

Assignment Date	18 November 2022
Student Name	B.Prabavathi
Student Roll Number	815819104015
Maximum Marks	

Utilization of algorithm dynamic programming, optimization

Example:

Given an array `arr[]` of `N` elements, the task is to find the minimum cost for reducing the array to a single element in `N-1` operations where in each operation:

- Delete the elements at indices `i` and `i+1` for some valid index `i`, replacing them with their sum.
- The cost of doing so is `arr[i] + arr[i+1]`, where `arr[]` is the array state just before the operation.
- This cost will be added to the final cost.

Examples:

Input: `arr[] = {3, 4, 2, 1, 7}`

Output: 37

Explanation:

Remove the elements at 0th and 1st index. `arr[] = {7, 2, 1, 7}`,

`Cost = 3 + 4 = 7`

Remove 1st and 2nd index elements. $arr[] = \{7, 3, 7\}$, $Cost = 2 + 1 = 3$

Remove 1st and 2nd index elements, $arr[] = \{7, 10\}$, $Cost = 3 + 7 = 10$

Remove the last two elements. $arr[] = \{17\}$, $Cost = 7 + 10 = 17$

Total cost = $7 + 3 + 10 + 17 = 37$

This is the minimum possible total cost for this array.

Input: $arr[] = \{1, 2, 3, 4\}$

Output: 19

Explanation:

Remove the 0th and 1st index elements. $arr[] = \{3, 3, 4\}$. $Cost = 1 + 2 = 3$

Remove the 0th and 1st index elements. $arr[] = \{6, 4\}$. $Cost = 3 + 3 = 6$

Remove the 0th and 1st index elements. $arr[] = \{10\}$. $Cost = 6 + 4 = 10$

Total cost = $3 + 6 + 10 = 19$.

This is the minimum possible cost.

Sub-optimal solution (using Range DP): The problem can be solved using the following idea:

- Let **arr[]** be the original array before any modifications are made.
- For an element in the array that has been derived from indices i to j of $a[]$, the cost of the final operation to form this single element will be the sum **arr[i] + arr[i+1] + . . . + arr[j]**. Let this value be denoted by the function $cost(i, j)$.
- To find the minimum cost for the section **arr[i, i+1, ... j]**, consider the cost of converting the pairs of sub-arrays **arr[i, i+1 . . . k]** & **arr[k+1, k+2 . . . j]** into single elements, and choose the minimum over all possible values of k from i to $j-1$ (both inclusive).

For implementing the above idea:

- The cost function can be calculated in constant time with preprocessing, using a [prefix sum array](#):
 - Calculate prefix sum (say stored in **pref[]** array).
 - So $cost(i, j)$ can be calculated as $(pref[j] - pref[i-1])$.
- Traverse from **i = 0 to N-1**:
 - Traverse **j = i+1 to N-1** to generate all the subarray of the main array:
 - Solve this problem for all these possible subarrays with the following dp transition -
 $dp[i][j] = cost(i, j) + \min_{i \leq k \leq j-1} (dp[i][k] + dp[k+1][j])$ as explained in the above idea.
- Here **dp[i][j]** is the minimum cost of applying **(j - i)**

operations on the sub-array **arr[i, i+1, . . . j]** to convert it to a single element.

cost(i, j) denotes the cost of the final operation i.e. the cost of adding the last two values to convert **arr[i, i+1, . . ., j]** to a single element.

- The final answer will be stored on **dp[0][N-1]**.

Below is the implementation of the above approach.

- C++

- Java

- C#

- Javascript

```
// C++ code to implement the approach

#include <bits/stdc++.h>

using namespace std;

// Function to find the minimum cost
int minCost(int arr[], int N)
{
    // Creating the prefix sum array
    int pref[N+1], dp[N][N];

    pref[0] = 0;

    memset(dp, 0, sizeof(dp));

    // Loop to calculate the prefix
sum
    for (int i = 0; i < N; i++) {
        pref[i + 1] = pref[i] +
arr[i];
    }

    // Iterating through all subarrays
    // of length 2 or greater
    for (int i = N - 2; i >= 0; i--) {
        for (int j = i + 1; j < N;
j++) {

            // Cost function = sum of
            // all elements in the
subarray

            int cost = pref[j + 1] -
pref[i];

            dp[i][j] = INT_MAX;

            for (int k = i; k < j;
```

```

k++) {

    // dp transition
    dp[i][j]
    = min(dp[i][j],
dp[i][k]
    + dp[k + 1][j] +
cost);
}

}

// Return answer
return dp[0][N - 1];
}

// Driver code
int main()
{
    int arr[] = { 3, 4, 2, 1, 7 };
    int N = sizeof(arr) /
sizeof(arr[0]);

    // Function call
    cout << minCost(arr, N);
    return 0;
}

```

Output