IBM **Quantum**

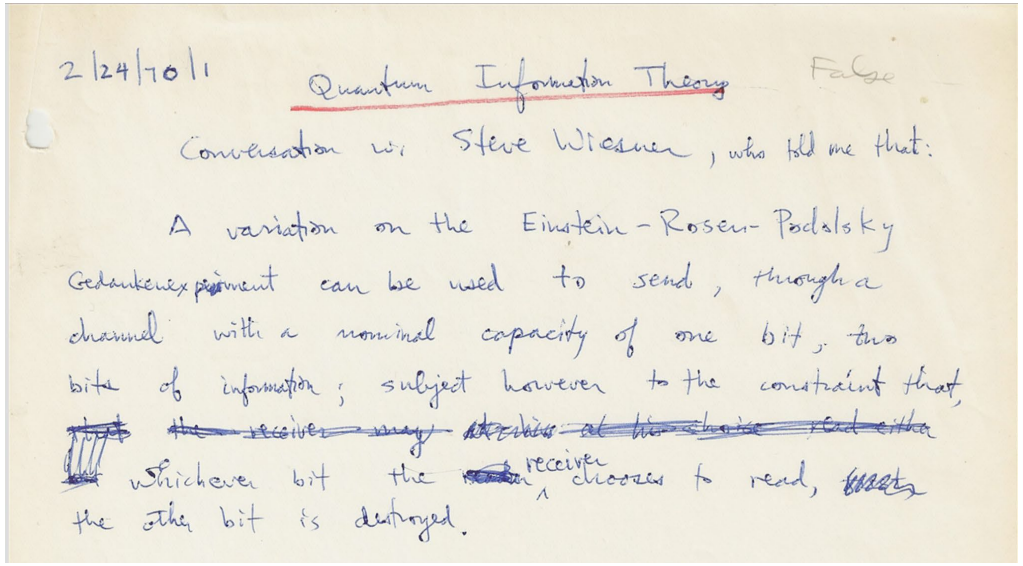
An overview of quantum computing

IBM

A brief history of quantum computing

- Quantum computing is the field of computation where we investigate the computational power and other properties of computation based on quantum mechanics
- These fundamental principles of quantum mechanics such as superposition, entanglement and interference are the main building blocks for the quantum computational theory
- The main ideas that built a foundation for quantum computing can be traced back to early 20th century (Planck, Bohr, Heisenberg, Schrodinger etc.)
- Starting in 1960s, there were some theoretical results, as well as earlier quantum algorithms (Simon's, Deutsch-Jozsa, Bernstein-Vazirani)

A brief history of quantum computing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Quantum Information Theory

Conversation w/ Steve Wiesner, who told me that:

A variation on the Einstein-Rosen-Podolsky Gedankenexperiment can be used to send, through a channel with a nominal capacity of one bit, two bits of information; subject however to the constraint that ~~the receiver may receive at his choice read either~~ whichever bit the receiver chooses to read, ~~reads~~ the other bit is destroyed.

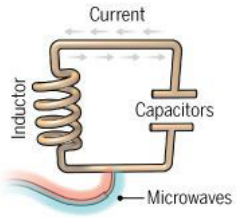
First usage of the word Quantum Information Theory in Bennett's notebook

IBM – MIT Conference on the Physics of Computation, 1981

- One big breakthrough was Shor's algorithm in 1994 about prime decomposition for RSA cryptography
- Since then, quantum computing has become a very impactful area at the intersection of physics, computer science, mathematics, chemistry and many other disciplines!

What does a quantum computer look like?

Superconducting loops



A resistance-free current oscillates back and forth around a circuit loop. An injected microwave signal excites the current into super-position states.

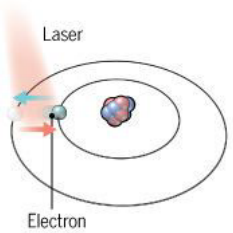
Longevity (seconds)	**0.00005**
Logic success rate	**99.4%**
Number entangled	**9**

Company support
Google, IBM, Quantum Circuits

Pros
Fast working. Build on existing semiconductor industry.

Cons
Collapse easily and must be kept cold.

Trapped ions



Electrically charged atoms, or ions, have quantum energies that depend on the location of electrons. Tuned lasers cool and trap the ions, and put them in super-position states.

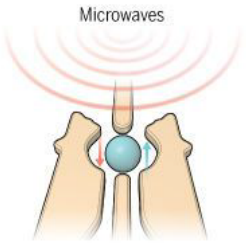
Longevity (seconds)	**>1000**
Logic success rate	**99.9%**
Number entangled	**14**

Company support
ionQ

Pros
Very stable. Highest achieved gate fidelities.

Cons
Slow operation. Many lasers are needed.

Silicon quantum dots



These "artificial atoms" are made by adding an electron to a small piece of pure silicon. Microwaves control the electron's quantum state.

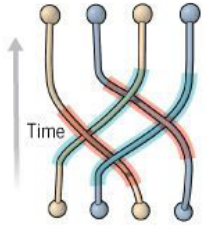
Longevity (seconds)	**0.03**
Logic success rate	**~99%**
Number entangled	**2**

Company support
Intel

Pros
Stable. Build on existing semiconductor industry.

Cons
Only a few entangled. Must be kept cold.

Topological qubits



Quasiparticles can be seen in the behavior of electrons channeled through semiconductor structures. Their braided paths can encode quantum information.

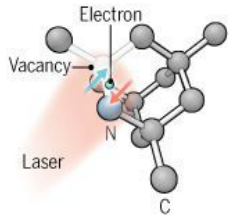
Longevity (seconds)	**N/A**
Logic success rate	**N/A**
Number entangled	**N/A**

Company support
Microsoft, Bell Labs

Pros
Greatly reduce errors.

Cons
Existence not yet confirmed.

Diamond vacancies



A nitrogen atom and a vacancy add an electron to a diamond lattice. Its quantum spin state, along with those of nearby carbon nuclei, can be controlled with light.

Longevity (seconds)	**10**
Logic success rate	**99.2%**
Number entangled	**6**

Company support
Quantum Diamond Technologies

Pros
Can operate at room temperature.

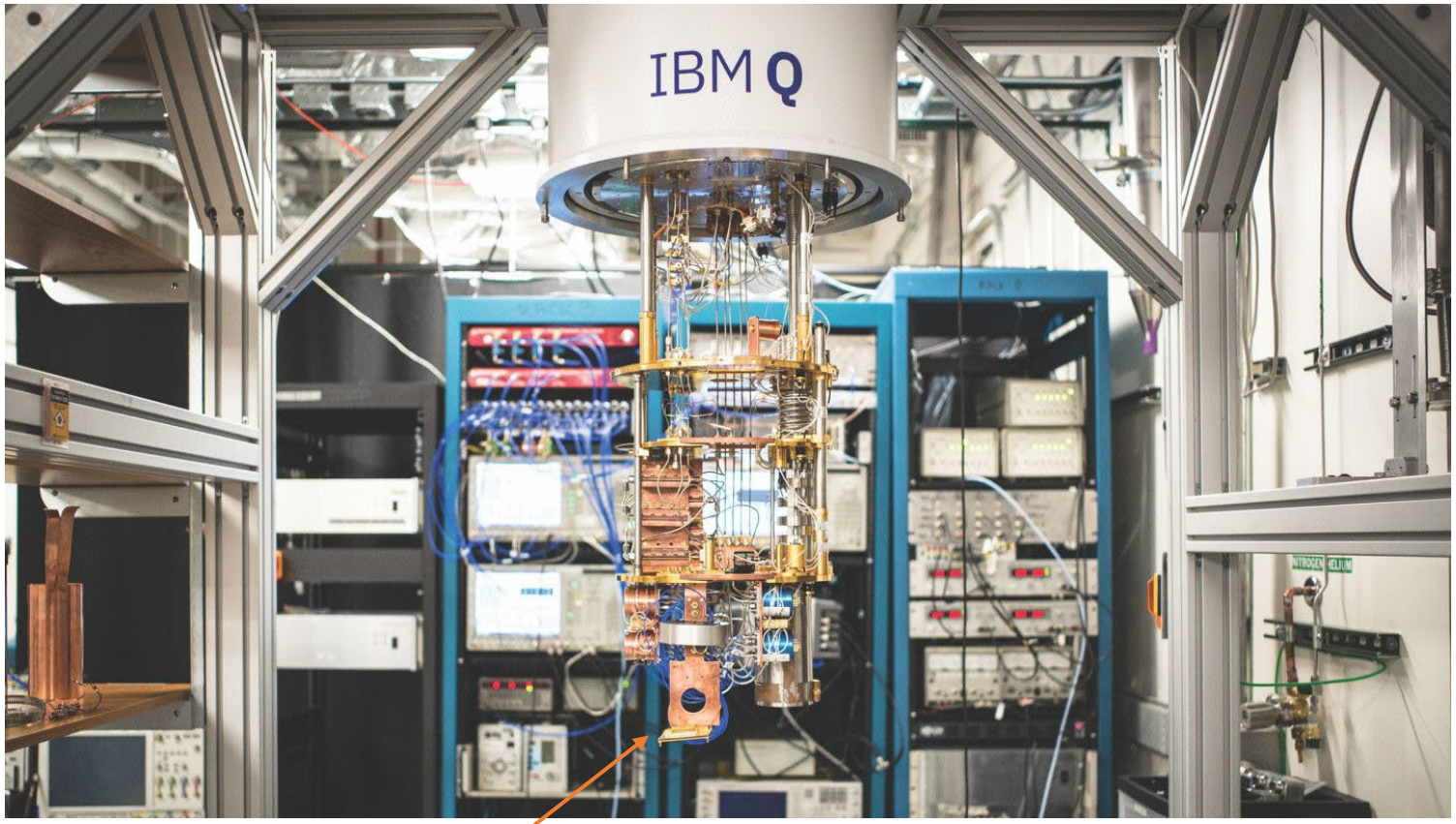
Cons
Difficult to entangle.

Note: Longevity is the record coherence time for a single qubit superposition state, logic success rate is the highest reported gate fidelity for logic operations on two qubits, and number entangled is the maximum number of qubits entangled and capable of performing two-qubit operations.

Image source: https://www.science.org/content/article/scientists-are-close-building-quantum-computer-can-beat-conventional-one

Superconducting qubits

IBM Quantum chip, 2021 (IBM Official)



IBM Quantum Computer, golden chandelier design

One of the largest quantum computers in the world

Image: IBM_Cleveland quantum system located at the Lerner Research Institute - Cleveland Clinic, 127 qubits

Classical computing

CLASSICAL COMPUTING

INPUT

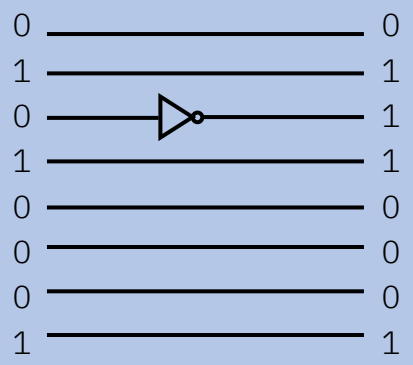
- Numerical values
- Vectors/matrices
- Graphs
- Images
- Text
- …

“Quantum”

01010001
01110101
01100001
01101110
01110100
01110101
01101101

COMPUTATION

- Basic arithmetic
- Linear/matrix algebra
- Regression
- Statistical analysis
- Machine learning
- …



OUTPUT

Solution to the computational task

01110001
01010101
01000001
01001110
01010100
01010101
01001101

“qUANTUM”

From bits to quantum bits (qubits)

- Qubits and in general quantum computations take place in a Hilbert space, that is a complete inner product space (a complex vector space)

- Qubits can be in the superposition of 0 and 1 states.

For basis states $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, we can have

$\alpha_0 |0\rangle + \alpha_1 |1\rangle$ where $|\alpha_0|^2 + |\alpha_1|^2 = 1$

- Polarized sunglasses is a good analogy for a quantum system where qubits are polarized photons. Say horizontal polarization is the qubit $|0\rangle$ and vertical polarization $|1\rangle$.

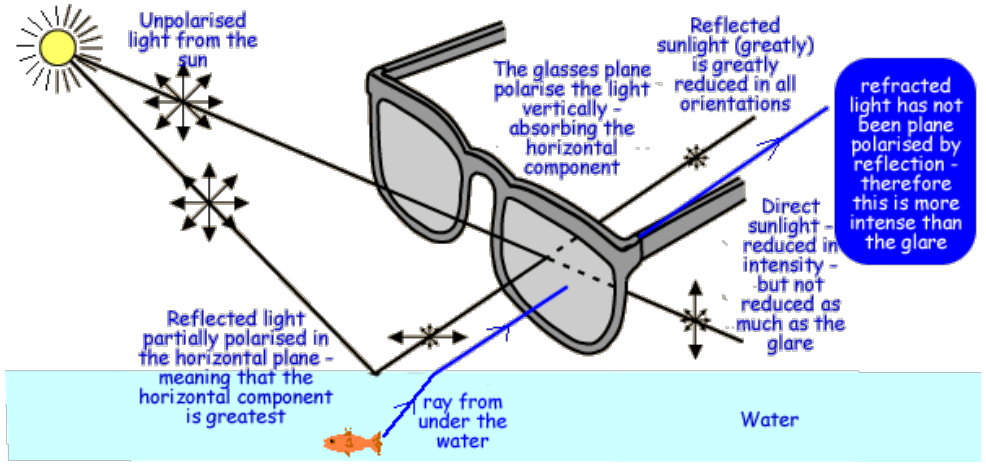
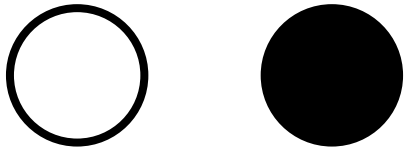


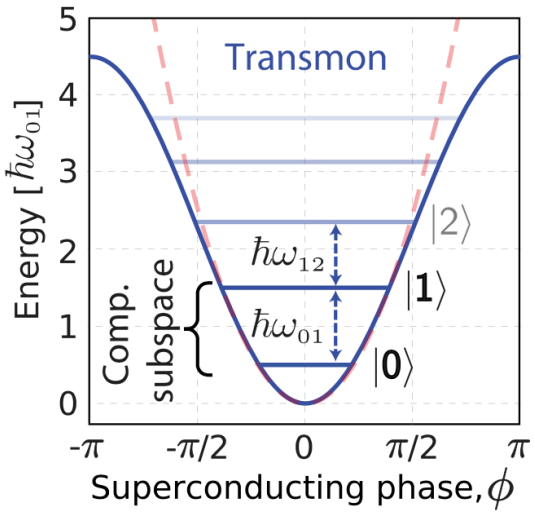
Image source: https://www.cyberphysics.co.uk/topics/light/polarised_spex.htm

Classical Bits



0 1

Quantum Qubits



Source: IBM Quantum Challenge 2021

$|0\rangle$ $|1\rangle$

$|\psi\rangle = a|0\rangle + b|1\rangle$ Superposition!

$P_0 = |\langle 0|\psi\rangle|^2 = |a\langle 0|0\rangle + b\langle 0|1\rangle|^2 = |a|^2$

$P_0 = |a|^2, P_1 = |b|^2$

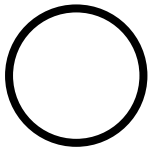
$|a|^2 + |b|^2 = 1 \quad a, b \in \mathbb{C}$

P-Classical Superposition

What about probabilistic classical systems (with $p_0, p_1 \in \mathbb{R}$)?

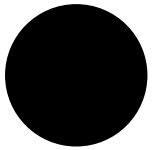
Sure, you can prepare a probabilistic "superposition", but using copies on more computational resources.

$$s = p_0(0) + p_1(1)$$



0

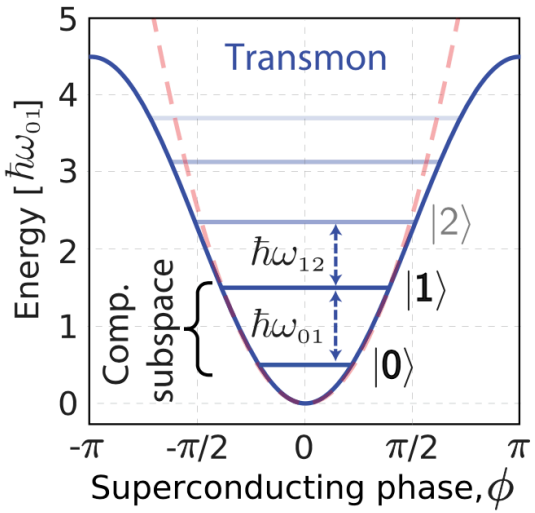
Bit A, in state 0, selected with probability p_0



1

Bit B, in state 0, selected with probability p_1

Quantum Superposition



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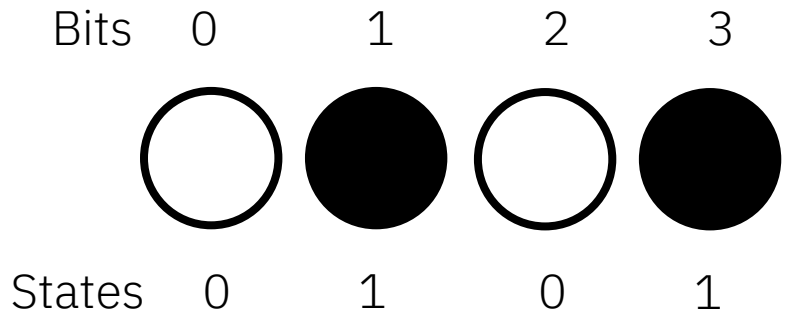
$P_0 = |\langle 0|\psi\rangle|^2 = |a\langle 0|0\rangle + b\langle 0|1\rangle|^2 = |a|^2$

$P_0 = |a|^2, P_1 = |b|^2$

$|a|^2 + |b|^2 = 1 \quad a, b \in \mathbb{C}$

Classical Entangled bits

Measuring bit 0 has no “effect” on bit 2



Quantum Entangled qubits

Qubits can be entangled:

If you measure q_0 to be in $|0\rangle$, you know q_2 is also in $|0\rangle$.

$$|\psi\rangle = \frac{1}{\sqrt{2}}(\,|0101\rangle + |1010\rangle)$$

Here, we use “littlendian” ordering:

$$|q_3 q_2 q_1 q_0\rangle$$

Classical Entanglement

Correlations exist in classical systems. You can prepare a state like this classically, but

(a) Using a copy of resources

(b) Measurement of bit 0 doesn't affect bit 2, it reveals which copy you have

$$s = p_0(0101) + p_1(1010)$$

4-bit copy A, in state 0101, selected with probability p_0

4-bit copy B, in state 1010, selected with probability p_1

Quantum Entanglement

Qubits can be entangled, with different entanglements in different superpositions on a single set of qubits:

If you measure q_0 to be in $|0\rangle$, you know q_2 is also in $|0\rangle$.

$$|\psi\rangle = \frac{1}{\sqrt{2}}(\,|0101\rangle + |1010\rangle)$$

Here, we use "littlendian" ordering:

$$|q_3 q_2 q_1 q_0\rangle$$

Classical Bits

A single set of N bits can be in any one of 2^N possible states.

$N = 4$ possible states

(0000), (0001), (0010), (0011)…

…(1100), (1101), (1110), (1111)

Quantum Qubits

A single set of N qubits can be in a superposition of ALL 2^N possible states, simultaneously.

$$|\psi\rangle = c_0\,|0001\rangle + c_1|0001\rangle + \cdots$$

$$+\, c_{14}\,|1110\rangle + c_{15}\,|1111\rangle \qquad c_i \in \mathbb{C}$$

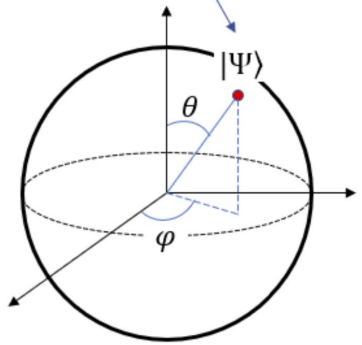
Visual representation of qubits

- A convenient way to picture these quantum states (single qubits) is Bloch sphere.

$$|\psi> = \alpha|0> + \beta|1> = \cos\frac{\theta}{2}\,|0> + e^{i\varphi}\sin\frac{\theta}{2}\,|1>$$

The absolute value (magnitude) of this term is always 1 regardless of the value φ.
(i.e, the magnitude of α and β is determined by θ only)

$$|\Psi\rangle = \cos(\tfrac{\theta}{2})|0\rangle + e^{i\varphi}sin(\tfrac{\theta}{2})|1\rangle$$



$0 \le \theta \le \pi$

$0 \le \varphi \le 2\pi$

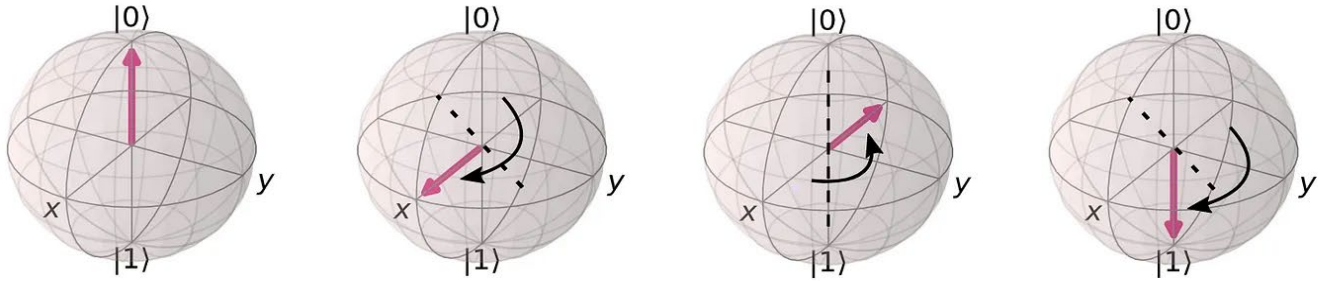


Image source: https://www.sharetechnote.com/html/QC/QuantumComputing_BlochSphere.html

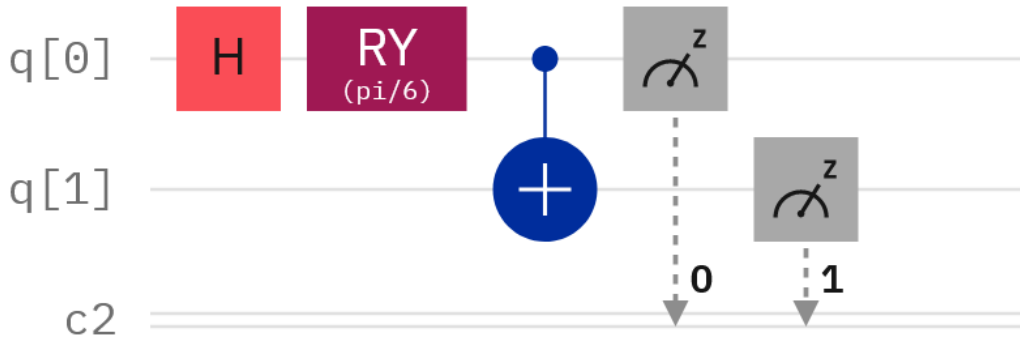
Measurement

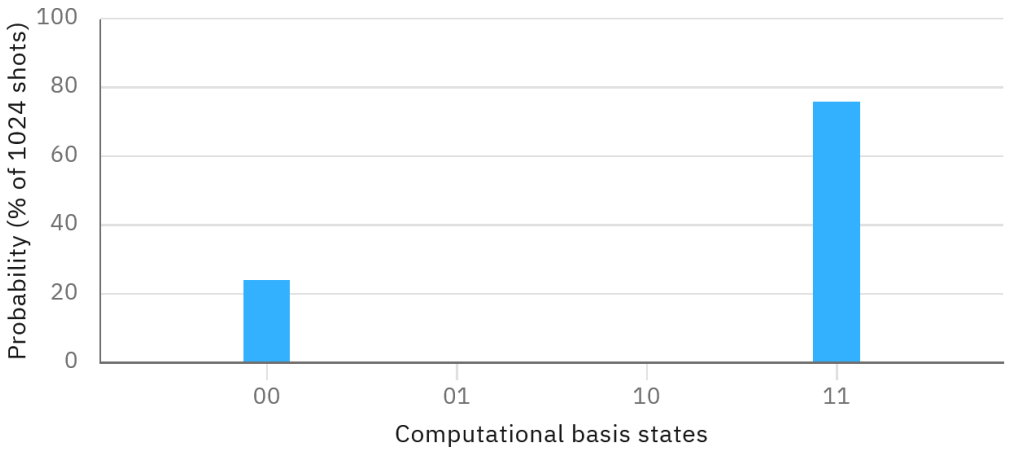
Measuring the state of a qubit, even one in superposition, yields a $|0\rangle$ or a $|1\rangle$.

The probability of measuring these states is related to the coefficients in the state vector.

The probabilities to the right are measured in the absence of noise.

$$CNOT_{0,1}R_{y,0}\left(\frac{\pi}{6}\right)H_0|\psi\rangle \approx 0.50|0\rangle|0\rangle + 0.866|1\rangle|1\rangle$$





H, above and in the diagram, is the Hadamard gate, not to be confused with the Hamiltonian.

Unitaries

Time evolution in quantum is described by the Schrödinger equation.

This means unitary matrices, which leads to unitary gates.

It also gives us complex coefficients.

Schrödinger equation:

$$i\hbar \frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle$$
$$\rightarrow |\psi(t)\rangle = \underbrace{e^{-iHt}}|\psi(t=0)\rangle$$

$U = e^{-iHt}$ is unitary!

Unitary operators:

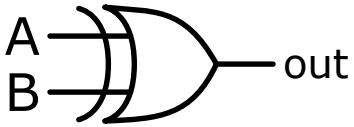
$U^\dagger U = e^{iH^\dagger t}e^{-iHt} = 1 \rightarrow$ reversibility

H is the Hamiltonian, the operator describing the energy of the system, different from case to case, not to be confused with the Hadamard gate.

Classical Gates

Classical gates may or may not be unitary

XOR[1]



A	B	A XOR B
0	0	0
1	0	1
0	1	1
1	1	0

[1]Source: Wikipedia Commons

Quantum Unitary Gates

Quantum gates are unitary.

CNOT

$|x\rangle|y\rangle \rightarrow |x, x \oplus y\rangle$

Reversible!

x	y	output
$\|0\rangle$	$\|0\rangle$	$\|00\rangle$
$\|1\rangle$	$\|0\rangle$	$\|11\rangle$
$\|0\rangle$	$\|1\rangle$	$\|01\rangle$
$\|1\rangle$	$\|1\rangle$	$\|10\rangle$

Operating on qubits

- Since we model qubits as complex vectors in Hilbert space, we operate on a quantum state with linear transformations, hence matrices!
- In this case, the matrices must be unitary matrices, that is $U^\dagger U = I$. So, potentially all the elements of SU(n).
- For single qubits, we have very commonly used matrices called Pauli matrices.

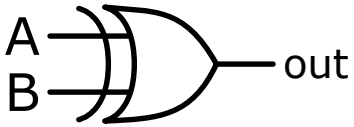
$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- We have larger unitary matrices for multi-qubit operations (4x4 for 2-qubits etc.)
- Now, we can think about these as quantum gates and build quantum circuits

Classical Gates

Classical gates may or may not be unitary

XOR[1]



A	B	A XOR B
0	0	0
1	0	1
0	1	1
1	1	0

[1]Source: Wikipedia Commons

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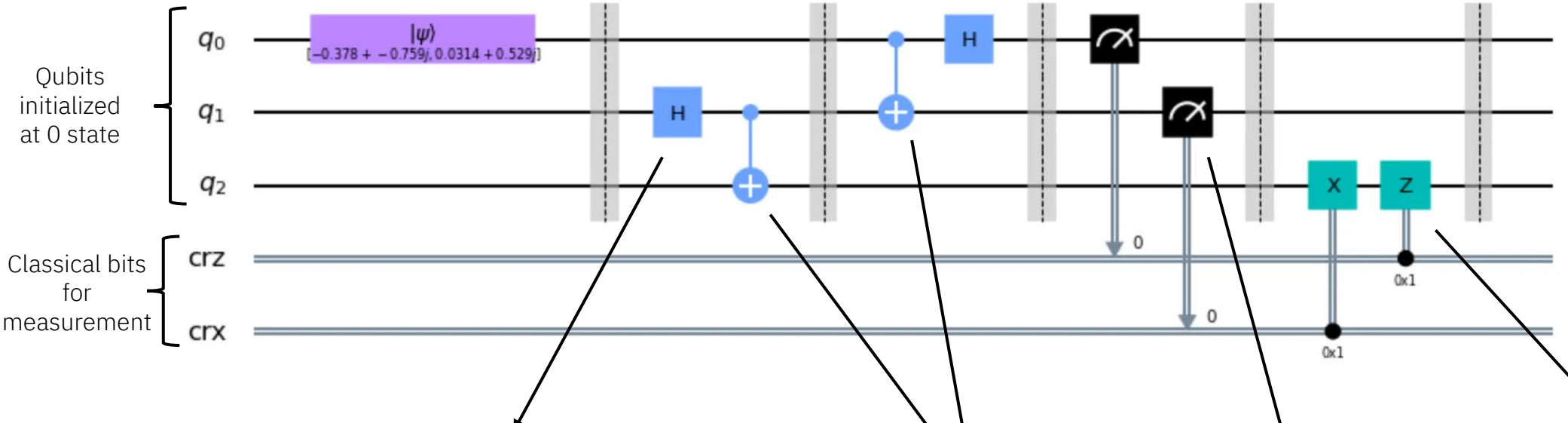
$\|0\rangle$	$\|1\rangle$	$\|10\rangle$
$\|1\rangle$	$\|1\rangle$	$\|01\rangle$

An example of quantum circuit



Hadamard gate, puts the qubit into uniform superposition of 0 and 1 states

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

An entangling gate that call CNOT gate

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Measuring the qubits onto classical bits to get the state 0 or 1.

Some extra post-measurement gates that are problem specific

Foundations - differences

Quantum	Classical
Superposition	On or off – probabilistic “superposition” has cost
Entanglement	Independent system states – “entanglement” possible
Interference	No interference
Measure a single state	Measure a single state
Unitary gates	Unitary & non-unitary gates
Complex coefficients	Real coefficients

Are these attributes of quantum *better* in all cases?

No. They’re different. So where can they bring value?

Understanding complexities

$$t(n) = c_0 + c_1 n + c_2 n^2 + \cdots c_m n^m$$

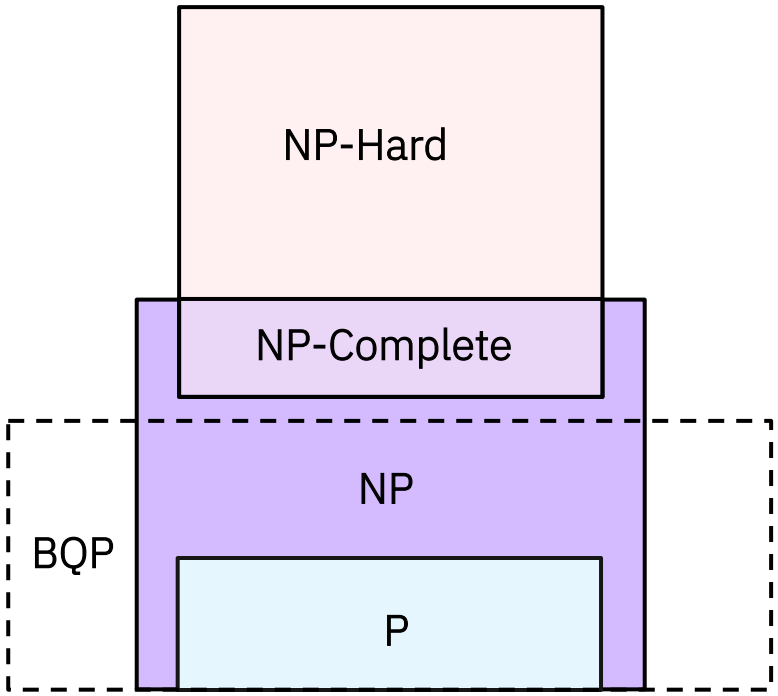
P (polynomial): problems that can be solved in polynomial time.

NP – (non-deterministic polynomial): Can check a solution in polynomial time, but can't find one in polynomial time.

NP Complete: NP-Hard problems also in NP, solutions of which map to solve all NP.

NP Hard: Problems as hard as the hardest problems in NP.

BQP (Bounded-error quantum polynomial): solvable by a quantum computer in polynomial time, with an error probability of at most 1/3



Some complexity classes, under the assumption that P is not equal to NP. Note all class assignments are subject to the uncertainty of complexity class structure.

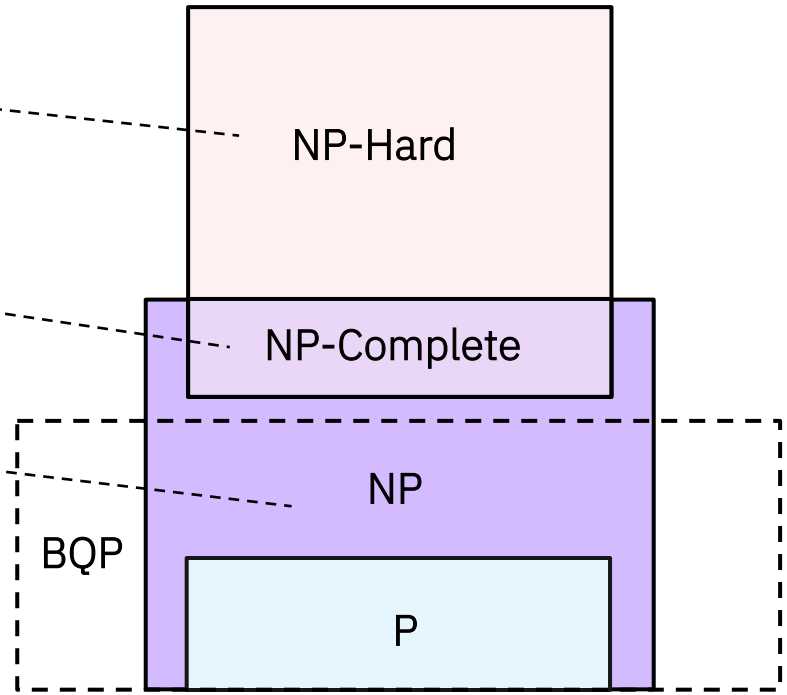
Understanding complexities

$$t(n) = c_0 + c_1 n + c_2 n^2 + \cdots c_m n^m$$

Max-cut problem & Travelling salesperson problem

Protein folding (hydrophobic/hydrophilic, self-avoiding model)

Prime factoring



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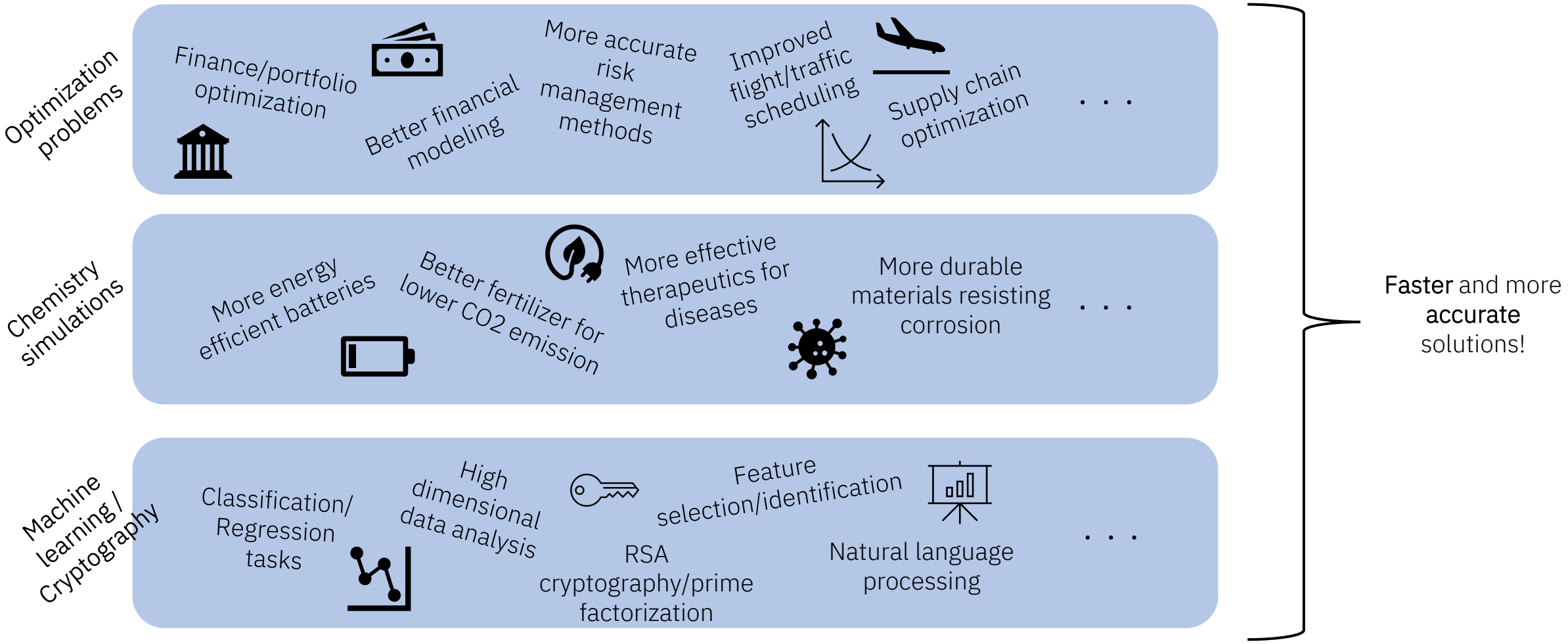
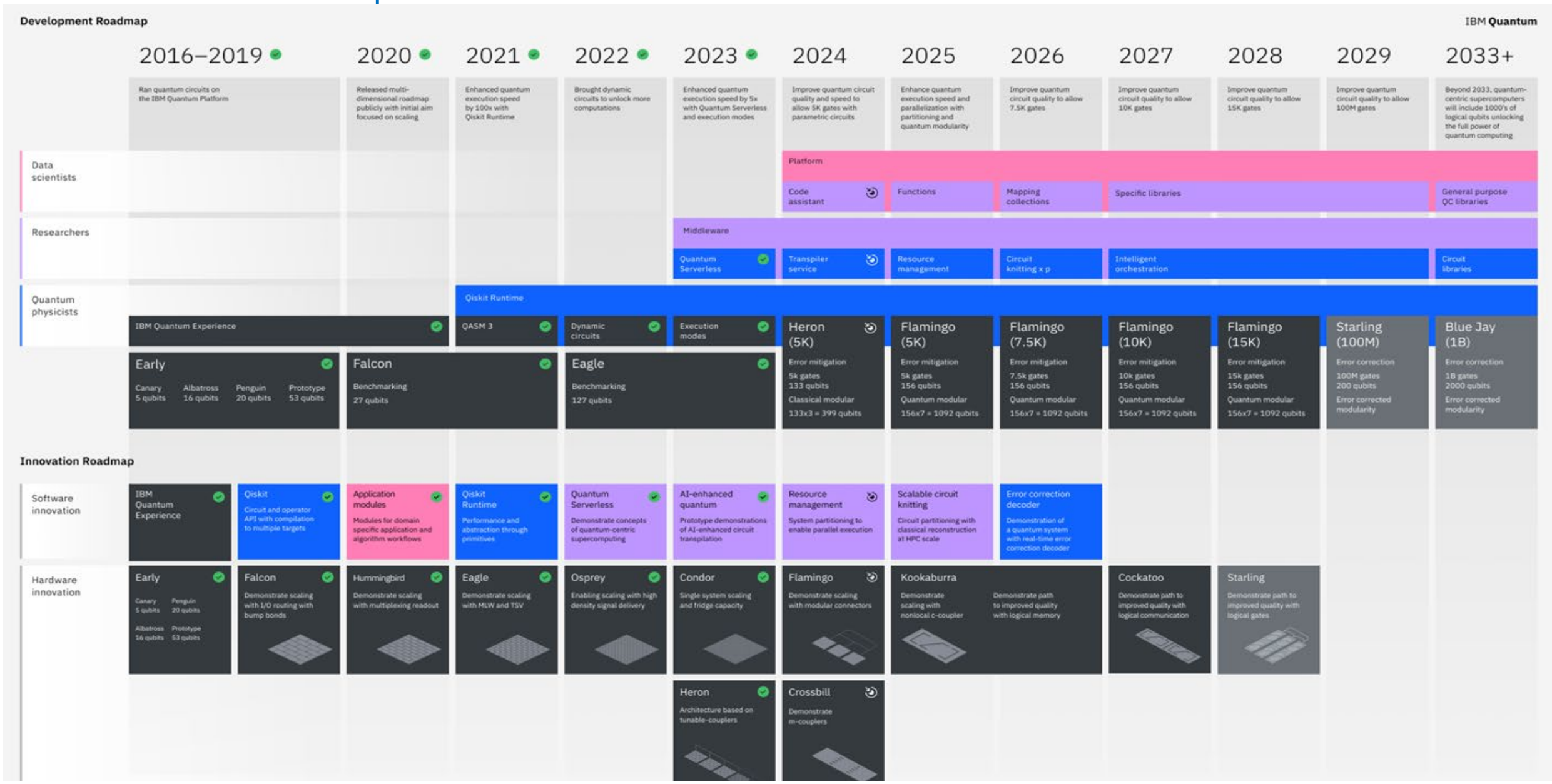
What can we do with quantum computers?



What's next in quantum?



Questions

Conclusions:

- Quantum computing is different from classical computing
- The differences are what make it valuable:
  - Superposition, entanglement
  - Unitary operations
- Groundbreaking research is already emerging at the utility scale