RAINFALL DATA USING FUNCTIONAL

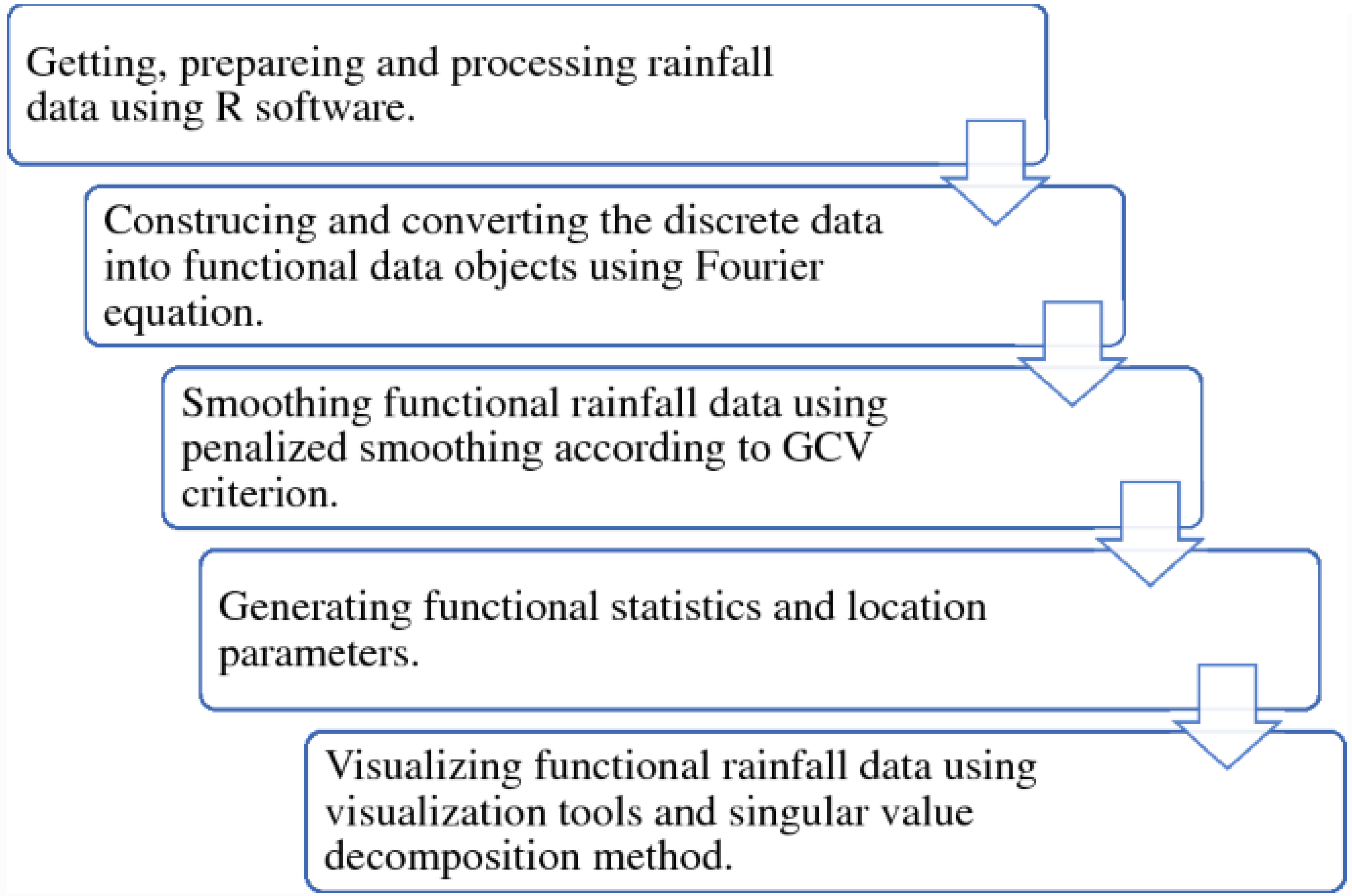
- In recent years, functional data analysis (FDA) has been used in many applications in order to analyze data that provide detail on curves, surfaces or other components of a continuum. The purpose of this research is mainly to incorporate and adapt visualization tools for the FDA of rainfall in the Taiz Region. Moreover, visualizing the temporal variations of rainfall in order to provide a clear understanding of the rainfall patterns and also to help in predicting the future. The current study has been conducted based on average monthly rainfall over the Taiz Region during the period of 1998–2018. The R software was used to process the data obtained from the Tropical Rainfall Measuring Mission in Taiz City. The functional rainfall data have been smoothed using penalized smoothing according to generalized cross-validation criteria. The results showed that the rainfall profiles in the Taiz Region depend significantly on their temporal locations due to the monsoonal influences, which reflect the distribution of rains in the spring and the summer seasons.

INTRODUCTION

- Recently, the field of functional data analysis (FDA) has seen phenomenal growth in numerous areas of applications such as environmental science, medical, engineering and biomaterials in order to enhance the planning methods and increase the efficiency of services or products Therefore, the FDA method is considered to be one of the most advanced techniques using all available data as practical measurements and curves This is due to the emergence of many modern technologies and software that have made mathematical computing possible in analyzing data by using the FDA So, the quantitative structure of the FDA approach could be viewed as an acceptable methodology with a specific perspective compared to traditional statistical analyses Many contributions cover a wide range of statistical problems involving the FDA such as mathematical foundations, covariance operator estimation, functional depth, functional autoregressive processes, linear regression semiparametric regression, nonparametric regression, spatial functional statistics, robust functional data analysis as well as sparsity in FDA Ramsay and Silverman offered an excellent overview of FDA bases and expanded the application of classical mathematical science, such as primary component analysis, regression analysis, linear models and confidence intervals, to the practical system. In another related book, Ferraty and Vieu provided studies on multiple parametric and nonparametric methods in order to produce useful tools for the FDA, while Kokoszka and Reimherr provided a detailed description of the current quantitative structure of the FDA. In order to improve the credit card payment system, Laukaitis studied the automatic regression model to obtain the cash flow index and transaction intensity using the FDA. Through this study, it is possible to predict and then solve the continuous stochastic problem for a full period of time. In another related study, Hyndman and Shang introduced new tools to form smooth curves of large amounts of visualization using the FDA. These tools included a bagplot and boxplot that use the first two rigorous factor scores: the data depth of Tukey and the regions with the highest density. According to their results, the proposed tools detect the extreme values at a to the traditional approaches. Therefore, The FDA approach has been used for the current work on the predictive the efficiency of due to the analysis as well as sparsity

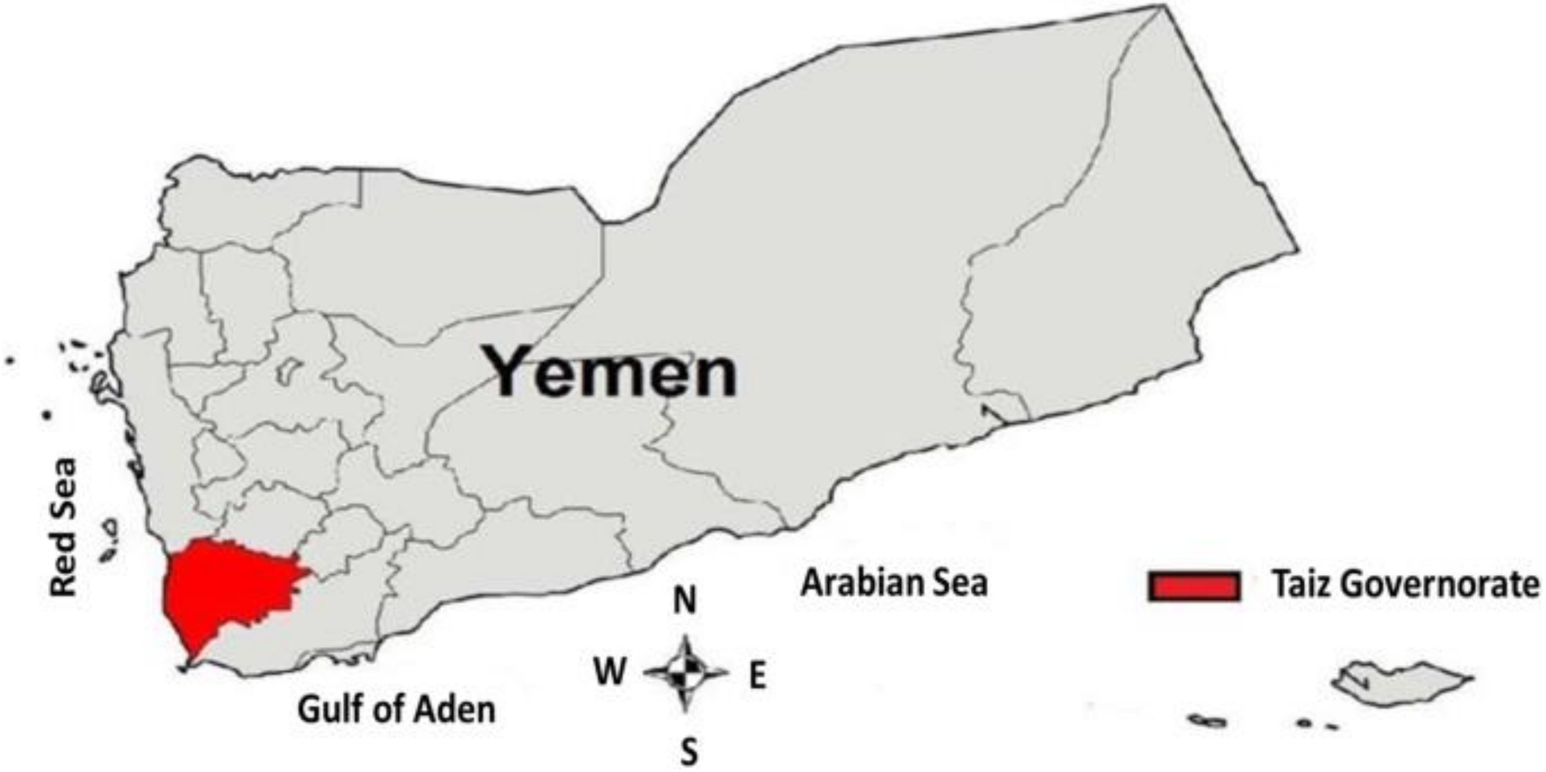
Methods and Materials

- This section presents the mathematical background of the FDA methodology that will be applied in the current work. The Eqs. (1)–(12) are employed to complete the research steps. The major step after getting data is to convert the discrete data into functional data, the next step is to smooth the functional data and the last two steps are to obtain the functional results of rainfall data.


Getting, prepareing and processing rainfall data using R software.
Construcing and converting the discrete data into functional data objects using Fourier equation.
Smoothing functional rainfall data using penalized smoothing according to GCV criterion.
Generating functional statistics and location parameters.
Visualizing functional rainfall data using visualization tools and singular value decomposition method.

The Singular Value Decomposition Technique

- Singular value decomposition (SVD) is a useful method in visualizing patterns of functional and multivariate data. This technique was proposed by Zhang et al. [29]; they employed the idea of projection pursuit by revealing just low-dimensional projections that show interesting features of the high-dimensional point. SVD technique decomposes high-dimensional smoothed multivariate data into singular rows, singular columns and singular values ordered according to the amount of interpreted variance.


Yemen
Red Sea
Gulf of Aden
Arabian Sea
N
W
E
S
Taiz Governorate

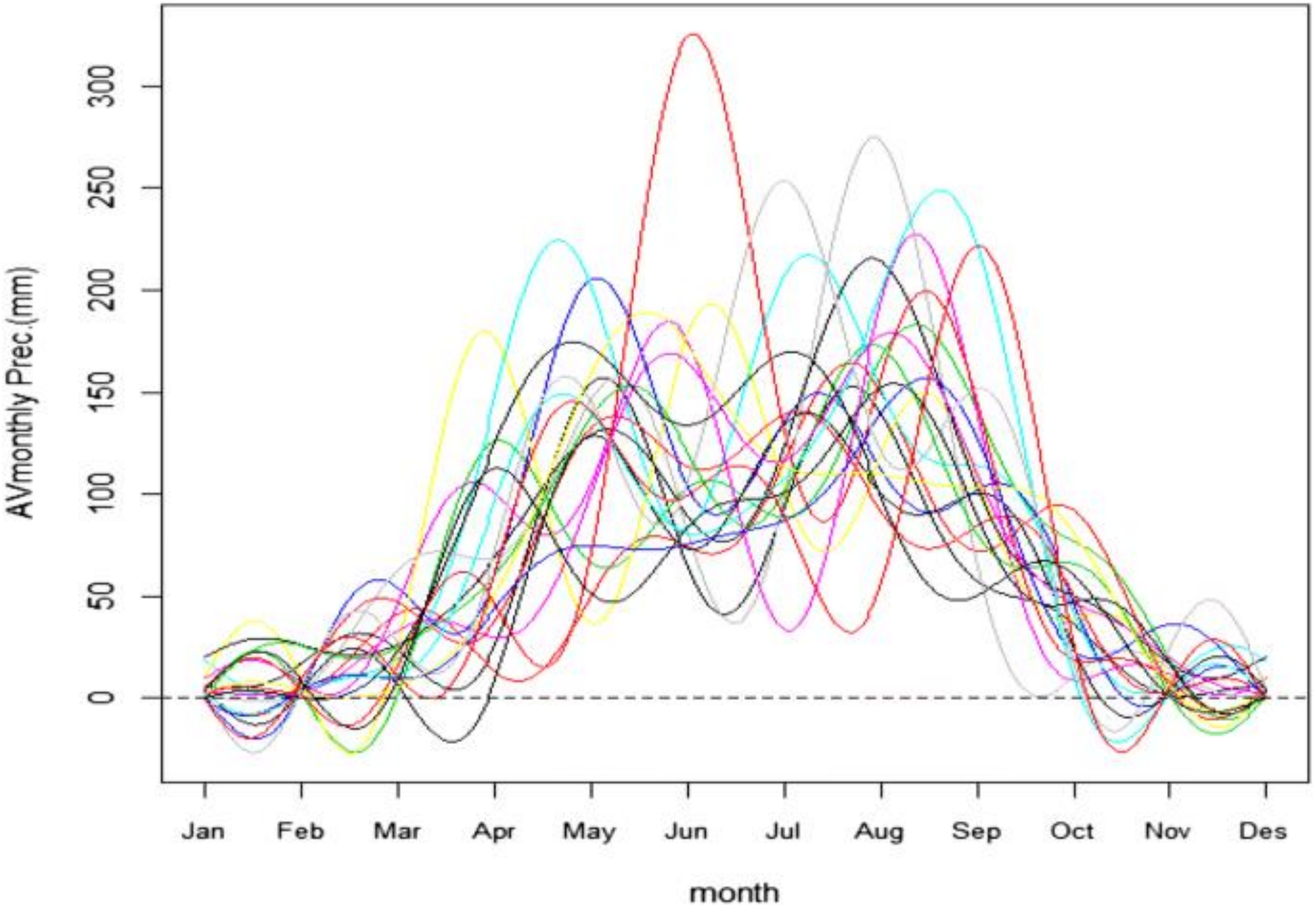
RESULTS AND DISCUSSION

- This section is divided into four sections. Constructing functional data will be displayed in the first section, while penalized smoothing will be demonstrated in the second section. Then, functional statistics with visualization will be presented in the third section. The fourth section will display the results of singular value decomposition visualization.

CONSTRUCTING FUNCTIONAL DATA

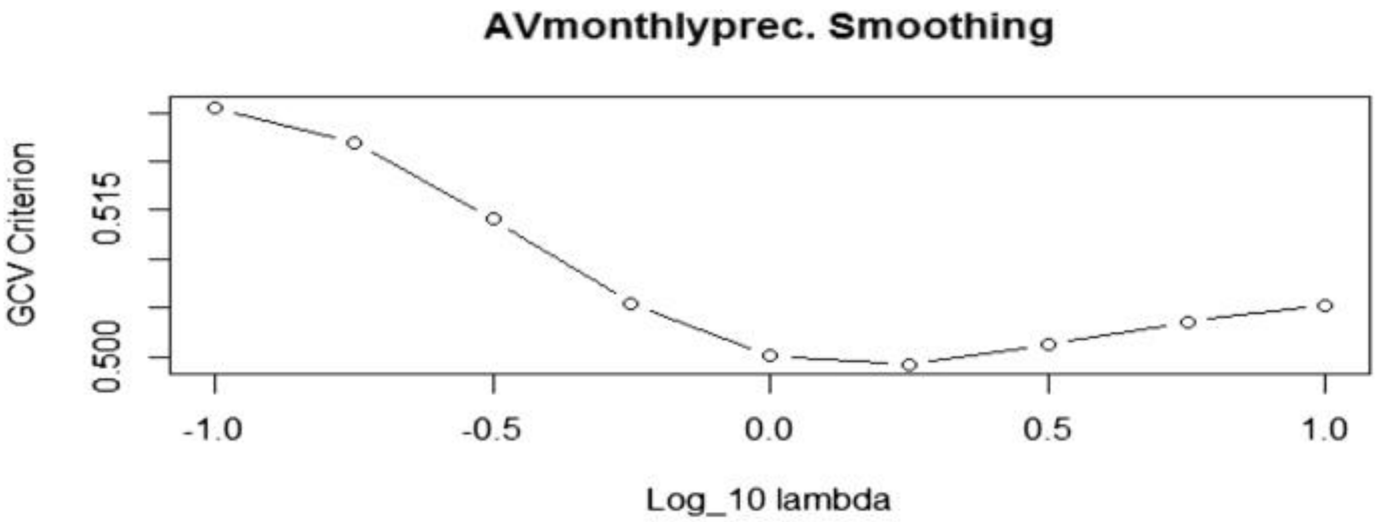
- The first step is to transform the discrete rainfall details into continuous functions or curves. Seasonal fluctuations and periodicity of the rainfall data of the entire series are often shown throughout the annual cycle. Then, Fourier basis functions are preferred. The choice of $k = 12$ can be justified to capture the precipitation variation within a month. In R software (FDA package), the discrete precipitation data were converted into functional data objects by using Fourier bases. All of the obtained functional data observations of the average monthly

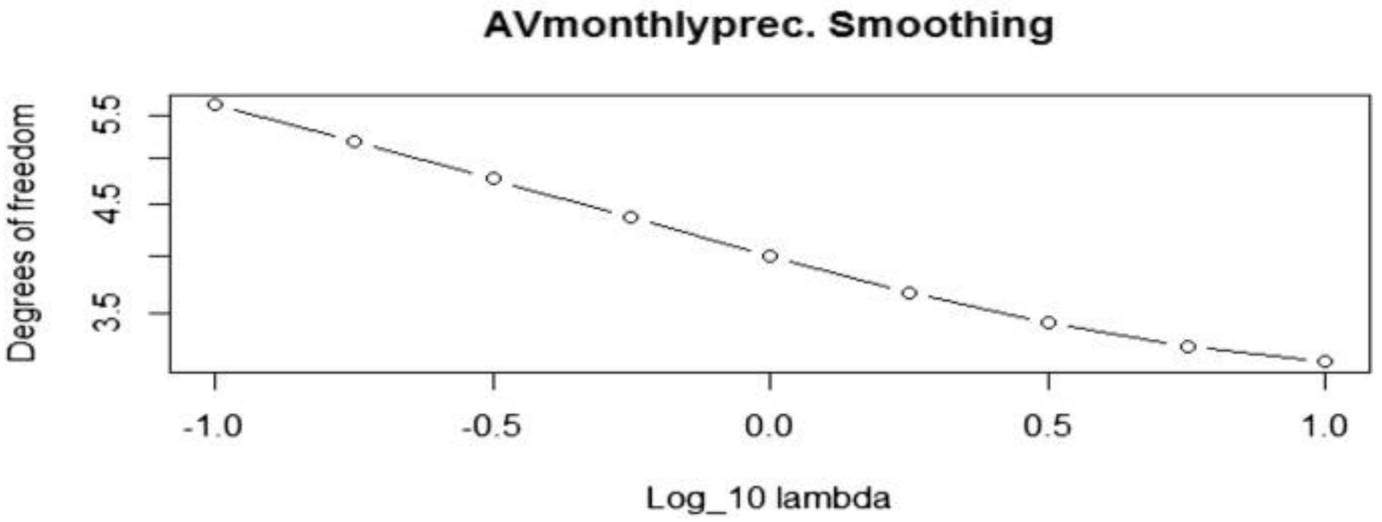
FD curves by Fourier basis
of Average Monthly Precipitation in Taiz city

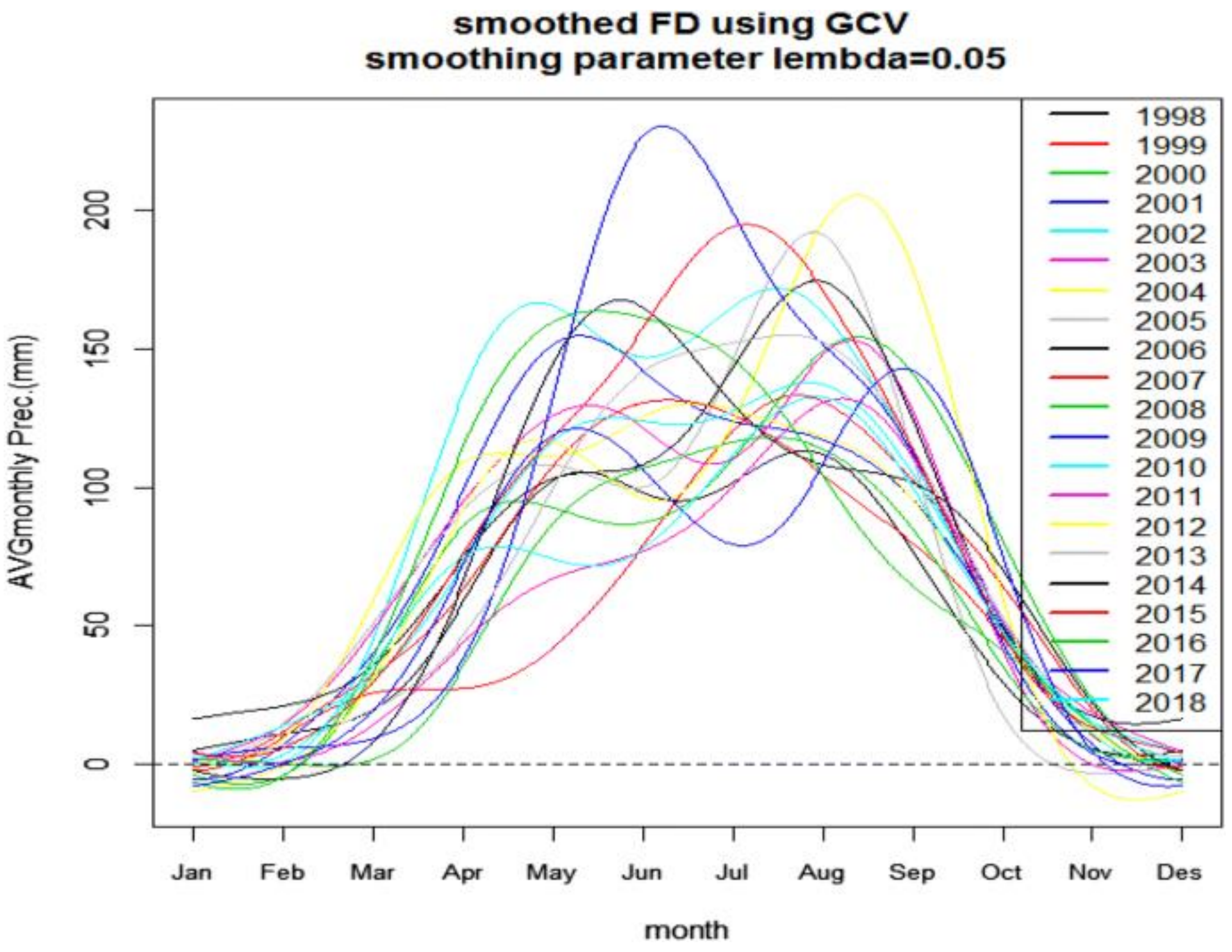


Penalized Smoothing for Functional Precipitation Data

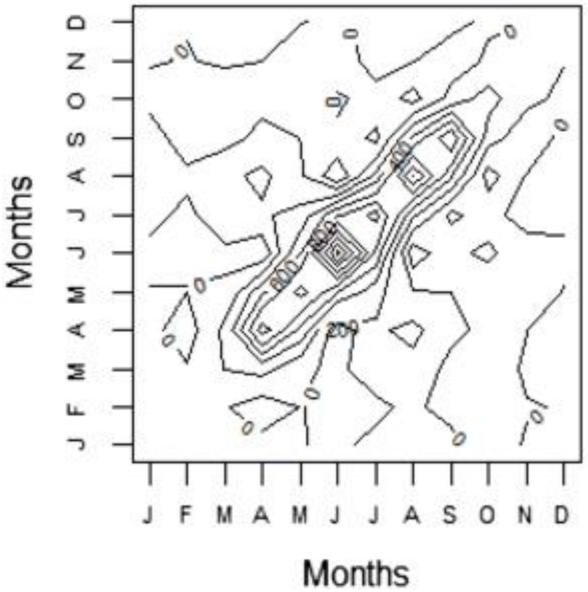
In the previous section, functional data objects are explained and constructed by using the Fourier basis expansion. The goal of penalized smoothing is to eliminate the contribution of the errors and noise and obtain the best estimate of curves x(t)x(t). The generalized cross-validation (GCV) criterion should be applied to choose the level of smoothing by arranging the harmonic acceleration operator with setting up a saturated basis that is capable of interpolating all the functional data of precipitation


AVmonthlyprec. Smoothing
GCV Criterion
0.500
0.515
-1.0
-0.5
0.0
0.5
1.0
Log_10 lambda

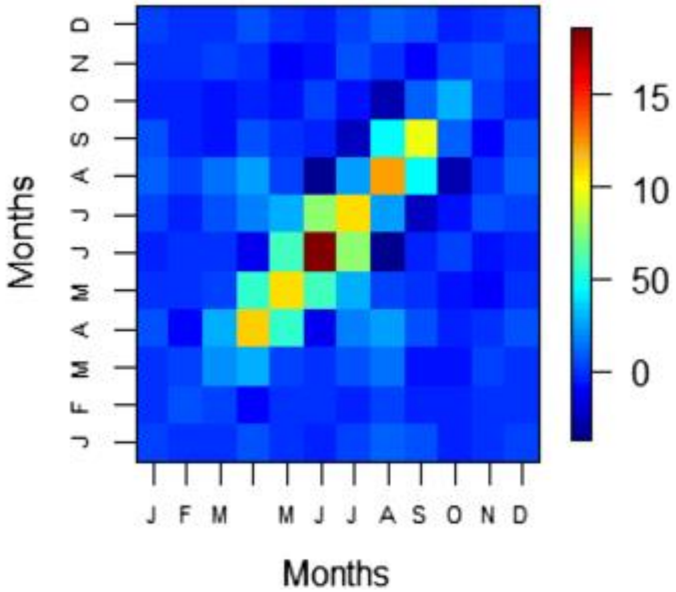

AVmonthlyprec. Smoothing
Degrees of freedom
3.5
4.5
5.5
-1.0
-0.5
0.0
0.5
1.0
Log_10 lambda


smoothed FD using GCV
smoothing parameter lembda=0.05
AVGmonthly Prec.(mm)
0
50
100
150
200
Jan
Feb
Mar
Apr
May
Jun
Jul
Aug
Sep
Oct
Nov
Des
month
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015
2016
2017
2018

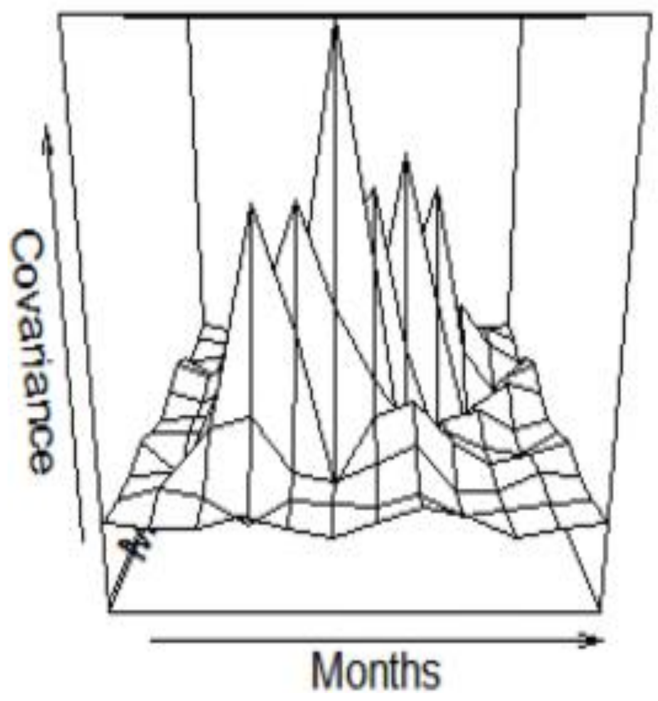
Variance-Covariance function across years for Anual precipitation Cycle in Taiz city



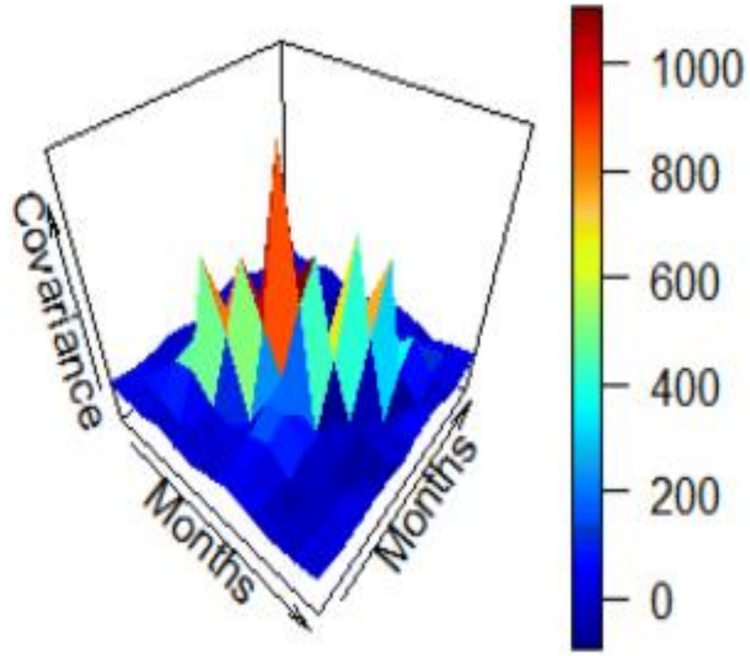
Variance-Covariance image across years for Anual precipitation Cycle in Taiz city



co-variance function across years
for Anual precipitation Cycle in Taiz city



co-variance image across years
for Anual precipitation Cycle in Taiz city



CONCLUSIONS

This research analyzes and visualizes the average monthly rainfall throughout the last two decades for Taiz City. FDA approaches with emphasis on smoothing and visualization were modified and applied for the rainfall measurements as an important step in a full FDA. This methodology included penalized smoothing with generalized cross-validation criteria, functional parameters such as location scales, variance–covariance surfaces and correlation functions with visualization and singular value decomposition technique. Based on the results, the following main conclusions can be drawn from this work