

In[16]:=

In[17]:= **InverseMetric**[g_] := **Simplify**[**Inverse**[g]]

ChristoffelSymbol[g_, xx_] := **Block**[{n, ig, res}, n = 4;

ig = **InverseMetric**[g];
res = **Table**[(1/2) * **Sum**[**ig**[[i, s]] *
 (-**D**[g[[j, k]], xx[[s]]] + **D**[g[[j, s]], xx[[k]]] + **D**[g[[s, k]], xx[[j]]]),
 {s, 1, n}], {i, 1, n}, {j, 1, n}, {k, 1, n}];
Simplify[res]

RiemannTensor[g_, xx_] := **Block**[{n, Chr, res}, n = 4;

Chr = **ChristoffelSymbol**[g, xx];
res = **Table**[**D**[Chr[[i, k, m]], xx[[1]]] - **D**[Chr[[i, k, 1]], xx[[m]]] +
Sum[Chr[[i, s, 1]] * Chr[[s, k, m]], {s, 1, n}] - **Sum**[Chr[[i, s, m]] * Chr[[s, k, 1]],
 {s, 1, n}], {i, 1, n}, {k, 1, n}, {1, 1, n}, {m, 1, n}];
Simplify[res]

RicciTensor[g_, xx_] := **Block**[{Rie, res, n}, n = 4;

Rie = **RiemannTensor**[g, xx];
res = **Table**[**Sum**[Rie[[s, i, s, j]], {s, 1, n}], {i, 1, n}, {j, 1, n}];
Simplify[res]

RicciScalar[g_, xx_] := **Block**[{Ricc, ig, res, n}, n = 4;

Ricc = **RicciTensor**[g, xx];
ig = **InverseMetric**[g];
res = **Sum**[**ig**[[s, i]] * **Ricc**[[s, i]], {s, 1, n}, {i, 1, n}];
Simplify[res]

xx = {t, x, θ , ϕ };

g = {{-(c[t])^2, 0, 0, 0}, {0, (a[t])^2, 0, 0},
 {0, 0, (ρ b[t]/a[t])^2, 0}, {0, 0, 0, (ρ b[t] $\sin[\theta]$ /a[t])^2}};

RicciScalar[g, xx]

Out[24]=
$$2 \left(\frac{a[t]^2}{\rho^2 b[t]^2} + \frac{3 a'[t]^2}{a[t]^2 c[t]^2} + \frac{b[t] a'[t] c'[t] - c[t] (4 a'[t] b'[t] + b[t] a''[t])}{a[t] \times b[t] c[t]^3} + \frac{-2 b[t] b'[t] c'[t] + c[t] (b'[t]^2 + 2 b[t] b''[t])}{b[t]^2 c[t]^3} \right)$$

$$\text{In[25]:= } 2 \left(\frac{a[t]^2}{\rho^2 b[t]^2} + \frac{3 a'[t]^2}{a[t]^2 c[t]^2} + \frac{b[t] a'[t] c'[t] - c[t] (4 a'[t] b'[t] + b[t] a''[t])}{a[t] \times b[t] c[t]^3} + \frac{-2 b[t] b'[t] c'[t] + c[t] (b'[t]^2 + 2 b[t] b''[t])}{b[t]^2 c[t]^3} \right)$$

RiemannTensor[g, xx]

$$\text{Out[25]= } 2 \left(\frac{a[t]^2}{\rho^2 b[t]^2} + \frac{3 a'[t]^2}{a[t]^2 c[t]^2} + \frac{b[t] a'[t] c'[t] - c[t] (4 a'[t] b'[t] + b[t] a''[t])}{a[t] \times b[t] c[t]^3} + \frac{-2 b[t] b'[t] c'[t] + c[t] (b'[t]^2 + 2 b[t] b''[t])}{b[t]^2 c[t]^3} \right)$$

$$\begin{aligned} \text{Out[26]= } & \{ \{ \{ \{ 0, 0, 0, 0 \}, \{ 0, 0, 0, 0 \}, \{ 0, 0, 0, 0 \}, \{ 0, 0, 0, 0 \} \}, \\ & \{ \{ 0, \frac{a[t] (-a'[t] c'[t] + c[t] a''[t])}{c[t]^3}, 0, 0 \}, \\ & \{ \frac{a[t] (a'[t] c'[t] - c[t] a''[t])}{c[t]^3}, 0, 0, 0 \}, \{ 0, 0, 0, 0 \}, \{ 0, 0, 0, 0 \} \}, \\ & \{ \{ 0, 0, \frac{1}{a[t]^4 c[t]^3} \rho^2 b[t] (b[t] (a[t] a'[t] c'[t] + c[t] (2 a'[t]^2 - a[t] a''[t])) + \\ & a[t] (-a[t] b'[t] c'[t] + c[t] (-2 a'[t] b'[t] + a[t] b''[t])) \}, 0 \}, \{ 0, 0, 0, 0 \}, \\ & \{ \frac{1}{a[t]^4 c[t]^3} \rho^2 b[t] (b[t] (-a[t] a'[t] c'[t] + c[t] (-2 a'[t]^2 + a[t] a''[t])) + \\ & a[t] (a[t] b'[t] c'[t] + c[t] (2 a'[t] b'[t] - a[t] b''[t])) \}, 0, 0, 0 \}, \\ & \{ 0, 0, 0, 0 \} \}, \{ \{ 0, 0, 0, \frac{1}{a[t]^4 c[t]^3} \rho^2 b[t] \text{Sin}[\theta]^2 \\ & (b[t] (a[t] a'[t] c'[t] + c[t] (2 a'[t]^2 - a[t] a''[t])) + \\ & a[t] (-a[t] b'[t] c'[t] + c[t] (-2 a'[t] b'[t] + a[t] b''[t])) \}, \\ & \{ 0, 0, 0, 0 \}, \{ \frac{1}{a[t]^4 c[t]^3} \rho^2 b[t] \text{Sin}[\theta]^2 \\ & (b[t] (-a[t] a'[t] c'[t] + c[t] (-2 a'[t]^2 + a[t] a''[t])) + \\ & a[t] (a[t] b'[t] c'[t] + c[t] (2 a'[t] b'[t] - a[t] b''[t])) \}, 0, 0, 0 \} \}, \\ & \{ \{ \{ 0, \frac{-a'[t] c'[t] + c[t] a''[t]}{a[t] \times c[t]}, 0, 0 \}, \{ \frac{a'[t] c'[t] - c[t] a''[t]}{a[t] \times c[t]}, 0, 0, 0 \}, \\ & \{ 0, 0, 0, 0 \}, \{ 0, 0, 0, 0 \} \}, \\ & \{ \{ 0, 0, 0, 0 \}, \{ 0, 0, 0, 0 \}, \{ 0, 0, 0, 0 \}, \{ 0, 0, 0, 0 \} \}, \\ & \{ \{ 0, 0, 0, 0 \}, \{ 0, 0, \frac{\rho^2 b[t] a'[t] (-b[t] a'[t] + a[t] b'[t])}{a[t]^4 c[t]^2}, 0 \}, \\ & \{ 0, \frac{\rho^2 b[t] a'[t] (b[t] a'[t] - a[t] b'[t])}{a[t]^4 c[t]^2}, 0, 0 \}, \{ 0, 0, 0, 0 \} \}, \\ & \{ \{ 0, 0, 0, 0 \}, \{ 0, 0, 0, \frac{\rho^2 b[t] \text{Sin}[\theta]^2 a'[t] (-b[t] a'[t] + a[t] b'[t])}{a[t]^4 c[t]^2} \}, \\ & \{ 0, 0, 0, 0 \}, \{ 0, \frac{\rho^2 b[t] \text{Sin}[\theta]^2 a'[t] (b[t] a'[t] - a[t] b'[t])}{a[t]^4 c[t]^2}, 0, 0 \} \}, \\ & \{ \{ \{ 0, 0, \frac{1}{a[t]^2 b[t] \times c[t]} (b[t] (a[t] a'[t] c'[t] + c[t] (2 a'[t]^2 - a[t] a''[t])) + \\ & a[t] (-a[t] b'[t] c'[t] + c[t] (-2 a'[t] b'[t] + a[t] b''[t])) \}, 0 \}, \{ 0, 0, 0, 0 \}, \end{aligned}$$

$$\begin{aligned}
& \left\{ \frac{1}{a[t]^2 b[t] \times c[t]} \left(b[t] \left(-a[t] a'[t] c'[t] + c[t] \left(-2 a'[t]^2 + a[t] a''[t] \right) \right) + \right. \right. \\
& \quad \left. \left. a[t] \left(a[t] b'[t] c'[t] + c[t] \left(2 a'[t] b'[t] - a[t] b''[t] \right) \right) \right), \{0, 0, 0\}, \{0, 0, 0, 0\} \right\}, \\
& \left\{ \{0, 0, 0, 0\}, \left\{ 0, 0, \frac{a'[t] \left(b[t] a'[t] - a[t] b'[t] \right)}{b[t] c[t]^2}, 0 \right\} \right\}, \\
& \left\{ 0, \frac{a'[t] \left(-b[t] a'[t] + a[t] b'[t] \right)}{b[t] c[t]^2}, 0, 0 \right\}, \{0, 0, 0, 0\} \right\}, \\
& \{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \}, \\
& \{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \\
& \quad \{0, 0, 0, \frac{\text{Sin}[\theta]^2 \left(a[t]^4 c[t]^2 + \rho^2 \left(b[t] a'[t] - a[t] b'[t] \right)^2 \right)}{a[t]^4 c[t]^2} \}, \\
& \quad \{0, 0, -\frac{\text{Sin}[\theta]^2 \left(a[t]^4 c[t]^2 + \rho^2 \left(b[t] a'[t] - a[t] b'[t] \right)^2 \right)}{a[t]^4 c[t]^2}, 0 \} \} \}, \\
& \left\{ \left\{ \left\{ 0, 0, 0, \frac{1}{a[t]^2 b[t] \times c[t]} \left(b[t] \left(a[t] a'[t] c'[t] + c[t] \left(2 a'[t]^2 - a[t] a''[t] \right) \right) + \right. \right. \right. \right. \\
& \quad \left. \left. \left. a[t] \left(-a[t] b'[t] c'[t] + c[t] \left(-2 a'[t] b'[t] + a[t] b''[t] \right) \right) \right) \right\}, \{0, 0, 0, 0\}, \{0, 0, \right. \right. \\
& \quad \left. \left. 0, 0 \right\}, \left\{ \frac{1}{a[t]^2 b[t] \times c[t]} \left(b[t] \left(-a[t] a'[t] c'[t] + c[t] \left(-2 a'[t]^2 + a[t] a''[t] \right) \right) + \right. \right. \right. \right. \\
& \quad \left. \left. \left. a[t] \left(a[t] b'[t] c'[t] + c[t] \left(2 a'[t] b'[t] - a[t] b''[t] \right) \right) \right) \right\}, \{0, 0, 0\} \right\}, \\
& \left\{ \{0, 0, 0, 0\}, \left\{ 0, 0, 0, \frac{a'[t] \left(b[t] a'[t] - a[t] b'[t] \right)}{b[t] c[t]^2} \right\}, \{0, 0, 0, 0\} \right\}, \\
& \left\{ 0, \frac{a'[t] \left(-b[t] a'[t] + a[t] b'[t] \right)}{b[t] c[t]^2}, 0, 0 \right\}, \{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \\
& \quad \{0, 0, 0, \frac{a[t]^4 c[t]^2 \text{Cot}[\theta]^2 - a[t]^4 c[t]^2 \text{Csc}[\theta]^2 - \rho^2 \left(b[t] a'[t] - a[t] b'[t] \right)^2}{a[t]^4 c[t]^2} \}, \\
& \quad \{0, 0, \frac{a[t]^4 c[t]^2 + \rho^2 b[t]^2 a'[t]^2 - 2 \rho^2 a[t] \times b[t] a'[t] b'[t] + \rho^2 a[t]^2 b'[t]^2}{a[t]^4 c[t]^2}, 0 \} \}, \\
& \{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \} \} \\
\text{In[27]:= } & \left\{ \{ \{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \}, \right. \\
& \left\{ 0, \frac{a[t] \left(-a'[t] c'[t] + c[t] a''[t] \right)}{c[t]^3}, 0, 0 \right\}, \\
& \left\{ \frac{a[t] \left(a'[t] c'[t] - c[t] a''[t] \right)}{c[t]^3}, 0, 0, 0 \right\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \}, \\
& \left\{ \{0, 0, \frac{1}{a[t]^4 c[t]^3} \rho^2 b[t] \left(b[t] \left(a[t] a'[t] c'[t] + c[t] \left(2 a'[t]^2 - a[t] a''[t] \right) \right) + \right. \right. \right. \\
& \quad \left. \left. \left. a[t] \left(-a[t] b'[t] c'[t] + c[t] \left(-2 a'[t] b'[t] + a[t] b''[t] \right) \right) \right) \right\}, \{0, 0, 0, 0\}, \right. \\
& \left\{ \frac{1}{a[t]^4 c[t]^3} \rho^2 b[t] \left(b[t] \left(-a[t] a'[t] c'[t] + c[t] \left(-2 a'[t]^2 + a[t] a''[t] \right) \right) + \right. \right. \\
& \quad \left. \left. a[t] \left(a[t] b'[t] c'[t] + c[t] \left(2 a'[t] b'[t] - a[t] b''[t] \right) \right) \right) \right\}, 0, 0, 0 \}, \\
& \{0, 0, 0, 0\} \}, \left\{ \left\{ 0, 0, 0, \frac{1}{a[t]^4 c[t]^3} \rho^2 b[t] \text{Sin}[\theta]^2 \right. \right. \\
& \quad \left. \left. \left(b[t] \left(a[t] a'[t] c'[t] + c[t] \left(2 a'[t]^2 - a[t] a''[t] \right) \right) + \right. \right. \right. \\
& \quad \left. \left. \left. a[t] \left(-a[t] b'[t] c'[t] + c[t] \left(-2 a'[t] b'[t] + a[t] b''[t] \right) \right) \right) \right\}, \right. \\
& \left\{ 0, 0, 0, 0 \right\}, \{0, 0, 0, 0\}, \left\{ \frac{1}{a[t]^4 c[t]^3} \rho^2 b[t] \text{Sin}[\theta]^2 \right. \\
& \quad \left. \left. \left(b[t] \left(a[t] a'[t] c'[t] + c[t] \left(2 a'[t]^2 - a[t] a''[t] \right) \right) + \right. \right. \right. \\
& \quad \left. \left. \left. a[t] \left(-a[t] b'[t] c'[t] + c[t] \left(-2 a'[t] b'[t] + a[t] b''[t] \right) \right) \right) \right\}, \right. \\
& \left\{ 0, 0, 0, 0 \right\}, \{0, 0, 0, 0\} \} \}
\end{aligned}$$

$$\begin{aligned}
& \left(b[t] \left(-a[t] a'[t] c'[t] + c[t] \left(-2 a'[t]^2 + a[t] a''[t] \right) \right) + \right. \\
& \quad \left. a[t] \left(a[t] b'[t] c'[t] + c[t] \left(2 a'[t] b'[t] - a[t] b''[t] \right) \right) \right), \{0, 0, 0\}\}, \\
& \left\{ \left\{ \left\{ 0, \frac{-a'[t] c'[t] + c[t] a''[t]}{a[t] \times c[t]}, 0, 0 \right\}, \left\{ \frac{a'[t] c'[t] - c[t] a''[t]}{a[t] \times c[t]}, 0, 0, 0 \right\}, \right. \right. \\
& \quad \left. \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\}, \\
& \left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\}, \\
& \left\{ \{0, 0, 0, 0\}, \left\{ 0, 0, \frac{\rho^2 b[t] a'[t] \left(-b[t] a'[t] + a[t] b'[t] \right)}{a[t]^4 c[t]^2}, 0 \right\}, \right. \\
& \quad \left. \left\{ 0, \frac{\rho^2 b[t] a'[t] \left(b[t] a'[t] - a[t] b'[t] \right)}{a[t]^4 c[t]^2}, 0, 0 \right\}, \{0, 0, 0, 0\} \right\}, \\
& \left\{ \{0, 0, 0, 0\}, \left\{ 0, 0, 0, \frac{\rho^2 b[t] \text{Sin}[\theta]^2 a'[t] \left(-b[t] a'[t] + a[t] b'[t] \right)}{a[t]^4 c[t]^2} \right\}, \right. \\
& \quad \left. \{0, 0, 0, 0\}, \left\{ 0, \frac{\rho^2 b[t] \text{Sin}[\theta]^2 a'[t] \left(b[t] a'[t] - a[t] b'[t] \right)}{a[t]^4 c[t]^2}, 0, 0 \right\} \right\}, \\
& \left\{ \left\{ \left\{ 0, 0, \frac{1}{a[t]^2 b[t] \times c[t]} \left(b[t] \left(a[t] a'[t] c'[t] + c[t] \left(2 a'[t]^2 - a[t] a''[t] \right) \right) + \right. \right. \right. \right. \\
& \quad \left. \left. a[t] \left(-a[t] b'[t] c'[t] + c[t] \left(-2 a'[t] b'[t] + a[t] b''[t] \right) \right) \right), 0 \right\}, \{0, 0, 0, 0\}, \right. \\
& \quad \left. \left\{ \frac{1}{a[t]^2 b[t] \times c[t]} \left(b[t] \left(-a[t] a'[t] c'[t] + c[t] \left(-2 a'[t]^2 + a[t] a''[t] \right) \right) + \right. \right. \right. \\
& \quad \left. \left. a[t] \left(a[t] b'[t] c'[t] + c[t] \left(2 a'[t] b'[t] - a[t] b''[t] \right) \right) \right), 0, 0, 0 \right\}, \{0, 0, 0, 0\} \right\}, \\
& \left\{ \{0, 0, 0, 0\}, \left\{ 0, 0, \frac{a'[t] \left(b[t] a'[t] - a[t] b'[t] \right)}{b[t] c[t]^2}, 0 \right\}, \right. \\
& \quad \left. \left\{ 0, \frac{a'[t] \left(-b[t] a'[t] + a[t] b'[t] \right)}{b[t] c[t]^2}, 0, 0 \right\}, \{0, 0, 0, 0\} \right\}, \\
& \left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\}, \\
& \left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \right. \\
& \quad \left. \left\{ 0, 0, 0, \frac{\text{Sin}[\theta]^2 \left(a[t]^4 c[t]^2 + \rho^2 \left(b[t] a'[t] - a[t] b'[t] \right)^2 \right)}{a[t]^4 c[t]^2} \right\}, \right. \\
& \quad \left. \left\{ 0, 0, -\frac{\text{Sin}[\theta]^2 \left(a[t]^4 c[t]^2 + \rho^2 \left(b[t] a'[t] - a[t] b'[t] \right)^2 \right)}{a[t]^4 c[t]^2}, 0 \right\} \right\}, \\
& \left\{ \left\{ \left\{ 0, 0, 0, \frac{1}{a[t]^2 b[t] \times c[t]} \left(b[t] \left(a[t] a'[t] c'[t] + c[t] \left(2 a'[t]^2 - a[t] a''[t] \right) \right) + \right. \right. \right. \right. \right. \\
& \quad \left. \left. a[t] \left(-a[t] b'[t] c'[t] + c[t] \left(-2 a'[t] b'[t] + a[t] b''[t] \right) \right) \right), \{0, 0, 0, 0\}, \{0, 0, \right. \\
& \quad \left. 0, 0\}, \left\{ \frac{1}{a[t]^2 b[t] \times c[t]} \left(b[t] \left(-a[t] a'[t] c'[t] + c[t] \left(-2 a'[t]^2 + a[t] a''[t] \right) \right) + \right. \right. \right. \\
& \quad \left. \left. a[t] \left(a[t] b'[t] c'[t] + c[t] \left(2 a'[t] b'[t] - a[t] b''[t] \right) \right) \right), 0, 0, 0 \right\}, \right. \\
& \quad \left. \left\{ \{0, 0, 0, 0\}, \left\{ 0, 0, 0, \frac{a'[t] \left(b[t] a'[t] - a[t] b'[t] \right)}{b[t] c[t]^2} \right\}, \{0, 0, 0, 0\}, \right. \right. \\
& \quad \left. \left\{ 0, \frac{a'[t] \left(-b[t] a'[t] + a[t] b'[t] \right)}{b[t] c[t]^2}, 0, 0 \right\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \right. \\
& \quad \left. \left\{ 0, 0, 0, \frac{a[t]^4 c[t]^2 \text{Cot}[\theta]^2 - a[t]^4 c[t]^2 \text{Csc}[\theta]^2 - \rho^2 \left(b[t] a'[t] - a[t] b'[t] \right)^2}{a[t]^4 c[t]^2} \right\}, \right. \\
& \quad \left. \left\{ 0, 0, \frac{a[t]^4 c[t]^2 + \rho^2 b[t]^2 a'[t]^2 - 2 \rho^2 a[t] \times b[t] a'[t] b'[t] + \rho^2 a[t]^2 b'[t]^2}{a[t]^4 c[t]^2}, 0 \right\}, \right. \\
& \quad \left. \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\}
\end{aligned}$$

RicciTensor[g, xx]

Out[27]=
$$\left\{ \left\{ \left\{ \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\}, \right. \right.$$

$$\left\{ \left\{ 0, \frac{a[t] (-a'[t] c'[t] + c[t] a''[t])}{c[t]^3}, 0, 0 \right\}, \right.$$

$$\left\{ \frac{a[t] (a'[t] c'[t] - c[t] a''[t])}{c[t]^3}, 0, 0, 0 \right\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\},$$

$$\left\{ \left\{ 0, 0, \frac{1}{a[t]^4 c[t]^3} \rho^2 b[t] (b[t] (a[t] a'[t] c'[t] + c[t] (2 a'[t]^2 - a[t] a''[t])) + \right. \right.$$

$$\left. a[t] (-a[t] b'[t] c'[t] + c[t] (-2 a'[t] b'[t] + a[t] b''[t])) \right\}, 0 \right\}, \{0, 0, 0, 0\},$$

$$\left\{ \frac{1}{a[t]^4 c[t]^3} \rho^2 b[t] (b[t] (-a[t] a'[t] c'[t] + c[t] (-2 a'[t]^2 + a[t] a''[t])) + \right.$$

$$\left. a[t] (a[t] b'[t] c'[t] + c[t] (2 a'[t] b'[t] - a[t] b''[t])) \right\}, 0, 0, 0 \right\},$$

$$\{0, 0, 0, 0\} \right\}, \left\{ \left\{ 0, 0, 0, \frac{1}{a[t]^4 c[t]^3} \rho^2 b[t] \sin[\theta]^2 \right. \right.$$

$$\left. (b[t] (a[t] a'[t] c'[t] + c[t] (2 a'[t]^2 - a[t] a''[t])) + \right.$$

$$\left. a[t] (-a[t] b'[t] c'[t] + c[t] (-2 a'[t] b'[t] + a[t] b''[t])) \right\},$$

$$\{0, 0, 0, 0\} \right\}, \left\{ \frac{1}{a[t]^4 c[t]^3} \rho^2 b[t] \sin[\theta]^2 \right.$$

$$\left. (b[t] (-a[t] a'[t] c'[t] + c[t] (-2 a'[t]^2 + a[t] a''[t])) + \right.$$

$$\left. a[t] (a[t] b'[t] c'[t] + c[t] (2 a'[t] b'[t] - a[t] b''[t])) \right\}, 0, 0, 0 \right\} \right\},$$

$$\left\{ \left\{ \left\{ 0, \frac{-a'[t] c'[t] + c[t] a''[t]}{a[t] \times c[t]}, 0, 0 \right\}, \left\{ \frac{a'[t] c'[t] - c[t] a''[t]}{a[t] \times c[t]}, 0, 0, 0 \right\}, \right. \right.$$

$$\left. \{0, 0, 0, 0\}, \{0, 0, 0, 0\} \right\},$$

$$\left\{ \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\} \right\},$$

$$\left\{ \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{\rho^2 b[t] a'[t] (-b[t] a'[t] + a[t] b'[t])}{a[t]^4 c[t]^2}, 0 \right\}, \right.$$

$$\left. \left\{ 0, \frac{\rho^2 b[t] a'[t] (b[t] a'[t] - a[t] b'[t])}{a[t]^4 c[t]^2}, 0, 0 \right\}, \{0, 0, 0, 0\} \right\},$$

$$\left\{ \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, \frac{\rho^2 b[t] \sin[\theta]^2 a'[t] (-b[t] a'[t] + a[t] b'[t])}{a[t]^4 c[t]^2} \right\}, \right.$$

$$\left. \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, \frac{\rho^2 b[t] \sin[\theta]^2 a'[t] (b[t] a'[t] - a[t] b'[t])}{a[t]^4 c[t]^2}, 0, 0 \right\} \right\},$$

$$\left\{ \left\{ \left\{ 0, 0, \frac{1}{a[t]^2 b[t] \times c[t]} (b[t] (a[t] a'[t] c'[t] + c[t] (2 a'[t]^2 - a[t] a''[t])) + \right. \right. \right.$$

$$\left. a[t] (-a[t] b'[t] c'[t] + c[t] (-2 a'[t] b'[t] + a[t] b''[t])) \right\}, 0 \right\}, \{0, 0, 0, 0\},$$

$$\left\{ \frac{1}{a[t]^2 b[t] \times c[t]} (b[t] (-a[t] a'[t] c'[t] + c[t] (-2 a'[t]^2 + a[t] a''[t])) + \right.$$

$$\left. a[t] (a[t] b'[t] c'[t] + c[t] (2 a'[t] b'[t] - a[t] b''[t])) \right\}, 0, 0, 0 \right\}, \{0, 0, 0, 0\} \right\},$$

$$\left\{ \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, \frac{a'[t] (b[t] a'[t] - a[t] b'[t])}{b[t] c[t]^2}, 0 \right\}, \right.$$

$$\left. \left\{ 0, \frac{a'[t] (-b[t] a'[t] + a[t] b'[t])}{b[t] c[t]^2}, 0, 0 \right\}, \{0, 0, 0, 0\} \right\},$$

$$\left\{ \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\} \right\},$$

$$\left\{ \left\{ 0, 0, 0, 0 \right\}, \left\{ 0, 0, 0, 0 \right\}, \right.$$

$$\left. \left\{ 0, 0, 0, \frac{\sin[\theta]^2 (a[t]^4 c[t]^2 + \rho^2 (b[t] a'[t] - a[t] b'[t])^2)}{a[t]^4 c[t]^2} \right\} \right\},$$

$$\begin{aligned} & \left\{ \emptyset, \emptyset, -\frac{\text{Sin}[\theta]^2 \left(a[t]^4 c[t]^2 + \rho^2 \left(b[t] a'[t] - a[t] b'[t] \right)^2 \right)}{a[t]^4 c[t]^2}, \emptyset \right\} \}, \\ & \left\{ \left\{ \emptyset, \emptyset, \emptyset, \frac{1}{a[t]^2 b[t] \times c[t]} \left(b[t] \left(a[t] a'[t] c'[t] + c[t] \left(2 a'[t]^2 - a[t] a''[t] \right) \right) + \right. \right. \right. \\ & \quad \left. \left. a[t] \left(-a[t] b'[t] c'[t] + c[t] \left(-2 a'[t] b'[t] + a[t] b''[t] \right) \right) \right) \right\}, \{ \emptyset, \emptyset, \emptyset, \emptyset \}, \{ \emptyset, \emptyset, \right. \\ & \quad \left. \emptyset, \emptyset \right\}, \left\{ \frac{1}{a[t]^2 b[t] \times c[t]} \left(b[t] \left(-a[t] a'[t] c'[t] + c[t] \left(-2 a'[t]^2 + a[t] a''[t] \right) \right) + \right. \right. \\ & \quad \left. \left. a[t] \left(a[t] b'[t] c'[t] + c[t] \left(2 a'[t] b'[t] - a[t] b''[t] \right) \right) \right) \right\}, \emptyset, \emptyset, \emptyset \right\}, \\ & \left\{ \{ \emptyset, \emptyset, \emptyset, \emptyset \}, \{ \emptyset, \emptyset, \emptyset, \frac{a'[t] \left(b[t] a'[t] - a[t] b'[t] \right)}{b[t] c[t]^2} \}, \{ \emptyset, \emptyset, \emptyset, \emptyset \}, \right. \\ & \quad \left\{ \emptyset, \frac{a'[t] \left(-b[t] a'[t] + a[t] b'[t] \right)}{b[t] c[t]^2}, \emptyset, \emptyset \right\}, \{ \{ \emptyset, \emptyset, \emptyset, \emptyset \}, \{ \emptyset, \emptyset, \emptyset, \emptyset \}, \\ & \quad \{ \emptyset, \emptyset, \emptyset, \frac{a[t]^4 c[t]^2 \text{Cot}[\theta]^2 - a[t]^4 c[t]^2 \text{Csc}[\theta]^2 - \rho^2 \left(b[t] a'[t] - a[t] b'[t] \right)^2}{a[t]^4 c[t]^2} \}, \\ & \quad \{ \emptyset, \emptyset, \frac{a[t]^4 c[t]^2 + \rho^2 b[t]^2 a'[t]^2 - 2 \rho^2 a[t] \times b[t] a'[t] b'[t] + \rho^2 a[t]^2 b'[t]^2}{a[t]^4 c[t]^2}, \emptyset \} \}, \\ & \left. \{ \{ \emptyset, \emptyset, \emptyset, \emptyset \}, \{ \emptyset, \emptyset, \emptyset, \emptyset \}, \{ \emptyset, \emptyset, \emptyset, \emptyset \}, \{ \emptyset, \emptyset, \emptyset, \emptyset \} \} \right\} \\ \text{Out[28]=} & \left\{ \left\{ \frac{1}{a[t]^2 b[t] \times c[t]} \left(b[t] \left(-a[t] a'[t] c'[t] + c[t] \left(-4 a'[t]^2 + a[t] a''[t] \right) \right) + \right. \right. \right. \\ & \quad \left. \left. 2 a[t] \left(a[t] b'[t] c'[t] + c[t] \left(2 a'[t] b'[t] - a[t] b''[t] \right) \right) \right) \right\}, \emptyset, \emptyset, \emptyset \right\}, \\ & \left\{ \emptyset, \frac{-a[t] a'[t] c'[t] + c[t] \left(-2 a'[t]^2 + \frac{2 a[t] a'[t] b'[t]}{b[t]} + a[t] a''[t] \right)}{c[t]^3}, \emptyset, \emptyset \right\}, \\ & \left\{ \emptyset, \emptyset, \frac{1}{a[t]^4 c[t]^3} \left(a[t]^4 c[t]^3 + 2 \rho^2 b[t]^2 c[t] a'[t]^2 + \right. \right. \\ & \quad \left. \left. \rho^2 a[t] \times b[t] \left(b[t] a'[t] c'[t] - c[t] \left(3 a'[t] b'[t] + b[t] a''[t] \right) \right) + \right. \right. \\ & \quad \left. \left. \rho^2 a[t]^2 \left(-b[t] b'[t] c'[t] + c[t] \left(b'[t]^2 + b[t] b''[t] \right) \right) \right) \right\}, \emptyset \right\}, \\ & \left\{ \emptyset, \emptyset, \emptyset, \frac{1}{a[t]^4 c[t]^3} \text{Sin}[\theta]^2 \left(a[t]^4 c[t]^3 + 2 \rho^2 b[t]^2 c[t] a'[t]^2 + \right. \right. \\ & \quad \left. \left. \rho^2 a[t] \times b[t] \left(b[t] a'[t] c'[t] - c[t] \left(3 a'[t] b'[t] + b[t] a''[t] \right) \right) + \right. \right. \\ & \quad \left. \left. \rho^2 a[t]^2 \left(-b[t] b'[t] c'[t] + c[t] \left(b'[t]^2 + b[t] b''[t] \right) \right) \right) \right\} \right\} \end{aligned}$$

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In[29]:=
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In[30]:=
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In[31]:=

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In[32]:=
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