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Homework 1 Theory and Practice For Deep Learning

1. Theory Homework : known $P(x, y)$

$p(y=0 | x, c(x)) = p(y=0 | c(x))$

$p(y=0 | x, c(x)=1) + p(y=1 | x, c(x)=1) = 1$ starting

$p(y=0 | x, c(x)=2) + p(y=1 | x, c(x)=2) = 1$

$p(x, y) \Rightarrow$ as function of $P(c(x)=1)$, $P(c(x)=2)$, $f(x | c(x)=1)$, $f(x | c(x)=2)$, and $p(y=0 | x, c(x)=1)$, $p(y=0 | x, c(x)=2)$

$\rightarrow$ Since disjoint event $\Rightarrow$ $p(y, x) = p(y, x, \{c(x)=1 \text{ or } c(x)=2\})$

$\rightarrow$ y is dependent on $c(x)$, y is independent of x $\Big\}$ $P(y=0 | x, c(x)) = p(y=0 | c(x))$ $\downarrow$ example from the question

$p(y, x) = p(y, x, c(x)=1) + p(y, x, c(x)=2)$

$$P(y, x, c(x)=1) = p(y | x, c(x)=1) \; P(x, c(x)=1)$$
$$= p(y | c(x)=1) \; P(x, c(x)=1)$$
$$= P(y | c(x)=1) \left[f(x | c(x)=1) \; P(c(x)=1) \right]$$

$$P(y, x, c(x)=2) = p(y | x, c(x)=2) \; P(x, c(x)=2)$$
$$= P(y | c(x)=2) \left[f(x | c(x)=2) \; P(c(x)=2) \right]$$

$$P(x,y) = P(y \mid c(x)=1)\left[f(x \mid c(x)=1)\, P(c(x)=1)\right] + P(y \mid c(x)=2)\left[f(x \mid c(x)=2)\, P(c(x)=2)\right]$$

$$P(y=0) = P(y=0 \mid x, c(x)=1)\, P(x, c(x)=1) + P(y=0 \mid x, c(x)=2)\, P(x, c(x)=2)$$

Since we are considering only 2 possibilities (binary) and y is dependent on $c(x)$

$\rightarrow$ $P(y=1 \mid c(x)) = 1 - P(y=0 \mid c(x))$

$\rightarrow$ $P(y=1) = P(y=1 \mid x, c(x)=1)\, P(x, c(x)=1) + P(y=1 \mid x, c(x)=2)\, P(x, c(x)=2)$

$$= (1 - P(y=0 \mid c(x)=1))\, P(x, c(x)=1) + (1 - P(y=1 \mid c(x)=2))\, P(x, c(x)=2)$$

$\rightarrow$ $y=0 \rightarrow p(x, y=0) = P(y=0 \mid c(x)=1)\left[f(x \mid c(x)=1)\, P(c(x)=1)\right] + P(y=0 \mid c(x)=2)\left[f(x \mid c(x)=2)\, P(c(x)=2)\right]$

$$= 0.2\left[f(x \mid c(x)=1)\, P(c(x)=1)\right] + 0.7\left[f(x \mid c(x)=2)\, P(c(x)=2)\right]$$

$$= 0.2\,(0.5)\, f(x \mid c(x)=1) + 0.7\,(0.5)\, f(x \mid c(x)=2)$$

$$= 0.1\, f(x \mid c(x)=1) + 0.35\, f(x \mid c(x)=2)$$

$\rightarrow$ $y=1 \rightarrow p(x, y=1) = (1-0.2)\left[f(x \mid c(x)=1)\, 0.5\right] + (1-0.7)(0.5)\left[f(x \mid c(x)=2)\right]$

$$= 0.4\, f(x \mid c(x)=1) + 0.15\, f(x \mid c(x)=2)$$

2. Theory Homework → matrix as just vectors

Inner product $v \cdot w = \sum_{d=1}^{D} v_d w_d$

we know → $A \cdot B := tr(A^T B)$ we know → $tr(Z) := \sum_{i} Z_{ii}$

Let $A = (a_{ij})$

Let $B = (b_{ij})$

let $C = A^T B \Rightarrow (C_{ij})$

$$(C)_{ij} = \sum_{k=1}^{m} a_{ki} b_{kj}$$

Equal

$$tr(A^T B) = \sum_{i=1}^{m} C_{ii} = \sum_{i=1}^{n} \sum_{k=1}^{m} a_{ki} b_{ki}$$

Prove $A \cdot B = B \cdot A$

$$\sum_{j=1}^{n} \sum_{i=1}^{m} (a_{ij})(b_{ij}) = A \cdot B$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} (a_{ij})(b_{ij}) = A \cdot B \longrightarrow \sum_{i=1}^{m} \sum_{j=1}^{n} (b_{ij})(a_{ij}) = B \cdot A$$

Equivalent due to commutative properties

$A^T \qquad B$

$$\begin{bmatrix} 1 & -2 \\ -2 & 3 \end{bmatrix} \cdot \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix} = -1 + (-2) + 0 - 6$$

$$= -9$$

$$-9 = \|v\| \, \|w\| \cos\angle(v,w)$$

$$\frac{-9}{\|v\| \, \|w\|} = \cos\angle(v,w)$$

$$\arccos\left[\frac{-9}{\|v\| \, \|w\|}\right] = \angle(v,w)$$

$$\|v\| = \sqrt{\langle v, v \rangle}$$

$$= \sqrt{tr(v^T v)}$$

$$= \sqrt{tr\begin{pmatrix} 5 & -8 \\ -8 & 13 \end{pmatrix}}$$

$$= \sqrt{18}$$

$$\begin{bmatrix} 1 & -2 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ -2 & 3 \end{bmatrix} =$$

$$\begin{bmatrix} 5 & -2-6 \\ -2-6 & 4+9 \end{bmatrix} = \begin{pmatrix} 5 & -8 \\ -8 & 13 \end{pmatrix}$$

$$\|w\| = \sqrt{\langle w, w \rangle}$$

$$= \sqrt{tr(w^T w)}$$

$$= \sqrt{tr\left(\begin{bmatrix} 1 & -1 \\ -1 & 5 \end{bmatrix}\right)}$$

$$= \sqrt{6}$$

$$\begin{bmatrix} -1 & 0 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix} :$$

$$\begin{bmatrix} 1 & -1 \\ -1 & 5 \end{bmatrix}$$

$$\arccos\left[\frac{-9}{\sqrt{18}\sqrt{6}}\right] = \angle(v,w)$$

$$\arccos\left[-\frac{9}{6\sqrt{3}}\right] = \angle(v,w) = 150^\circ$$