IB 120/201 - Lab 2

Discrete Time Modeling

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Background

Yeast Model

In this example you will model the logistic growth of yeast. Below is data from an experiment in which brewer's yeast (*S. cerevisiae*) was grown over a 47 hour period. The population of yeast for this study was measured as a volume.

Time	0	1.5	9	10	18	23	25.5	27	34	38	42	45.5	47
Population	0.37	1.63	6.2	8.87	10.66	12.5	12.6	12.9	13.27	12.77	12.87	12.9	12.7

You may recall that the general equation for logistic growth (in discrete time) is:

$$N(t+1) = N(t) + rN(t)\left(1 - \frac{N(t)}{K}\right) \tag{1}$$

This can also be written as:

$$\frac{dN}{dt} = rN(t)\left(1 - \frac{N(t)}{K}\right) \tag{2}$$

Assignment

For this assignment, you'll need to use the JupyterHub python notebook 'hw2.ipynb' in the folder 'code' within the 'assignments' folder. Open it, and when instructed to, edit and add code. When you're done editing the notebook to answer the questions below, download the notebook onto your local computer with the 'Notebook (.ipynb)' format, and upload the file to bCourses. Refer to the Python notebook for more detailed instructions to answer the questions.

1. Plot the yeast data given in the table.
2. What is the approximate carrying capacity of yeast? (Eyeball it)
3. The next few cells in the python notebook will guide you through a different numerical approach to solving logistic growth to further your understanding of using differential equations as models.
4. Write a function for the differential equation (2). Through trial and error, fit a logistic model of growth to the data presented in the table. To do this use the carrying capacity you approximated from part 2, the initial condition from the data and try the following reproductive factors ($r = 1.5, r = 2.0, r = 0.4, r = 0.7$). Which reproductive factor fits the data best?
5. Suppose you have decided to go into the yeast selling business. Modify the logistic growth equation to include a constant daily harvest amount, denote this amount h.

6. Create a function to find the new equilibrium points for this new model. (Hint: You will have to use the quadratic equation, $N = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, to find the equilibrium points.) Have the function take in h, K, and r, as arguments, so that it can output a *list* of both equilibrium population values for any combination of the three parameters.

7. Set $h = 1$. Starting at the carrying capacity you found in part 2. (i.e. set the initial population size to K) simulate 100 days of yeast growth. Make sure to plot your results to help visualize what is happening. Do the equilibria you notice in the graph match the equilibria you calculate using your function from part 6? Is this a sustainable harvest amount?

8. If you found $h = 1$ to be unsustainable in the long term, change the harvest amount so that it becomes sustainable.