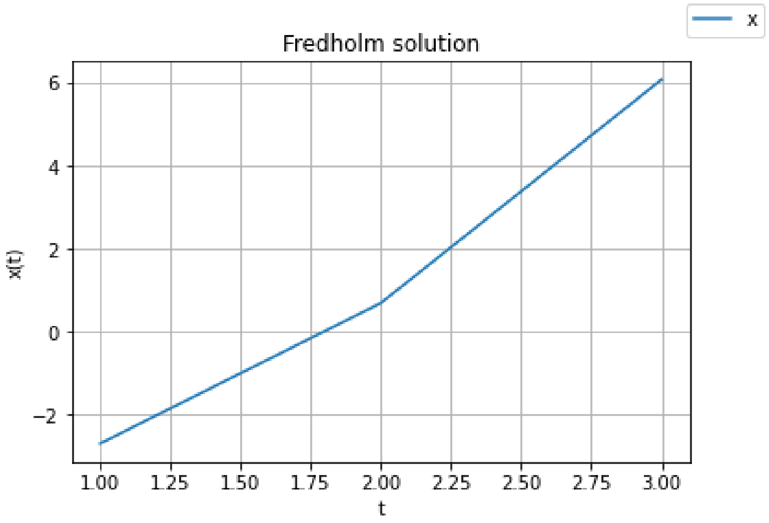
In [7]:

```
def gauss_3(K: Callable, func: Callable) -> list and list:
    x = np.linspace(low_lim, up_lim, 3)
    xt = [(low_lim + up_lim) / 2] * 3
    w = [- math.sqrt(3. / 5), 0, math.sqrt(3. / 5)]
    cj, ct = 5. / 9, 8. / 9
    h = (up_lim - low_lim) / 2

    for i in range(3):
        xt[i] += w[i] * h

    size = len(xt)
    A = np.zeros((size, size))

    for i in range(0, size):
        A[i][0] = -h * cj * K(xt[i], xt[1])
        for j in range(2, size - 1):
            A[i][j] = -h * ct * K(xt[i], xt[j])
        A[i][size - 1] = -h * cj * K(xt[i], xt[size - 1])
        A[i][i] += 1

    B = np.zeros((size, 1))
    for j in range(0, size):
        B[j][0] = func(x[j])
    return np.linalg.solve(A, B), x


def gauss_10(K: Callable, func: Callable) -> list and list:
    x = np.linspace(low_lim, up_lim, 10)
    xt = [(low_lim + up_lim) / 2] * 10
    w = [
        -0.973906528517172,
        -0.865063366688985,
        -0.679409568299024,
        -0.433395394129244,
        -0.148874338981631,
        +0.148874338981631,
        +0.433395394129244,
        +0.679409568299024,
        +0.865063366688985,
        +0.973906528517172
    ]

    c = [
        0.066671344308688,
        0.149451349159581,
        0.219086362515982,
        0.269266719309996,
        0.295524224714753,
        0.295524224714753,
        0.269266719309996,
        0.219086362515982,
        0.149451349159581,
        0.066671344308688
    ]

    h = (up_lim - low_lim) / 9
    for i in range(10):
        xt[i] += w[i] * (up_lim - low_lim) / 2

    size = len(xt)
    A = np.zeros((size, size))

    for i in range(0, size):
        for j in range(0, size):
            A[i][j] = -(up_lim - low_lim) / 2 * c[j] * K(xt[i], xt[j])
        A[i][i] += 1

    B = np.zeros((size, 1))
    for j in range(0, size):
        B[j][0] = func(x[j])
    return np.linalg.solve(A, B), x
```

Результаты

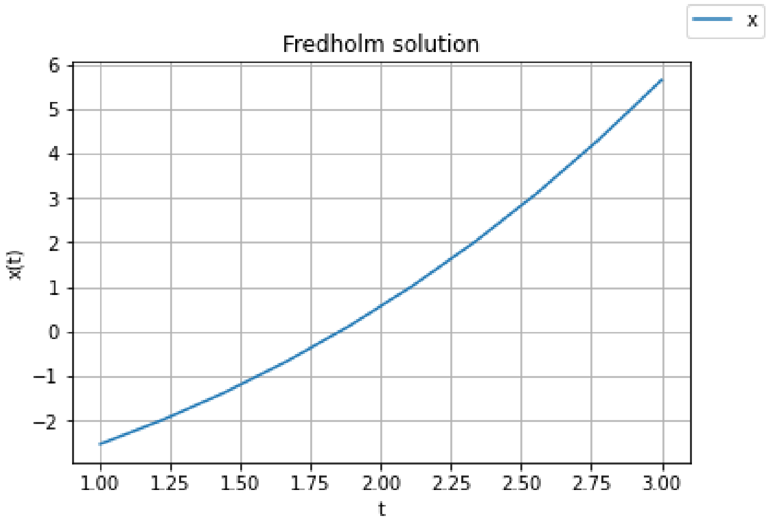
In [8]:

```
x, t = gauss_3(K=K, func=func) # для 3х узлов
draw(t, x)
```



In [9]:

```
x, t = gauss_10(K=K, func=func) # для 10 узлов
draw(t, x)
```


x
Fredholm solution
x(t)
t