Introduction to Artificial Intelligence with Python

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Inference Rules

Modus Ponens

If it is raining, then Harry is inside.

It is raining.

Harry is inside.

Modus Ponens

$$\alpha \to \beta$$

$$\alpha$$

$$\beta$$

And Elimination

Harry is friends with Ron and Hermione.

Harry is friends with Hermione.

And Elimination

$$\frac{\alpha \wedge \beta}{\alpha}$$

Double Negation Elimination

It is not true that Harry did not pass the test.

Harry passed the test.

Double Negation Elimination

$$\frac{\neg(\neg\alpha)}{\alpha}$$

Implication Elimination

If it is raining, then Harry is inside.

It is not raining or Harry is inside.

Implication Elimination

$$\frac{\alpha \to \beta}{\neg\alpha \vee \beta}$$

Biconditional Elimination

It is raining if and only if Harry is inside.

If it is raining, then Harry is inside,
and if Harry is inside, then it is raining.

Biconditional Elimination

$$\frac{\alpha \leftrightarrow \beta}{(\alpha \rightarrow \beta) \land (\beta \rightarrow \alpha)}$$

De Morgan's Law

It is not true that both
Harry and Ron passed the test.

Harry did not pass the test
or Ron did not pass the test.

De Morgan's Law

$$\frac{\neg(\alpha \wedge \beta)}{\neg\alpha \vee \neg\beta}$$

De Morgan's Law

It is not true that
Harry or Ron passed the test.

Harry did not pass the test
and Ron did not pass the test.

De Morgan's Law

$$\frac{\neg(\alpha \lor \beta)}{\neg\alpha \land \neg\beta}$$

Distributive Property

$$\frac{(\alpha \wedge (\beta \vee \gamma))}{(\alpha \wedge \beta) \vee (\alpha \wedge \gamma)}$$

Distributive Property

$$\frac{(\alpha \vee (\beta \wedge \gamma))}{(\alpha \vee \beta) \wedge (\alpha \vee \gamma)}$$

Search Problems

- initial state
- actions
- transition model
- goal test
- path cost function

Theorem Proving

- initial state: starting knowledge base
- actions: inference rules
- transition model: new knowledge base after inference
- goal test: check statement we're trying to prove
- path cost function: number of steps in proof

Resolution

(Ron is in the Great Hall) ∨ (Hermione is in the library)

Ron is not in the Great Hall

Hermione is in the library

$$P \lor Q$$

$$\neg P$$

$$Q$$

$$\frac{\begin{array}{c} P \lor Q_1 \lor Q_2 \lor \ldots \lor Q_n \\ \neg P \end{array}}{Q_1 \lor Q_2 \lor \ldots \lor Q_n}$$

(Ron is in the Great Hall) ∨ (Hermione is in the library)

(Ron is not in the Great Hall) ∨ (Harry is sleeping)

(Hermione is in the library) ∨ (Harry is sleeping)

$$\frac{P \lor Q \qquad \neg P \lor R}{Q \lor R}$$

$$\frac{P \lor Q_1 \lor Q_2 \lor \ldots \lor Q_n \qquad \neg P \lor R_1 \lor R_2 \lor \ldots \lor R_m}{Q_1 \lor Q_2 \lor \ldots \lor Q_n \lor R_1 \lor R_2 \lor \ldots \lor R_m}$$

clause

a disjunction of literals

e.g. $P \lor Q \lor R$

conjunctive normal form

logical sentence that is a conjunction of clauses

e.g. $(A \vee B \vee C) \wedge (D \vee \neg E) \wedge (F \vee G)$

Conversion to CNF

- Eliminate biconditionals
  - turn $(\alpha \leftrightarrow \beta)$ into $(\alpha \rightarrow \beta) \wedge (\beta \rightarrow \alpha)$
- Eliminate implications
  - turn $(\alpha \rightarrow \beta)$ into $\neg\alpha \vee \beta$
- Move $\neg$ inwards using De Morgan's Laws
  - e.g. turn $\neg(\alpha \wedge \beta)$ into $\neg\alpha \vee \neg\beta$
- Use distributive law to distribute $\vee$ wherever possible

Conversion to CNF

$$(P \vee Q) \rightarrow R$$

$$\neg(P \vee Q) \vee R$$

eliminate implication

$$(\neg P \wedge \neg Q) \vee R$$

De Morgan's Law

$$(\neg P \vee R) \wedge (\neg Q \vee R)$$

distributive law

Inference by Resolution

$$P \lor Q$$

$$\neg P \lor R$$

$$(Q \lor R)$$

$$P \vee Q \vee S$$

$$\neg P \vee R \vee S$$

$$(Q \vee S \vee R \vee S)$$

$$\frac{\begin{array}{c} P \vee Q \vee S \\ \neg P \vee R \vee S \end{array}}{(Q \vee R \vee S)}$$

$$\frac{P \qquad \neg P}{(\,)}$$

Inference by Resolution

- To determine if $\mathrm{KB} \vDash \alpha$:
  - Check if $(\mathrm{KB} \wedge \neg\alpha)$ is a contradiction?
    - If so, then $\mathrm{KB} \vDash \alpha$.
    - Otherwise, no entailment.

Inference by Resolution

- To determine if $\mathrm{KB} \vDash \alpha$:
  - Convert $(\mathrm{KB} \land \neg\alpha)$ to Conjunctive Normal Form.
  - Keep checking to see if we can use resolution to produce a new clause.
    - If ever we produce the **empty** clause (equivalent to False), we have a contradiction, and $\mathrm{KB} \vDash \alpha$.
    - Otherwise, if we can't add new clauses, no entailment.

Inference by Resolution

Does $(A \vee B) \wedge (\neg B \vee C) \wedge (\neg C)$ entail A?

$$(A \vee B) \wedge (\neg B \vee C) \wedge (\neg C) \wedge (\neg A)$$

$(A \vee B)$ $\quad$ $(\neg B \vee C)$ $\quad$ $(\neg C)$ $\quad$ $(\neg A)$

Inference by Resolution

Does $(A \vee B) \wedge (\neg B \vee C) \wedge (\neg C)$ entail A?

$$(A \vee B) \wedge (\neg B \vee C) \wedge (\neg C) \wedge (\neg A)$$

$(A \vee B)$ $\quad$ $\underline{(\neg B \vee C)}$ $\quad$ $\underline{(\neg C)}$ $\quad$ $(\neg A)$

Inference by Resolution

Does $(A \lor B) \land (\neg B \lor C) \land (\neg C)$ entail A?

$$(A \lor B) \land (\neg B \lor C) \land (\neg C) \land (\neg A)$$

$(A \lor B)$ $\quad$ $(\neg B \lor C)$ $\quad$ $(\neg C)$ $\quad$ $(\neg A)$ $\quad$ $(\neg B)$

Inference by Resolution

Does $(A \vee B) \wedge (\neg B \vee C) \wedge (\neg C)$ entail A?

$$(A \vee B) \wedge (\neg B \vee C) \wedge (\neg C) \wedge (\neg A)$$

$(A \vee B) \quad (\neg B \vee C) \quad (\neg C) \quad (\neg A) \quad (\neg B)$

Inference by Resolution

Does $(A \vee B) \wedge (\neg B \vee C) \wedge (\neg C)$ entail A?

$$(A \vee B) \wedge (\neg B \vee C) \wedge (\neg C) \wedge (\neg A)$$

$\underline{(A \vee B)}$ $\quad (\neg B \vee C) \quad (\neg C) \quad (\neg A) \quad$ $\underline{(\neg B)}$

Inference by Resolution

Does $(A \lor B) \land (\neg B \lor C) \land (\neg C)$ entail A?

$$(A \lor B) \land (\neg B \lor C) \land (\neg C) \land (\neg A)$$

$\underline{(A \lor B)}$ $(\neg B \lor C)$ $(\neg C)$ $(\neg A)$ $\underline{(\neg B)}$ (A)

Inference by Resolution

Does $(A \vee B) \wedge (\neg B \vee C) \wedge (\neg C)$ entail A?

$$(A \vee B) \wedge (\neg B \vee C) \wedge (\neg C) \wedge (\neg A)$$

$(A \vee B)$ $(\neg B \vee C)$ $(\neg C)$ $(\neg A)$ $(\neg B)$ (A)

Inference by Resolution

Does $(A \lor B) \land (\neg B \lor C) \land (\neg C)$ entail A?

$$(A \lor B) \land (\neg B \lor C) \land (\neg C) \land (\neg A)$$

$(A \lor B)$ $\quad (\neg B \lor C)$ $\quad (\neg C)$ $\quad \underline{(\neg A)}$ $\quad (\neg B)$ $\quad \underline{(A)}$

Inference by Resolution

Does $(A \vee B) \wedge (\neg B \vee C) \wedge (\neg C)$ entail A?

$$(A \vee B) \wedge (\neg B \vee C) \wedge (\neg C) \wedge (\neg A)$$

$(A \vee B)$ $(\neg B \vee C)$ $(\neg C)$ $\underline{(\neg A)}$ $(\neg B)$ $\underline{(A)}$ $(\ )$

Inference by Resolution

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$(A \vee B)$ $(\neg B \vee C)$ $(\neg C)$ $(\neg A)$ $(\neg B)$ (A) $(\ )$

First-Order Logic

Propositional Logic

Propositional Symbols

MinervaGryffindor

MinervaHufflepuff

MinervaRavenclaw

MinervaSlytherin

…

First-Order Logic

Constant Symbol

Minerva
Pomona
Horace
Gilderoy
Gryffindor
Hufflepuff
Ravenclaw
Slytherin

Predicate Symbol

Person
House
BelongsTo

First-Order Logic

$Person(Minerva)$ Minerva is a person.

$House(Gryffindor)$ Gryffindor is a house.

$\neg House(Minerva)$ Minerva is not a house.

$BelongsTo(Minerva, Gryffindor)$

Minerva belongs to Gryffindor.

Universal Quantification

Universal Quantification

$$\forall x.\ BelongsTo(x, Gryffindor) \rightarrow \neg BelongsTo(x, Hufflepuff)$$

For all objects x, if x belongs to Gryffindor,
then x does not belong to Hufflepuff.

Anyone in Gryffindor is not in Hufflepuff.

Existential Quantification

Existential Quantification

$$\exists x.\ House(x) \land BelongsTo(Minerva, x)$$

There exists an object x such that
x is a house and Minerva belongs to x.

Minerva belongs to a house.

Existential Quantification

$$\forall x.\ Person(x) \rightarrow (\exists y.\ House(y) \land BelongsTo(x, y))$$

For all objects x, if x is a person, then
there exists an object y such that
y is a house and x belongs to y.

Every person belongs to a house.

Knowledge

Thanks