

# Home work 3

$$1. C_{j,k} = \sum_i A_{ijk} b_i$$

$$\text{Einsum: } ijk, i \rightarrow jk, [A, b]$$

$$C_j = \sum_{i,k} A_{ijk} B_{ik}$$

$$\text{Einsum: } ijk, ik \rightarrow j, [A, B]$$

$$A_{ik} = \sum_{j,l} A_{ijkl}$$

$$\text{Einsum } ijkl \rightarrow ik, [A]$$

$$A_{ki} = \sum_{j,l} A_{ijkl}$$

$$\text{einsum } ijkl \rightarrow ki, [A]$$

$$C_i = \sum_{j,k} A_{ijk} A_{ijk}$$

$$\text{einsum } ijk, ijk \rightarrow i, [A, A]$$

$$C = x^T A x, \quad \left. \begin{array}{l} x \in \mathbb{R}^{d \times 1}, \text{ 1-tensor} \\ A \in \mathbb{R}^{d \times d}, \text{ 2-tensor} \end{array} \right\}$$

$$\text{einsum } i, ij, j \rightarrow [x, A, x]$$

$$C = A G^T B$$

$$A \in \mathbb{R}^{d \times e} \rightarrow \text{2-tensor}$$

$$G \in \mathbb{R}^{f \times e} \rightarrow \text{2-tensor} \Rightarrow G^T \in \mathbb{R}^{e \times f} \rightarrow \text{2-tensor}$$

$$B \in \mathbb{R}^{f \times l} \rightarrow \text{2-tensor}$$

$$\text{einsum } ij, jk, kl = [A, G, B] \rightarrow \text{The result}$$



$$C_{????} = \sum_{a,b,c,d} A_{abcd} B_{bcde} E_{cdef}$$

Einsum  
abcd, bcde, cdef  $\rightarrow$  abef, [A, B, E]

2 Overfitting with more and more dimensions

Classification  $\rightarrow$   $(x, y)$  samples  $\rightarrow N/2$  independent samples  $\rightarrow -1$

$y \in \{-1, 1\} \rightarrow$  we would know that

$$x_i = (x_i^{(i)})$$

$$P(Y=1) + P(Y=-1) = 1$$

$$P(x^{(1)} < 0 \mid Y = -1) = 0.5$$

$$P(x \geq 0 \mid Y = +1) = 0.5$$

$$P(x^{(1)} < 0 \mid Y = +1) = 0.5$$

$$P(x' \geq 0 \mid Y = +1) = 0.5$$

$$f_0(x) = 2I[x^{(1)} \geq 0] - 1 = \begin{cases} -1 & x^{(1)} < 0 \\ +1 & x^{(1)} \geq 0 \end{cases}$$

Compute  $E_{(x,y) \sim P} [I[f_0(x) \neq y]] \rightarrow$  This works for any  $P(Y = \pm 1)$

$$f_0(x) = -1 \text{ when } x^{(1)} < 0$$

$$f_0(x) = +1 \text{ when } x^{(1)} \geq 0$$

$$P(f_0(x) = -1 \mid Y = +1) = 0.5$$

$$P(f_0(x) = +1 \mid Y = -1) = 0.5$$

$$E_{(x,y) \sim P} [I[f_0(x) \neq y]] = P(f_0(x) = -1, y = 1) + P(f_0(x) = +1, y = -1)$$

$$= P(f_0(x) = -1 \mid y = 1) P(Y = 1) + P(f_0(x) = +1 \mid y = -1) P(Y = -1)$$

$$= 0.5 \times 0.5 + 0.5 \times 0.5$$

$$= 0.5$$