Designing Math Fonts

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BachoTEX 2004

Text Fonts Suitable for Mathematics

Characters and Glyphs

Font Dimensions

NewMath

LatinModern Math Fonts

Which text fonts are suitable for math?

A text font should meet the following requirements:

- The font design should be calm and unobtrusive.
- Italics should
  - be really italic (not oblique Roman): a a f f
  - have letterforms clearly distinct from Roman: S S X X
  - have a distinct slant: A A c c
- The font should have at least two weights.
  The difference must be recognizable in a single letter:

  $A\ \boldsymbol{A}\quad x\ \boldsymbol{x}\quad f\ \boldsymbol{f}$

SansSerif and Math

SansSerif fonts are in general less suitable for math.

- Many letters are too similar to certain symbols.
- For many letters there is only a slight difference between upper- and lowercase.

A few examples of confusable letters and symbols:

$\mathrm{C\ c}\ C\ c\ \subset \qquad \mathrm{O\ o}\ O\ o\ \circ\ 0 \qquad \mathrm{U\ u}\ U\ u\ \cup$

$\mathsf{C\ c}\ C\ c\ \subset \qquad \mathsf{O\ o}\ O\ o\ \circ\ 0 \qquad \mathsf{U\ u}\ U\ u\ \cup$

$\mathrm{I\ l}\ I\ l\ 1 \qquad \mathrm{T}\ T\ \top \qquad \mathrm{X\ x}\ X\ x\ \times$

$\mathsf{I\ l}\ I\ l\ 1 \qquad \mathsf{T}\ T\ \top \qquad \mathsf{X\ x}\ X\ x\ \times$

Special Requirements

Most text fonts will fail on some of the following requirements:

- Optical Sizes
- Width
- Greeks
- Special Letterforms

Special Requirements

- Optical Sizes
  - Very desirable for math typesetting:

    A^{A^A} i_{i_i} x^{x_x} $(a+b)^{(a+b)^{(a+b)}}$

    A^{A^A} i_{i_i} x^{x_x} $(a+b)^{(a+b)^{(a+b)}}$

  - Some OpenType fonts offer (typically 4) optical sizes:
    - Caption 6 – 8 point $[abcxyz)$
    - Regular 9 – 13 point $[abcxyz)$
    - Subhead 14 – 24 point $[abcxyz)$
    - Display 25 – 72 point $[abcxyz)$
  - But still a 5-point size (“scriptscript”) is missing here

Special Requirements

- Optical Sizes
- Width
  - For better differentiation from text italic, math italic should be a bit wider (about 5–10%)
  - Only ComputerModern has a particularly designed math italic
  - Very few fonts come with different widths which could be employed here

Special Requirements

- Optical Sizes
- Width
- Greeks
  - Mathematics needs a complete set of Greek letters
  - Even in fonts which come with a Greek alphabet some variant letterforms are missing

Special Requirements

- Optical Sizes
- Width
- Greeks
- Special Letterforms
  - Mathematics needs some special letters and letterforms
  - These letters are unlikely to pre-exist in text fonts
  - Some fonts come with a few math characters, but most of them are hardly suitable for mathematics

Characters and Glyphs

- Some Glyphs are not distinct enough for mathematics and must be redesigned
- Glyphs which do not pre-exist must be added

Greek Letters

Mathematics can't do without Greek letters.

- The Greek letters should
  - either come from the same font as the Latin letters
  - or should match the text font
    in style, general appearance, shape, and color
- Which letters are needed?
- Which letterforms are crucial?

All Greek letters should be available in Roman and italics:

$\Gamma\ \Delta\ \Theta\ \ldots\ \alpha\ \beta\ \gamma\ \ldots$
$\mathit{\Gamma}\ \mathit{\Delta}\ \mathit{\Theta}\ \ldots\ \mathit{\alpha}\ \mathit{\beta}\ \mathit{\gamma}\ \ldots$

The full set of variant letters is needed for math:

$\epsilon\ \ \theta\ \ \kappa\ \ \pi\ \ \rho\ \ \sigma\ \ \phi\ \ \ X\ \ \ \beta$
$\varepsilon\ \ \vartheta\ \ \varkappa\ \ \varpi\ \ \varrho\ \ \varsigma\ \ \varphi\ \ \ \mathit{X}\ \ \ \mathit{\beta}$

Some additional derived glyphs are needed:

$\nabla\ \ \amalg\ \ \upsilon\ \ \ \lambda\ \ \mathit{\lambda}\ \ \ \mathrm{\Im}\ \ \mathit{\Im}\ \ \ \mathrm{\imath}\ \ \mathit{\imath}$

Sometimes archaic Greek letters are needed:

$\mathit{F}\ \ \mathit{\varrho}\ \ \mathit{\eth}\ \ \mathit{\varsigma}$

Some (italic) Greek letterforms need special attention:

- a α
- κ $\varkappa$ x
- γ y
- δ δ
- The italic letters Greek “nu” and Latin “v” share the same form in many fonts: ν

  Here a special “round v” is needed
  (and a matching “round w”),
  also “nu” should have a clearly distinct shape:

  ν w ν (original glyph shapes in most fonts)
  υ w ν (new shapes suitable for math)

Special Letters

Any text font will lack some special Latin letters and “alphabetic symbols” needed in math.

At least these letters and symbols have to be designed and should match the overall design:

$$\flat \natural \sharp \text{þ} \eth \emptyset \jmath \nexists \propto \infty \prime \setminus * \hbar \hbar \wp \partial \partial$$

Also, Fraktur-R and Fraktur-I are needed and the first four Hebrew letters: $\mathfrak{R}\ \mathfrak{I}\ \aleph\ \beth\ \gimel\ \daleth$

In mathematics, the “open form” of *g* is preferred:

possible: g *good:* g

Math Symbols

Math symbols should match the letters and other “alphabetical” symbols in

- shape

$$a * b \quad i \longrightarrow \infty$$

$$\mathit{a} * \mathit{b} \quad \mathit{i} \longrightarrow \infty$$

- size

$$a + b \quad g \neq f \quad A \geq B$$

$$\mathit{a} + \mathit{b} \quad \mathit{g} \neq \mathit{f} \quad \mathit{A} \geq \mathit{B}$$

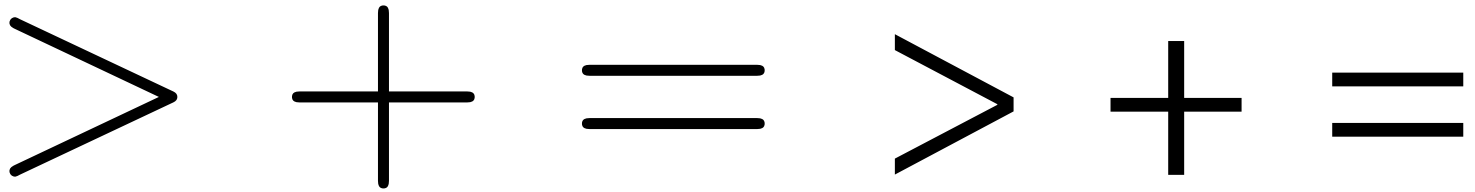
- color

$$a + b = c \quad \mathbf{a} + \mathbf{b} = \mathbf{c} \quad \boldsymbol{a} + \boldsymbol{b} = \boldsymbol{c}$$

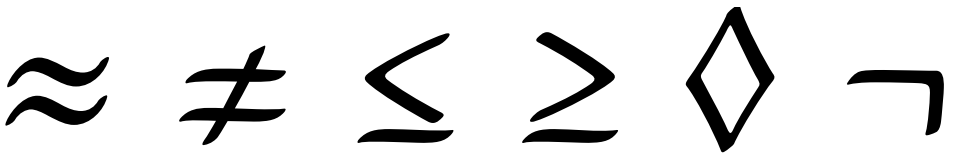
Math Symbols

There is little room for variation in the design of math symbols.

- The penshape could be
  round or slightly elliptic (but normally not rotated)
- Ends of lines could be rounded or cut-off:



- Handwritten appearance:
  rather exotic and of limited use



Glyphs in Extension Fonts

- Integral symbols and square root symbols should match the overall design $\int f(x)\,\mathrm{d}x \quad \sqrt{x}$
- Letter-derived operator symbols should match the design of their counterparts $\sum \Sigma \quad \prod \Pi$
- Other operator symbols should be slightly bolder than their base symbol $\bigcup \cup \quad \bigotimes \otimes$
- Larger parentheses and wider accents should follow the design of their base symbol

$$\left(\left(\ \right\}\right\} \quad \widehat{xyz}\ \widehat{xy}\ \hat{x}$$

Font Dimensions

- Fonts Dimensions explained in TEXbook's Appendix G
- Sidebearings and Kerning of Math Italic
- Super- and Subscript Positioning
- Accent Positioning

Basic Font Dimensions

The following fontdimen values can be taken from the text font:

- **fontdimen 1: slant per pt**
  determined by the slant of the italic font
  (usually given in degrees in a Type1 font)
- **fontdimen 5: x-height**
  given by the Roman (or italic) x's height

The following fontdimen values are **zero for math fonts:**

- **fontdimen 2: interword space**
- **fontdimen 3: interword stretch**
- **fontdimen 4: interword shrink**
- **fontdimen 7: extra space**

Basic Font Dimensions

fontdimen 6: quad width is a special case.

- It has a great influence on the appearance of formulas, as math units (mu) are calculated by it (1 mu = 1/18 quad).
- A good starting point is to set it to 10 pt (or to the value given by the text font).
- With optical sizes, it should be greater for smaller sizes and smaller for greater sizes.

Math symbol font dimensions: Math axis

fontdimen 22: axis_height is used in

- TEX's "vcenter" operation
- positioning fractions
  (fraction bars are centered on the axis)
- typesetting large delimiters, big operators and integrals
  (they all get vcentered):

$$f(z) = \frac{1}{2\pi \mathrm{i}} \int_{K_1} \left(\sum_{v=0}^{\infty} \frac{f(\zeta)(z-a)^v}{(\zeta-a)^{v+1}} \right) \mathrm{d}\zeta$$

Math symbol font dimensions: Math axis

The obvious choice for **axis_height**
is the *middle height of delimiters*
(as math delimiters should match text delimiters,
and math delimiters get centered on the axis).

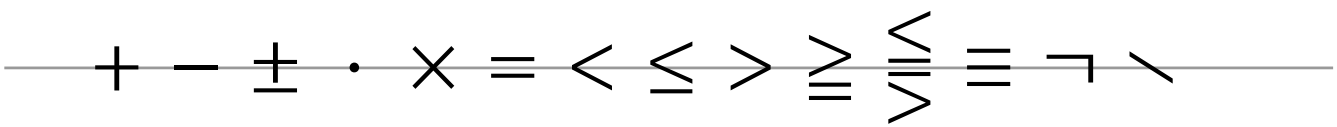
But this might be too high or too low in some fonts.

TEX centers the (tfm) bounding box, not the actual glyph.

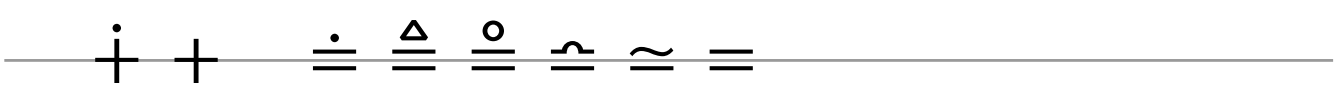
So – within narrow bounds – it is possible
to set axis_height to a slightly different value
(and to adjust the vertical metrics of delimiters accordingly).

Math symbol font dimensions: Math axis

Many mathematical symbols are vertically centered on the axis:



This is not done by TEX, but is a matter of font design.
In fact, it couldn't be done by TEX, as derived symbols should appear at the same height as their base symbol:



Therefore: One should choose axis_height first, and then start the actual font design.

Math symbol font dimensions: Fractions

In typesetting (generalized) fractions

- **fontdimens 8–10: num1, . . . , num3**
  are used to shift up the numerator,
- **fontdimens 11–12: denom1, denom2**
  to shift down the denominator

by a certain amount with respect to the current baseline.

fontdimens 20–21: delim1, delim2
specify a minimum height for delimiters
around generalized fractions

Math symbol font dimensions: Fractions

These fontdimens are quite font-dependent.

As a general rule, these should be chosen (for display) so that

- a numerator should (almost) stand so high as to allow for descenders or simple indices
- a denominator should (almost) stand so low as to allow for ascenders (or simple exponents)

(even if it doesn't have a descender or ascender, respectively)

$$\frac{\frac{n}{3}\cdot\frac{2}{3}\cdot\frac{a}{3}\cdot\frac{af}{3}\cdot\frac{af_2}{3}}{} \qquad \frac{\frac{2}{n}\cdot\frac{2}{3}\cdot\frac{2}{a}\cdot\frac{2}{af}\cdot\frac{2}{a^2}\cdot\frac{2}{af^2}}{}$$

For text style, the appearance can be a bit more cramped (so that simple fractions fit the normal baselineskip)

Math symbol font dimensions: Super- and subscripts

fontdimens 18–19: sup_drop, sub_drop
are not taken from the current main font,
but from the current “scriptfont”
(relevant with optical sizes).

They represent minimum (starting) amounts by which superscript and subscript will be moved up and down.

Their values are quite font-dependent
(**sup_drop** 0.35–0.5, **sub_drop** 0.02–0.1)

Math symbol font dimensions: Super- and subscripts

- **fontdimens 16–17: sub1, sub2**
  are used for subscript positioning:
  - sub1 for a subscript alone (empty superscript field).
    The value in ComputerModern seems too high:
    $a_1 x_2 f_2^1 \qquad a_1 x_2 f_2^1$
  - sub2 in positioning
    a joint superscript/subscript combination
- **fontdimens 13–15: sup1, . . . , sup3**
  are used for superscript positioning.
  They are chosen by TEX according to style:
  - sup3 in “cramped style” (radicands and denominators)
  - sup1 in display
  - sup2 otherwise

Math extension font dimensions: rule_thickness

fontdimen 8: default_rule_thickness

- should be set to the height (thickness) of horizontal bars in symbols like + − =
- must equal the height of all root symbols in extension fonts (due to TEX's "radical" operation)

Used in the typesetting of

- overline and underline
- root symbols (vinculum)
- generalized fractions, fraction bars
- positioning of joint subscript/superscript combinations

Math extension font dimensions: big_op_spacing

fontdimens 9–13: big_op_spacing1, . . . , big_op_spacing5

- used exclusively to typeset limits
  on big operator symbols (Appendix G, Rule 13a)
- fontdimens **9** and **11** for superscript positioning
- fontdimens **10** and **12** for subscript positioning
- fontdimen **13** for adding a kern
  above (superscript) or below (subscript)

Good starting values could be taken from
existing extension fonts like cmex10.

9	0.11–0.12	**10**	0.16–0.17	**13**	0.1–0.14
11	0.2–0.21	**12**	0.6–0.62		

Sidebearings of Math Italic

Italic (especially lowercase) letters from a text font typically protrude their bounding box.

$$|a|b|c|d|e|f|g|h|i|j|k|l|m|n|o|p|q|r|s|t|u|v|w|x|y|z|$$

- A starting point for math italic adjustment is to give all italic letters equal sidebearings.
- Further adjustment depends on font design and visual appearance of individual characters.
- Each character should “look right” (centered and non-touching) when enclosed in delimiters.

$$|a|b|c|d|e|f|g|h|i|j|k|l|m|n|o|p|q|r|s|t|u|v|w|x|y|z|$$

Kerning of Math Italic

This is highly font-dependent.

It also depends on the encoding
(NewMath's "MathCore" has math italic,
Roman and Italic Greek, and a Roman "d"
and some other letters in the same font).

Math italic letters should be recognizable
as individual letters,
not as part of words.

Typically, combinations with italic *f* need special attention.

Super- and Subscript Positioning

TEX (mis-)uses the italic correction for this.

$$\Gamma_2\ \Gamma^2\ \Gamma_2^2 \qquad \Delta_2\ \Delta^2\ \Delta_2^2 \qquad P_2\ P^2\ P_2^2 \qquad f_2\ f^2\ f_2^2$$

For most letters a zero value is just right.

These letters may need a non-zero value:

$$A\ F\ P\ T\ V\ W\ Y$$

$$\Gamma\ \Delta\ \Lambda\ \mathrm{P}\ \mathrm{T}\ \Upsilon\ \Psi\ \mathrm{Y}\ \nabla$$

$$\mathit{\Gamma}\ \mathit{\Delta}\ \mathit{\Lambda}\ \mathit{\Upsilon}\ \mathit{\Psi}$$

$$\lambda\ \wp\ \partial$$

Accent Positioning

This is done by TEX's **skewchar** mechanism.

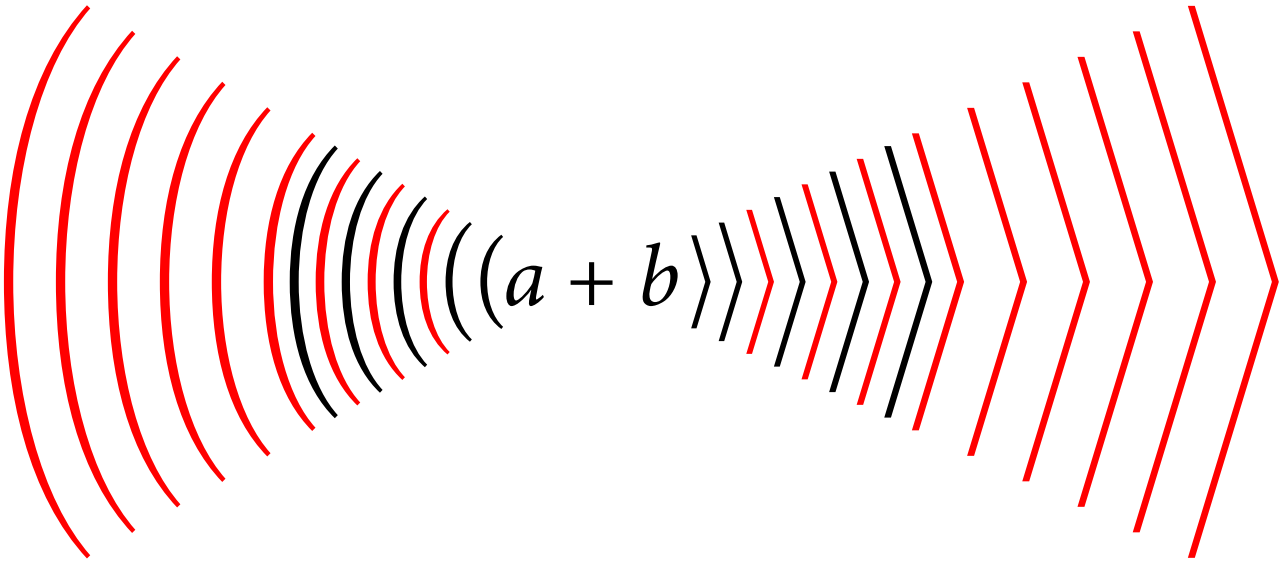
For each character that needs adjustment, there must be a kerning pair of (character + skewchar) in the tfm file.

The kerning value is then used for accent positioning.

$\vec{A}\ \overleftarrow{b}\ \hat{\imath}\ \tilde{k}$

NewMath

- Current version 0.59a
- Standard encodings for all math fonts
  (using virtual fonts to map to existing math fonts)
- 256 characters per font
- better kerning in math
- more glyphs – e.g. more sizes for delimiters and accents



NewMath Encodings

Currently characters are defined in 6 encodings:

- Math Core
- Math Symbol Principal
- Math Extension Principal

- Math Symbol One
- Math Symbol Two

- Math Extension One

NewMath Encodings – Math Core

Alphabetic and font-dependent symbols:

- Math Italic

  $ABCDEF \ldots XYZabcdef \ldots xyz$

- Roman and italic Greek letters

  $\Gamma\mathit{\Gamma}\Delta\mathit{\Delta}\Theta\mathit{\Theta} \ldots \alpha\mathit{\alpha}\beta\mathit{\beta}\gamma\mathit{\gamma} \ldots \nabla \amalg \ni \ldots$

- other “alphabetic” symbols

  $\aleph \beth \daleth \gimel \flat \natural \sharp \text{þ} \eth \emptyset \jmath \ell \propto \infty \prime \backslash * \hbar \hbar \wp \partial \partial$

- basic delimiters

  $([\{ \lfloor \lceil \langle / | \| \backslash \rangle \rceil \rfloor \}])$

NewMath Encodings – Math Symbol Principal

- Formal Script (or Calligraphic) alphabet (Latin upper- and lowercase and digits)

  $\mathscr{ABCDEF} \ldots \mathscr{XYZ} \ldots$

- the most common mathematical symbols

  $+ - = \pm \mp \cap \sqcup \wedge \oplus \subset \supseteq < \geq \ldots$

- accents

  $\grave{x}\ \acute{x}\ \vec{x}\ \hat{x}\ \tilde{x} \ldots$

NewMath Encodings – Math Symbol One

- Blackboard bold (or “Doublestroke”) alphabet (Latin upper- and lowercase and digits)

  $\mathbb{C}\mathbb{N}\mathbb{Q}\mathbb{R}\mathbb{Z}\ldots\mathbb{b}\mathbb{k}\mathbb{l}\ldots\mathbb{1}\ldots$

- additional mathematical symbols

  $\circledast\ \circlearrowleft\ \Uparrow\ \rfloor\ \nearrow\ \leqslant\ \lneqq\ \gneqq\ \leqslant\ \Subset\ \Mapsto\ \Cap\ \Doteqdot\ \because\ \vDash\ \nparallel\ \boxtimes\ \complement\ \diagdown$

NewMath Encodings – Math Symbol Two

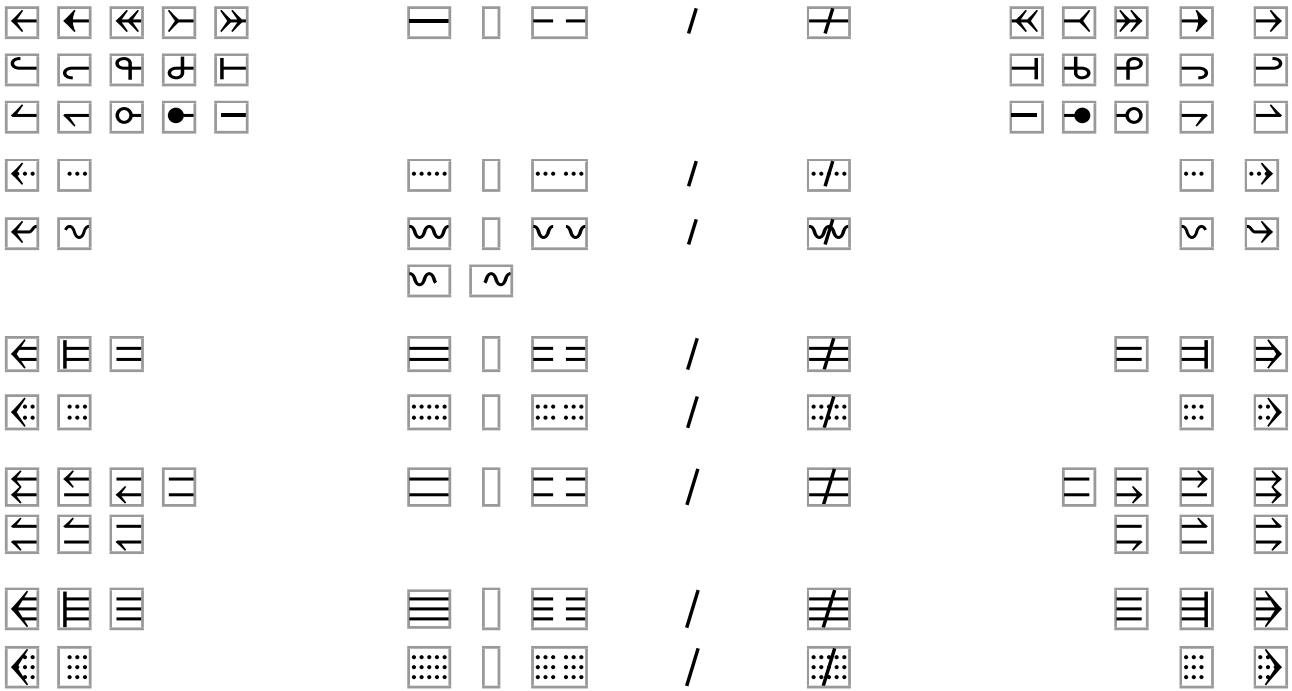
- Fraktur alphabet
  (Latin upper- and lowercase; oldstyle digits)

  $\mathfrak{ABC}\ldots\mathfrak{XYZ}\ldots\mathfrak{abc}\ldots\mathfrak{xyz}\ldots 0123456789$
- some additional accents, delimiters, and other symbols

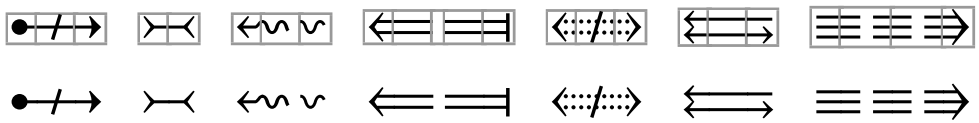
  $\hat{x}\ \check{x}\ \hat{\hat{x}}\ \check{\check{x}}\ \big(\ \big\langle\ \big\{\ \big[\!\big[\ \big]\!\big]\ \big\}\ \big\rangle\ \big)\ \ \biguplus\ \square\ \boxminus\ \circledcirc$
- an “Arrow Construction Kit” consisting of:
  - left and right endings
  - (repeatable) middle parts
  - negated middle parts
  - (repeatable) middle parts with a gap

Arrow Construction Kit (Math Symbol Two)

left middle right



Examples:

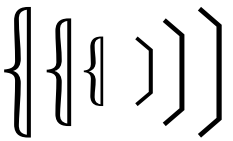
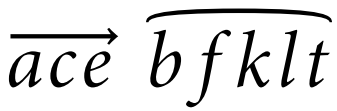


NewMath Encodings – Math Extension Principal

- big operators $\Sigma \sum \Pi \coprod \cap \bigcup \otimes \bigoplus$
- integral symbols $\int \iiint \oint \oiint$
- root symbols $\surd \surd \surd \surd$
- large delimiters $\left(\left(\left(\right\}\right\}\right\}$
- wide accents $\widetilde{ace}\ \widehat{bfklt}$
- vertical arrow parts $\updownarrow \Uparrow$
- over- and underbrace parts $\underbrace{ab\cdots c}\ \overbrace{uvwxyz}$

NewMath Encodings – Math Extension One

Additional Extension Font Characters:

- big operators 
- integral symbols 
- large delimiters 
- wide accents 

LatinModern Math

What will it be?

A set of math fonts

- accompanying LatinModern text fonts
- as a replacement for ComputerModern math fonts
- in Type1 format with open MetaType1 sources
- encoded in NewMath Standard, thus offering:
  - many additional symbols
  - improved kerning and positioning for many symbols
- with the final aim of complete Unicode math support

LatinModern Math

What should it be?

- (maybe) metrics-compatible with ComputerModern
- (definitely) not glyph-compatible
- consistently designed throughout
- maybe offering glyph variants for some symbols:
  - additional or enhanced symbols
    (e.g. upright integral symbols)
  - “old versions” kept for CM / AMS compatibility
    (e.g. Hebrew letters)

Conclusion

Main difficulties:

- Design of additional letters and “alphabetic” symbols
- Creation of extension fonts
  (restriction to 15 heights, 15 depths . . .)
- Fiddling with character widths, italic correction, skewchar

Minor difficulties:

- Design of “technical” math symbols
- Font dimensions