Utilization Of Algorithm,Dynamic Programming And Optimization

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1 Introduction

During last 20 years, the drive systems of trains had met an epochal innovation. Three-phase induction machines employing digital controlled variable-voltage-variable-frequency (VVVF) inverter have been used instead of conventional DC machines. This technical progress enables regenerative brake and pure electric brake. It will also be able to realize driverless operation with communication facilities.

Furthermore, use of fully digital-equipped control centers, digital drive control schemes on every train and communication systems between these components are indispensable for advanced intelligent railway transport systems in next generation. Such advanced systems enable real-time optimal operation of train group in practical use.

Energy conservation of advanced railway systems in next generation should be realized with intelligent train control systems. In recent years, social demands for global environmental protection like reduction of greenhouse effect gases and prevention of further water pollution are growing. One of the most important assignments is reduction of total energy consumption in many kinds of environmental problems. In transport sector, there is no doubt that railway system as a public transportation should play more principal role with its high transport capacity and efficiency in the near future. We should, therefore, make every possible effort to save amount of total energy consumption in railway systems.

Finding the optimal “energy-saving” train run-curve is one of the effective methods of energy conservation. The optimization is made by changing the speed-position profile with keeping the same running time.

Analysis of an optimal “energy-saving” train run-curve is one of difficult research topics of train operation. There were many studies on optimal train run-curve which assumed use of non-regenerative electric and mechanical brake in the past. However, this theory cannot be directly extended to a regenerative brake train, mainly because of regenerative power and dependency of regenerative ability on catenary voltage.

In this paper, the authors formulate optimal train control problem considering kinetic equation of a train employing electric motive force and regenerative brake blended with mechanical brake without considering feeder circuit. This optimal problem is much simpler than actual complicated one[1]. However, the problem formulation includes non-linear characteristic of motive/brake torque and complicated train profile like several local speed limit, local inclines. Many previous works on optimal control problems adopt the numerical techniques of calculus of variations, Pontryagin’s maximum principle and so on. These methods often meet some difficulties accounting for complicated actual train running condition. Bellman’s Dynamic Programming (DP) has a substantial advantage at these points, which can directly deal with such difficult constraints of optimal control problem, except terminal boundary condition[2].

2 Mathematical formulation

Let us define the following notation.

m, L, x_f	total mass and length of train, total running distance
$t, T, x(t), v(t)$	current time, trip time, train position and velocity
$u(t)$	control input $u \in [1\ 4]$: acceleration $u = 5$: coasting $u \in [6\ 10]$: deceleration
$p(t), J$	input/output power and total consumed energy of train
$\xi(u)$	motor/generator efficiency
$f(u,v)$	acceleration/deceleration with motor/generator
$r(x, v)$	deceleration with running and incline resistance
$C(x,v)$	objective function about speed constraints depend on train position

The following differential equations eqns (1), (2), (3) describe state equation of train motion and consumed energy.

$$\frac{dx}{dv} = v \tag{1}$$

$$\frac{dv}{dt} = f(u,v) - r(x,v) \tag{2}$$

$$\begin{aligned} \frac{dJ}{dt} &= \xi(u)\, m\, f(u,v)\, v, \\ &= p(t). \end{aligned} \tag{3}$$

Then optimal energy-saving control problem is formulated as follows.

$$\begin{aligned} &\min_u J \\ &subj\ to.\ eqns\ (1), (2), (3) \\ &\qquad x(0) = 0,\ v(0) = 0,\ x(T) = x_f,\ v(T) = 0 \\ &\qquad C(x,v) \le 0, \quad 1 \le u \le 10. \end{aligned} \tag{4}$$

3 Bellman's dynamic programming (DP) and improvements in numerical algorithm

Basic DP algorithm for optimal control problem[3]

When applying DP algorithm to an optimal control problem, it is necessary to transform original problem into multistage decision process. Generally, this conversion is accomplished by linearization and time-uniform discretization. It is also indispensable to divide objective state space into lattice. We can obtain discretized linearized state equations eqns (5), (6) using first-order Taylor expansion and trapezoidal rule for approximation of integral.

$$\psi_k = \left(\mathbf{I} - \frac{\mathbf{A}\Delta t}{2}\right)^{-1}\left(\mathbf{I} + \frac{\mathbf{A}\Delta t}{2}\right)\psi_{k-1} + \left(\mathbf{I} - \frac{\mathbf{A}\Delta t}{2}\right)^{-1}\mathbf{B}f_0\Delta t \tag{5}$$

$$J_k = J_{k-1} + \xi(u)m\frac{\Delta t}{2}\left(f(u,v_k)v_k + f(u,v_{k-1})v_{k-1}\right) \tag{6}$$

where $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ \alpha & \beta \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \psi = \begin{pmatrix} x \\ v \end{pmatrix}$

$f_0 = f(u_{k-1}, v_{k-1}) - r(x_{k-1}, v_{k-1}) - \alpha v_{k-1} + \beta x_{k-1}$

$\alpha = f_v(u_{k-1}, v_{k-1}) - r_v(x_{k-1}, v_{k-1})$

$\beta = -r_x(x_{k-1}, v_{k-1})$

$\Delta t = T / N$.

N : Number of division

In addition, DP needs to transform terminal boundary conditions to penalty function eqn (7).

$$\phi(x(T), v(T)) = \lambda_1(x(T) - L)^2 + \lambda_2(v(T))^2 \tag{7}$$

Thus, optimal problem eqn (4) is approximately converted into the following N-stage decision process eqn (8).

$$\begin{aligned} &\min_{\{u_k\}_{k=1}^N}\{J + \phi(\psi_N)\} \\ &subj\ to.\ (5),\ C(\psi_k) \le 0,\ 1 \le u_k \le 10. \end{aligned} \tag{8}$$

Finally, DP process can be executed with a digital computer as the following steps.

(a) k=N.

(b) determine optimal policy on every lattice point of state space with solution of eqn (5) and local valuation which is calculated by bilinear interpolation, if k=0 go to (d).
(c) k=k-1, go to (b).
(d) search forward from an origin along the trajectory created by optimal policies.

Improvements in numerical algorithm

Effective utilization of system memory for shorter computation time

Fortunately, the optimal train control problem does not involve explicitly time-dependent term in the object function shown in eqns (1), (2), (3). It means thatwe have only to solve these equations once in whole DP procedure by storing thesolved results into system memory of a digital computer. Of course, this methoddemands larger memory than the basic DP process. The required system memoryfor the improved method is approximately 256MB or larger.

Confined state space and irregular lattice

In general, conversion from optimal control problem into multistage decision process which DP algorithm can solve will enlarge the required amount of system memory and total calculation time related to spacing of admissible state space. On the contrary, assuring accuracy of final state needs fine lattice points. This antinomy will cause insolvability as a result. Therefore, it is essential to confine state space, divide into nonuniform spacing like fig. 2 and make up different admissible region depend on time as fig. 3. In proposed algorithm, it is also important to number split state spaces sequentially to avoid extra time cost.

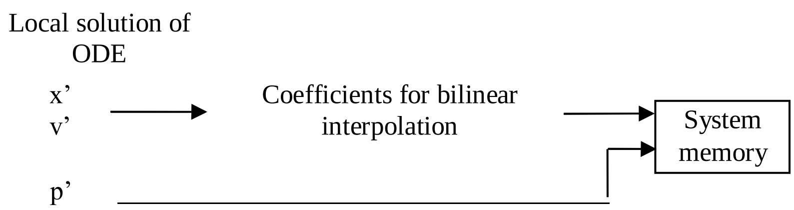

Figure 1: Applicable use of system memory for cutting down computation time.

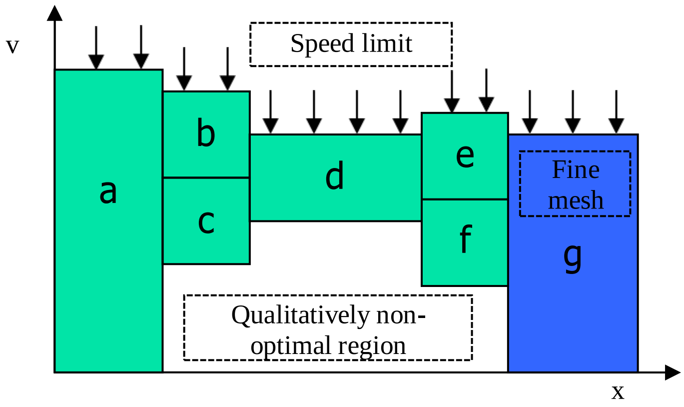

Figure 2: Confined state space and nonuniform spacing.

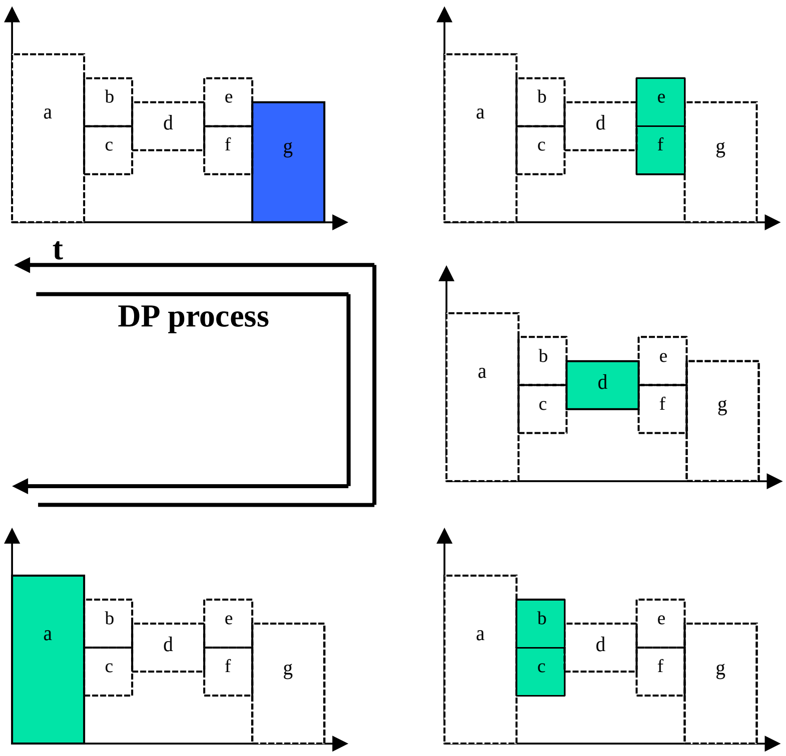

Figure 3: Example of time-dependent admissible region.

4 Numerical examples and their results

In this section, the authors show two numerical examples, one in simple case without any speed limits and inclines, the other in actual complicated train running profile. The specifications of train using in the examples are presented table 1 and fig. 4. They are based on typical values of a crowded commuter train whose congestion rate is 250%.

Fig. 5, 6 and table 2 show optimized results of two different track profiles. Terminal boundary condition is satisfied under 0.6m and 0.1m/sec. The results of total computation time cost show that the proposed method is feasible for practical use. In both cases, little chattering of control input caused by the discretization is observed. However, numerical error of the total energy consumption is small.

Table 1: Parameters of train.

Total weight [t]	Total length [m]	Maximum speed [km/h]	Efficiency of Motor/Generator	Organization
587.0	200.0	110.0	0.95/0.88	6M4T

Table 2: Optimization results.

No	Dest.	Terminal error of position	v(T)	J[MJ]	computation time[sec]
1	xf=3000	0.3	0.0837	111.2	20
2	xf=2180	0.6	0.0422	54.83	22

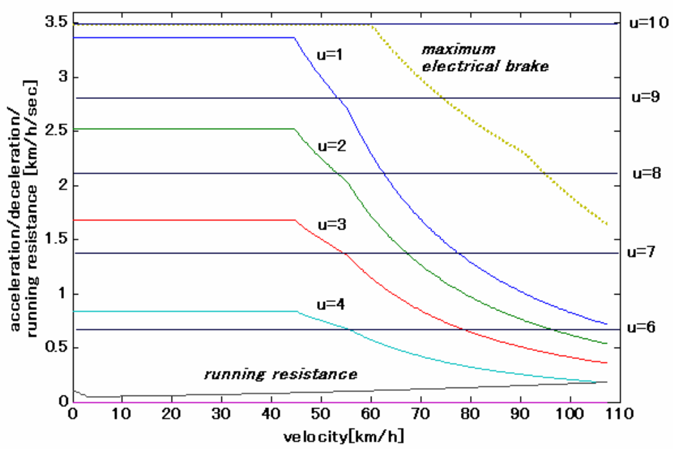

Figure 4: Acceleration/deceleration performance and running resistance.

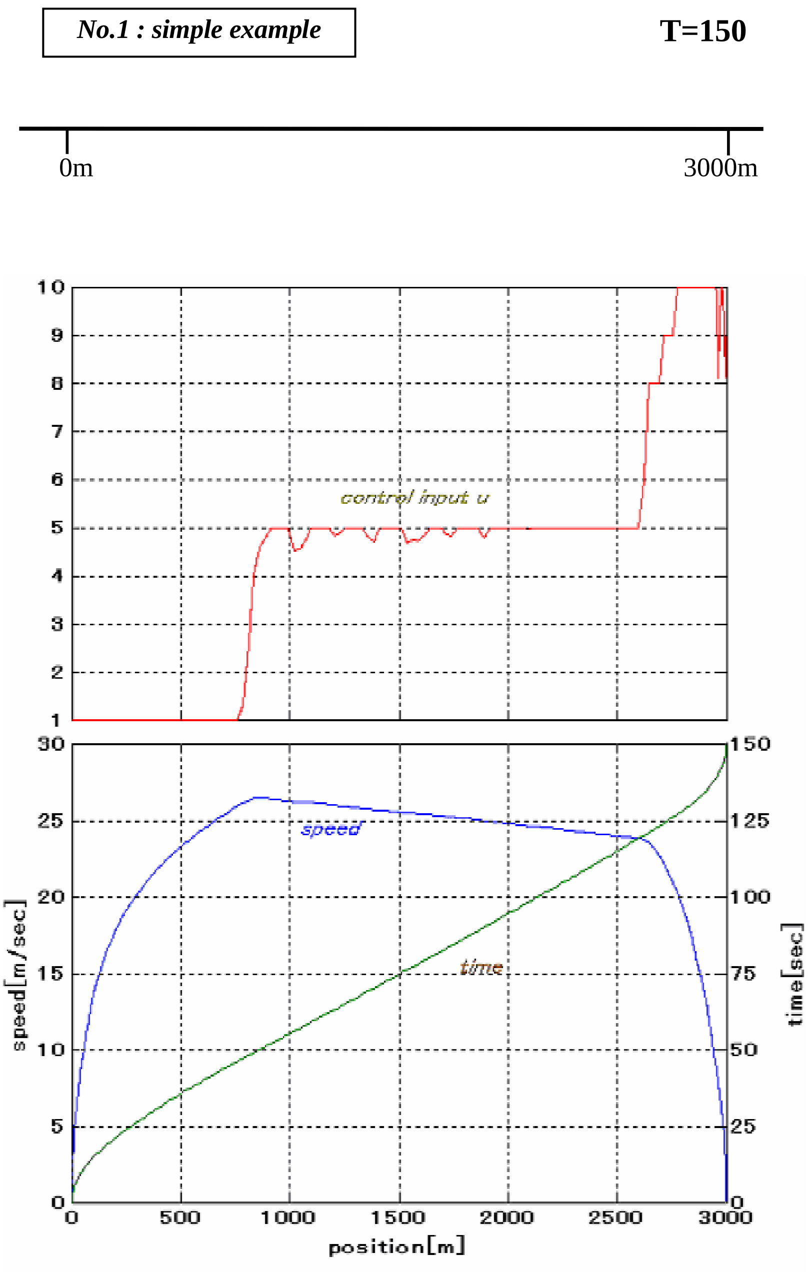

Figure 5: Track profile and optimization results on simple example.

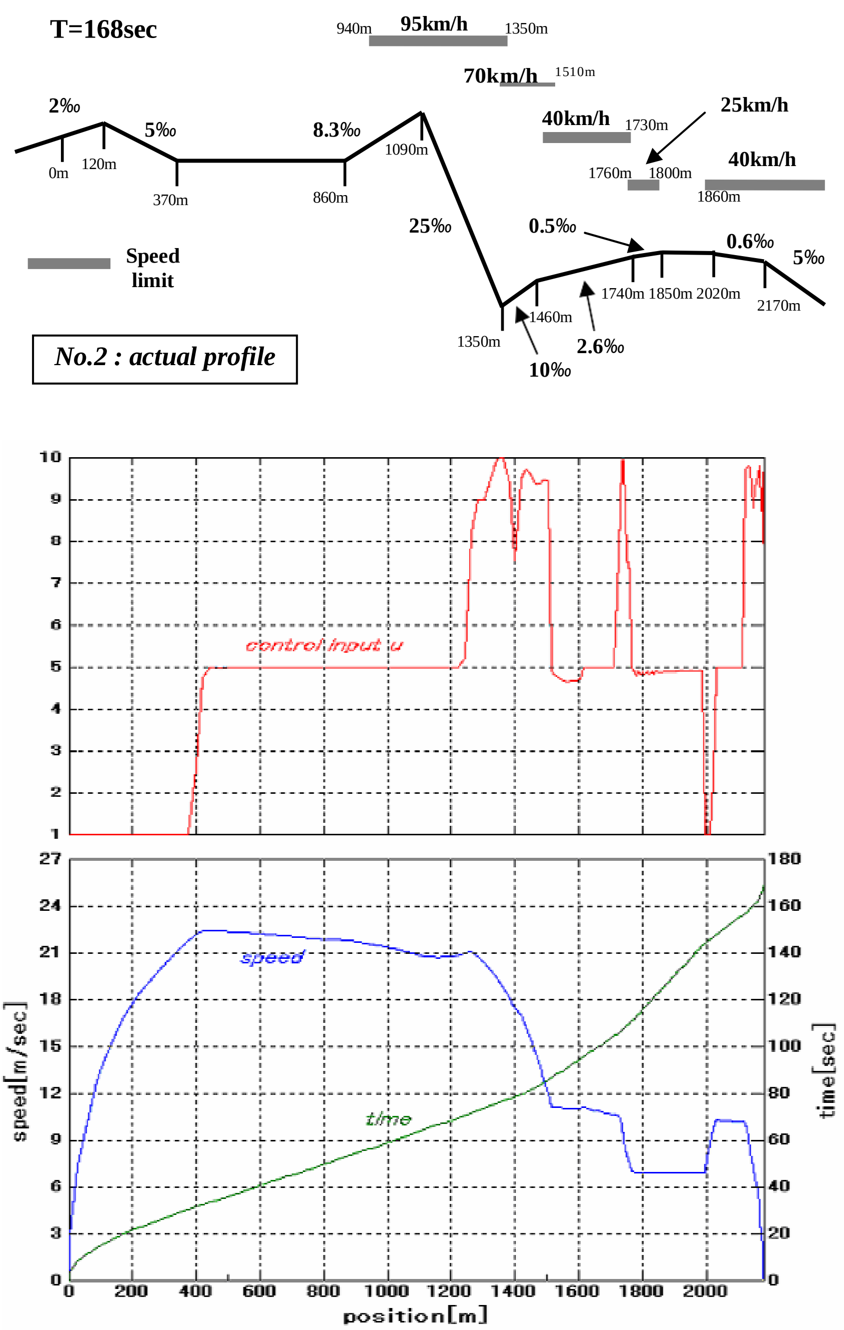

Figure 6: Track profile and optimization results on actual profile.

5 Conclusions

The optimal control problem for minimizing energy consumption by a train has been numerically solved with the proposed algorithm based on Bellman's Dynamic Programming in this paper. It can be applied to actual complicated running condition of a train.