1. Find A, B, C of the following quadrate formula

$$\int_0^1 f(x)\,dx = A f(0) + B f'(0) + C f'(1) + R(f)$$

2. Given

x	0	1	2	3	4	5	6
$f(x)$	8	5	3	2	3	1	4

use repeated trapezium rule with $h=2$ to find approx. of $\int_0^4 f(x)\,dx$

3. Show that Newton's interpolating polynomial of 1st degree passing through (x_0, y_0) and (x_1, y_1) may be written as.

$$p_1(x) = \frac{1}{x_1 - x_0}\begin{vmatrix} y_0 & x_0 - x \\ y_1 & x_1 - x \end{vmatrix}$$

4. Show that the Newton's iterates for the function $f(x) = \frac{1}{x} - 5$ are given by $x_{n+1} = 2x_n - 5x_n^2$. Approx. the zero of f using Newton's method with $x_0 = \frac{1}{4}$

① $\int_0^1 f(x)\,dx = A f(0) + B f'(0) + C f'(1)$

$B_a f(x) = b_{00}(x) f(0) + b_{01}(x) f'(0) + b_{11}(x) f'(1)$
$+ R_m f(x)$

$x_0 = 0 \quad x_1 = 1 \quad => m = 2 - 1 = 1$

$I_0 = \{0, 1\} \quad I_1 = \{1\} \quad n = |I_0| + |I_1|$

$b_{00}(x) = ax^2 + bx + c \quad = 1$

$$\begin{cases} b_{00}(0) = 1 \\ b_{00}^{(1)}(0) = 0 \\ b_{00}^{(1)}(1) = 0 \end{cases} => \begin{cases} c = 1 \\ b = 0. \\ 2a + b = 0 => a = 0 \end{cases}$$

$b_{01}(x) = ax^2 + bx + c \quad = -\frac{1}{2}x^2 + 1 \cdot x$

$$\begin{cases} b_{01}(0) = 0 \\ b_{01}^{(1)}(0) = 1 \\ b_{01}^{(1)}(1) = 0 \end{cases} => \begin{cases} c = 0 \\ b = 1 \\ 2a + b = 0 => a = -\frac{1}{2} \end{cases}$$

$b_{11}(x) = ax^2 + bx + c \quad = \frac{1}{2}x^2$

$$\begin{cases} b_{11}(0) = 0 \\ b_{11}^{(1)}(0) = 0 \\ b_{11}^{(1)}(1) = 1 \end{cases} => \begin{cases} c = 0 \\ b = 0 \\ 2a + b = 1 => a = \frac{1}{2} \end{cases}$$

$=> A = \int_0^1 b_{00}(x)\,dx = \int_0^1 dx = x\Big|_0^1 = 1$

$B = \int_0^1 b_{01}(x)\,dx = \int_0^1 \left(-\frac{1}{2}x^2 + 1 \cdot x\right) dx =$

$= -\frac{1}{2} \cdot \frac{x^3}{3}\Big|_0^1 + 1 \cdot \frac{x^2}{2}\Big|_0^1 = -\frac{1}{6} + \frac{1}{2} = \frac{1}{3}$

$$C = \int_0^1 b_{11}(x)\,dx = \int_0^1 \frac{1}{2} x^2\,dx = \frac{1}{2} \cdot \frac{x^3}{3}\Big|_0^1 = \frac{1}{6}$$

$$\Rightarrow \int_0^1 f(x)\,dx = 1 \cdot f(0) + \frac{1}{3} f'(0) + \frac{1}{6} f'(1) + R(f$$

(2) $a = 0$; $b = 4$, $h = 2 \Rightarrow h = \frac{b-a}{n}$, $n = 2$

$x_0 = 0$; $x_1 = x_0 + h = 2$, $x_2 = x_0 + 2h = 4$

$$\int_0^4 f(x)\,dx \simeq \frac{4-0}{2 \cdot 2}\left(f(0) + f(4) + 2 f(2)\right)$$

$$= 8 + 3 + 2 \cdot 3 = \underbrace{17}$$

(3) $f(x_0) = y_0$ $\quad f(x_1) = y_1$

$$L_1 f(x) = l_0(x) f(x_0) + l_1(x) f(x_1)$$

$$l_0(x) = \frac{u_0(x)}{u_0(x_0)} \qquad l_1(x) = \frac{u_1(x)}{u_1(x_1)}$$

$$u(x) = (x - x_0)(x - x_1)$$

$$u_0(x) = x - x_1 \qquad u_1(x) = x - x_0$$

$$L_1 f(x) = \frac{x - x_1}{x_0 - x_1} y_0 + \frac{x - x_0}{x_1 - x_0} y_1 =$$

$$= \frac{1}{x_1 - x_0}\left((x - x_0) y_1 - (x - x_1) y_0\right) =$$

$$= \frac{1}{x_1 - x_0} \begin{vmatrix} y_0 & x_0 - x \\ y_1 & x_1 - x \end{vmatrix}$$

$$\hookrightarrow = (x_1 - x) y_0 - (x_0 - x) y_1$$

$$= (x - x_0) y_1 - (x - x_1) y_0$$

$$N_1 f(x) = f(x_0) + (x - x_0) D^1 f(x_0)$$

x	f	$D^1 f$
x_0	y_0	$\frac{y_1 - y_0}{x_1 - x_0}$
x_1	y_1	

$$N_1 f(x) = y_0 + (x - x_0) \frac{y_1 - y_0}{x_1 - x_0}$$

(4) $f(x) = \frac{1}{x} - 5$ $\qquad x_{n+1} = 2x_n - 5x_n^2$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$f'(x) = -\frac{1}{x^2}$$

$$\Rightarrow x_{i+1} = x_i - \frac{\frac{1}{x_i} - 5}{-\frac{1}{(x_i)^2}}$$

$$= x_i + x_i^2 \left(\frac{1}{x_i} - 5 \right)$$

$$= 2x_i - 5x_i^2$$

$x_0 = \frac{1}{4}$ $\qquad x_1 = \frac{1}{2} - 5 \cdot \frac{1}{16} = \frac{3}{16}$