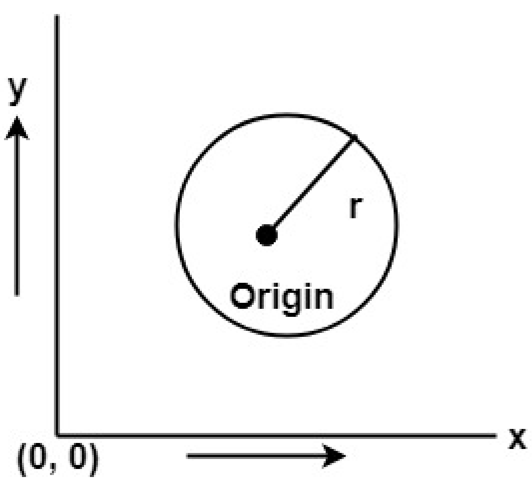
CIRCLE

Tara Qadr
2024-2025
3rd Stage
Computer department
College of science
University of sulaimani

Defining a Circle:

- Circle is an eight-way symmetric figure. The shape of circle is the same in all quadrants. In each quadrant, there are two octants. If the calculation of the point of one octant is done, then the other seven points can be calculated easily by using the concept of eight-way symmetry.
- For drawing, circle considers it at the origin. If a point is $P_1(x, y)$,

then the other seven points will be



P2 (x, -y)

P3(-x, -y)

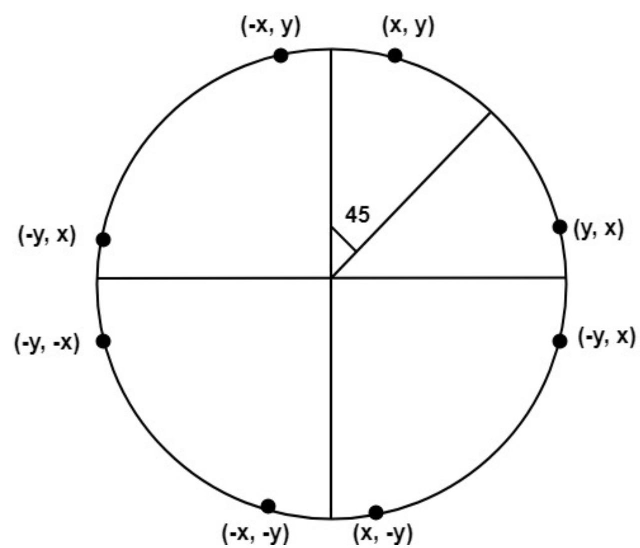
P4 (-x, y)

P5 (y, x)

P6 (y, -x)

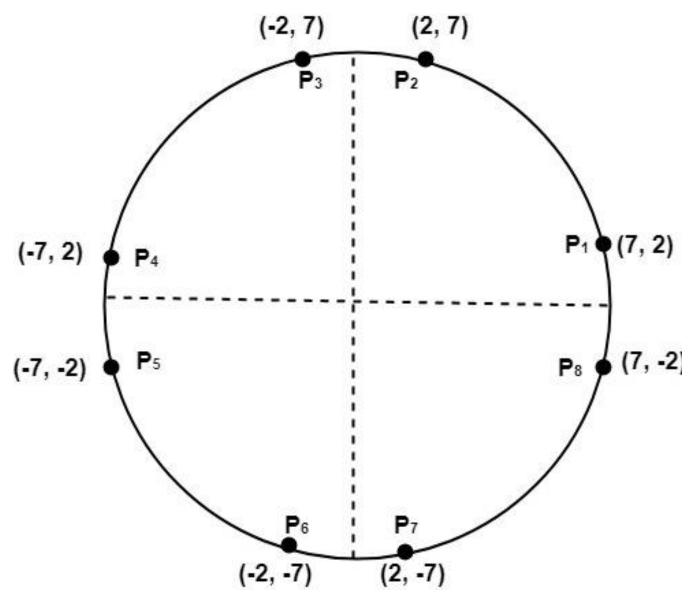
P7 (-y, -x)

P8 (-y, x)



Example:

- Let we determine a point (2, 7) of the circle .
- then other points will be (2, -7), (-2, -7), (-2, 7), (7, 2), (-7, 2), (-7, -2), (7, -2)

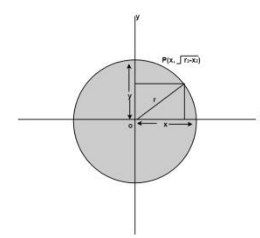


Eight way symmetry of a Circle

mathematical definition of a circle

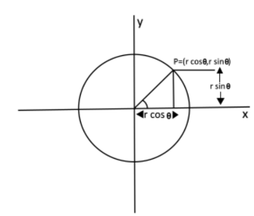
- There are two standards methods of mathematically defining a circle centered at the origin.
- 1 Defining a circle using Polynomial Method

$$y^2=r^2-x^2$$

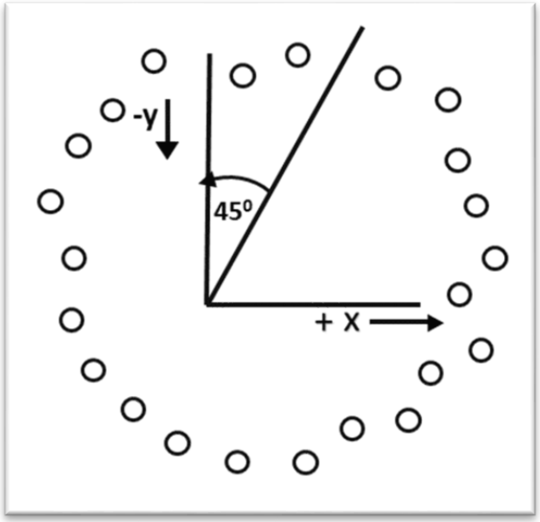


- 2 Defining a circle using Polar Co-ordinates

$$x=r\cos\theta \qquad y = r\sin\theta$$



Bresenham's Circle Algorithm:

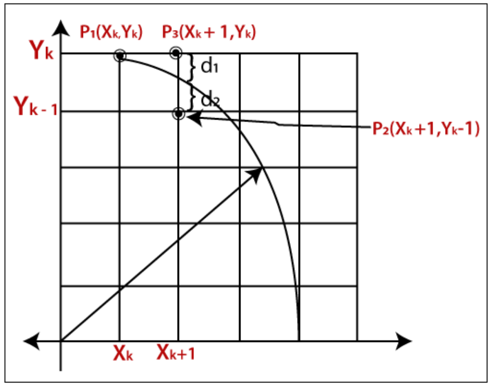


- **Formula :**
- $d0 = 3 - 2R \quad \begin{Bmatrix} x0 = 0 \\ y0 = R \end{Bmatrix}$
- $if\ \underline{di =< 0}\ \ then\ NC = (x0 + 1\ , y0)$

  $$di + 1 = di + 4X + 6$$

$else\ if\ \underline{di > 0}$ the n $\mathrm{NC} = (\mathrm{xi} + 1, \mathrm{yi} - 1)$

$$di + 1 = di + 4(xi - yi) + 10$$



Bresenham's Circle Algorithm

- Example : plot(draw) a circle with radiance= 8 by using Bresenham's algorithm .

Answer:

Iteration	X	Y	Desertion parameter
0	X0=0	Y0=8	**3-2R=3 -2 * 8=3-16= -13<0**

- **Formula :**
- $d0 = 3 - 2R \quad \begin{Bmatrix} x0 = 0 \\ y0 = R \end{Bmatrix}$
- $if\ di =< 0 \quad then\ NC = (x0 + 1\ , y0)$

$$di + 1 = di + 4X + 6$$

$else\ if\ di > 0\ \ \text{the n NC} = (\text{xi} + 1, \text{yi} - 1)$

$$di + 1 = di + 4(xi - yi) + 10$$

Bresenham's Circle Algorithm

- Example : plot(draw) a circle with radiance= 8 by using Bresenham's algorithm .

Answer:

Iter.	X	Y	Desertion parameter
0	X0=0	Y0=8	**d0=3-2R=3 -2 * 8=3-16=-13 <0**
1	X1=x0+1=1	Y1=y0=8	**d1= -13+4*1+6= -3<0**

- **Formula :**
- $d0 = 3 - 2R \quad \begin{Bmatrix} x0 = 0 \\ y0 = R \end{Bmatrix}$
- $if\ di =< 0 \quad then\ NC = (x0 + 1\ , y0)$

$$di + 1 = di + 4X + 6$$

$else\ if\ di > 0 \ \ \text{the n NC} = (\text{xi} + 1, \text{yi} - 1)$

$$di + 1 = di + 4(xi - yi) + 10$$

Bresenham's Circle Algorithm

- Example : plot(draw) a circle with radiance= 8 by using Bresenham's algorithm .

Answer:

Iter.	X	Y	Desertion parameter
0	X0=0	Y0=8	**d0=3-2R=3 -2 * 8=3-16=-13 <0**
1	X1=x0+1=1	Y1=y0=8	**d1= -13+4*1+6= -3<0**
2	X2=x1+1=2	Y2=y1=8	**d2= -3+4*2+6= 11>0**

- **Formula :**
- $d0 = 3 - 2R \quad \begin{Bmatrix} x0 = 0 \\ y0 = R \end{Bmatrix}$
- $if\ di =< 0 \quad then\ NC = (x0 + 1\ , y0)$

$$di + 1 = di + 4X + 6$$

$$else\ if\ di > 0 \ \ \text{the n NC} = (\text{xi} + 1, \text{yi} - 1)$$

$$di + 1 = di + 4(xi - yi) + 10$$

Bresenham's Circle Algorithm

- Example : plot(draw) a circle with radiance= 8 by using Bresenham's algorithm .

Answer:

Iter.	X	Y	Desertion parameter
0	X0=0	Y0=8	**d0=3-2R=3 -2 * 8= 3-16=-13 <0**
1	X1=x0+1=1	Y1=y0=8	**d1= -13+4*1+6= -3<0**
2	X2=x1+1=2	Y2=y1=8	**d2= -3+4*2+6= 11>0**
3	X3=x2+1=3	Y3=y2-1=7	**d3= 11+4 (3-7)+10= 5 >0**
	4	6	$d4=5+4(-2)+10$

$=15-8$

$=7$

- **Formula :**
- $d0 = 3 - 2R \quad \begin{Bmatrix} x0 = 0 \\ y0 = R \end{Bmatrix}$
- $if\ di =< 0\ then\ NC = (x0 + 1\ , y0)$

  $di + 1 = di + 4X + 6$

  $else\ if\ di > 0\ \text{then}\ \text{NC} = (\text{xi} + 1, \text{yi} - 1)$

  $di + 1 = di + 4(xi - yi) + 10$

5 5 $d5=7+0+16$

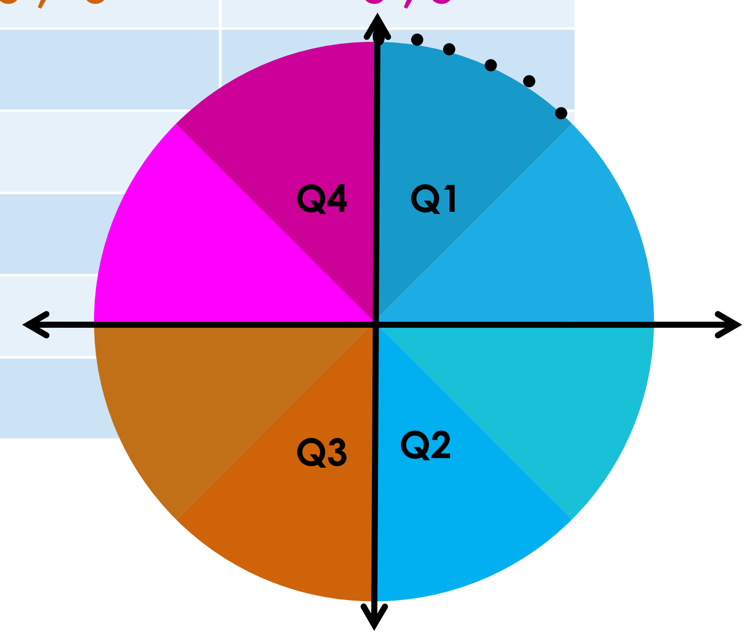
Bresenham's Circle Algorithm

- Example : plot(draw) a circle with radiance= 8 by using Bresenham's algorithm .

Answer:

Iter.	X	Y	Desertion parameter
0	X0=0	Y0=8	**d0=3-2R=3 -2 * 8=3-16=-13 <0**
1	X1=x0+1=1	Y1=y0=8	**d1= -13+4*1+6= -3<0**
2	X2=x1+1=2	Y2=y1=8	**d2= -3+4*2+6= 11>0**
3	X3=x2+1=3	Y3=y2-1=7	**d3= 11+4 (3-7)+10= 5 >0**
4	X4=x3+1=4	Y4=y2-1=6	**d4= 5+4 (4-6)+10= 7>0**
5	X5=x4+1=5	Y5=y4-1=5	**X=Y the algorithm stop here**

Q1(x , y)	Q2(x , -y)	Q3(-x , -y)	Q4(-x , y)
0 , 8	0 , -8	0 , -8	0 , 8
1 , 8	1 , -8	-1 , -8	-1 , 8
2 , 8	2 , -8	-2 , -8	-2 , 8
3 , 7	3 , -7	-3 , -7	-3 , 7
4 , 6	4 , -6	-4 , -6	-4 , 6
5 , 5	5 , -5	-5 , -5	-5 , 5
6 , 4	6 , -4		
7 , 3	7, -3		
8 , 2	8, -2		
8 , 1	8 ,-1		
8 , 0	8, 0		



Midpoint Circle Algorithm steps

1. Input radius r and circle center (x_c, y_c) and obtain the first point on the circumference of the circle centered on the origin as

   $(x_0, y_0) = (0, r)$

2. Calculate the initial value of the decision parameter as

$$p_0 = \frac{5}{4} - r \quad \text{or} \quad \left[p_0 = 1 - r \quad \text{for r an integer}\right]$$

3. At each x_k position starting at k = 0 , perform the following test. If $p_k < 0$, the next point along the circle centered on (0, 0) is (x_k+1, y_k) and

   $p_{k+1} = p_k + 2x_{k+1} + 1$

   Otherwise the next point along the circle is (x_k+1, y_k-1) and

   $p_{k+1} = p_k + 2x_{k+1} + 1 - 2y_{k+1}$

   Where $2x_{k+1} = 2x_k+2$ and $2y_{k+1} = 2y_k - 2$

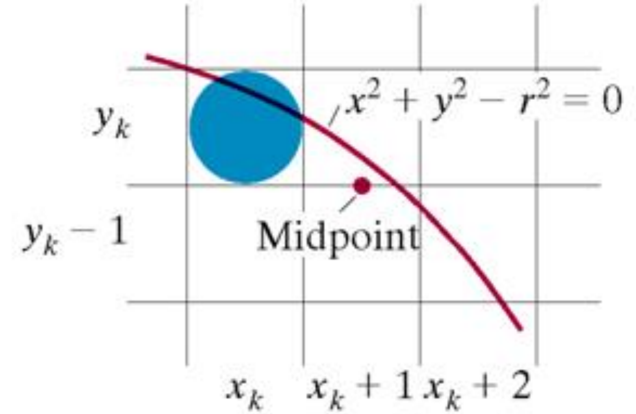
Midpoint Circle Algorithm steps (2)

4. Determine symmetry points in the other seven octants

5. Move each calculated pixel position (x, y) on to the circular path centered on (x_c, y_c) and plot the coordinate values

   $x = x + x_c$, $y = y + y_c$

6. Repeat steps 3 through 5 until x >= y



Midpoint between candidate pixels at sampling position $x_k + 1$ along a circular path.

Example: Midpoint Circle Algorithm

$r = 10$ $(x_0, y_0) = (0, 10)$ $p_0 = 1 - r = -9$, $2x_0 = 0$, $2y_0 = 20$

$-9<0$
$-9+2x+1=$
$-9+2*1+1=-6$

$-6<0$
$-6+2x+1=$
$-6+2*2+1=-1$

$-1<0$
$-1+2x+1=$
$-1+2*3+1=6$

$6>0$
$p_{k+1} = p_k + 2x_{k+1} + 1 - 2y_{k+1}$
$6+2x+1-2y=$
$6+2*4+1-2*9= -3$

k	pk	X_{k+1}	y_{k+1}	$2X_{k+1}$	$2y_{k+1}$
0	-9	0	10	0	20
1	-6	1	10	2	20
2	-1	2	10	4	20
3	6	3	10	6	20
4	-3	4	9	8	18
5	8	5	9	10	18
6	5	6	8	12	16
7		7	7	14	14

Example 2

$r = 8 \quad (x_0, y_0) = (0, 8)$

$p_0 = 1-8 = -7$, $2x_0 = 0$, $2y_0 = 16$

k	pk	X_{k+1}	y_{k+1}	$2X_{k+1}$	$2y_{k+1}$
0	-9	0	10	0	20
1	-6	1	10	2	20
2	-1	2	10	4	20
3	6	3	10	6	20
4	-3	4	9	8	18
5	8	5	9	10	18
6	5	6	8	12	16
7		7	7	14	14

Calculate of all quarters

Q1	Q2(-x, y)	Q3((-x,-y)	Q4(x.-y)
0,8	0,8	0,-8	0,-8
1,8	-1,8	-1,-8	1,-8
2,8	-2,8	-2,-8	2,-8
3,7	-3,7	-3,-7	3,-7
4,7	-4,7	-4,-7	4,-7
5,6	-5,6	-5,-6	5,-6
6,5	-6,5	-6,-5	6,-5
7,4	-7,4	-7,-4	7,-4
7,3	-7,3	-7,-3	7,-3
8,2	-8,2	-8,-2	8,-2
8,1	-8,1	-8,-1	8,-1
8,0	-8,0	-8,0	8,0

Example 3

r = 6 **(x_c ,y_c) = (5, 5)** use midpoint algorithm

k	P_k	X_{k+1}	y_{k+1}	$2X_{k+1}$	$2y_{k+1}$	$X_{k+1}+X_c$	$y_{k+1}+Y_c$
0	-5	0	6	0	12	5	11
1	-2	1	6	2	12	6	11
2	3	2	6	4	12	7	11
3	0	3	5	6	10	8	10
4		4	4	8	8	9	9

Thank you