MA 8551- ALGEBRA AND NUMBER THEORY

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1) If G is a non empty set and * is a binary operation on G, then (G, *) is called a group if it satisfies the following conditions.

i) Closure : For all $a, b, c \in G$, $a * (b * c) = (a * b) * c$

ii) Associative : For all $a, b, c \in G$, $a * (b * c) = (a * b) * c$

iii) Identity : There exists $e \in G$ with $a * e = e * a = a$ for all $a \in G$

iv) Inverse : There exists $a^{-1} \in G$ with $a * a^{-1} = a^{-1} * a = e$ for all $a \in G$.

2) Given $S = \{-1, 1\}$

To verify S is a group under multiplication.

0	-1	1
-1	1	-1
1	-1	1

i) S is closed under multiplication

ii) S is associative under multiplication

iii) 1 is the identity

iv) inverse of 1 is 1

inverse of -1 is -1

3) Let e_1 and e_2 be identities

To prove $e_1 = e_2$

Let e_1 be any element and e_2 be identity

$e_1 * e_2 = e_2 * e_1 = e_1$ — ①

Let e_2 be any element and e_1 be identity

$e_2 * e_1 = e_1 * e_2 = e_2$ —— ②

From ① & ② $e_1 = e_2$ Hence proved.

4) Let $(G, *)$ be a group. Let $a \in h$ and e be the identity of G.

Let a_1^{-1} and a_2^{-1} be the inverse of a

To prove : $a_1^{-1} = a_2^{-1}$

If a_1^{-1} be the inverse of a, then

$a * a_1^{-1} = a_1^{-1} * a = e$

If a_2^{-1} be the inverse of a then

$a * a_2^{-1} = a_2^{-1} * a = e$

LHS : $a_1^{-1} = a_1^{-1} * e$

$= a_1^{-1} * (a * a_2^{-1})$

$= (a_1^{-1} * a) * a_2^{-1}$

$= e * a_2^{-1}$

$= a_2^{-1} = R.H.S$ Hence proved.

5) Assume G is abelian

$ab = ba$ (Commutative property)

So $(ab)^{-1} = (ba)^{-1} = a^{-1} b^{-1}$

Assume $(ab)^{-1} = a^{-1} b^{-1}$

To prove : $ab = ba$

Since G is a group. $(ab)^{-1} = b^{-1}a^{-1}$ — (1)

$(ba)^{-1} = a^{-1}b^{-1}$ — (2)

From (1) & (2)

$(ab)^{-1} = (ba)^{-1}$

$ab = ba$

G is abelian.