Terminology & variables

Element → a data point in a matrix denoted by ~~index~~ a specific set of indices.

Control volume → region in space surrounding a node

Node → A fixed point in space

~~Tg~~ **face** → boundary of CV

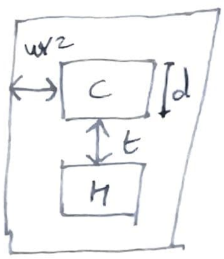
variables

T_g → matrix (3d), stores nodal temperature } → ~~size~~ a, ~~b~~ ~~c~~
P_g → " " " " Pressure
P → " " " face "
T_{xy}, T_{yz}, T_{zx} → " " " Temperature
h_g → ~~nodes~~ matrix, nodal specific enthalpy
S_g → " " " entropy

T_Hot_in ≡ $T_{HOT,in}$ → Hot fluid inlet temp (BC)
T_cold_in ≡ $T_{cold,in}$ → Cold " " " (BC)
P_Hot_in ≡ $P_{HOT,in}$ → Hot " " Pressure (BC)
P_cold_in ≡ $P_{cold,in}$ → Cold " " " "

geometric parameter

t, w, d
↳ hydraulic diameter



L → length of Hex
a → number of nodes in x direction
b → " " " y " = 4 b_ch + 1
c → " " " z " = 2 b_ch + 1
b_ch → no. of pairs of hot and cold channels in one column
c_ch → " " channels along z, ie is no. of rows.

TRN formulation

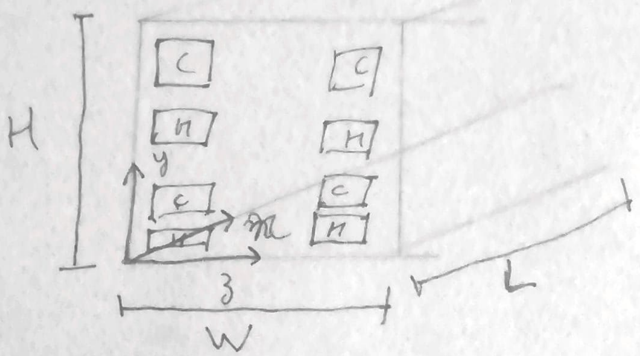
~~Matrix~~ T → [

No. of elements

$(x, y, z) \to (a, b, c)$

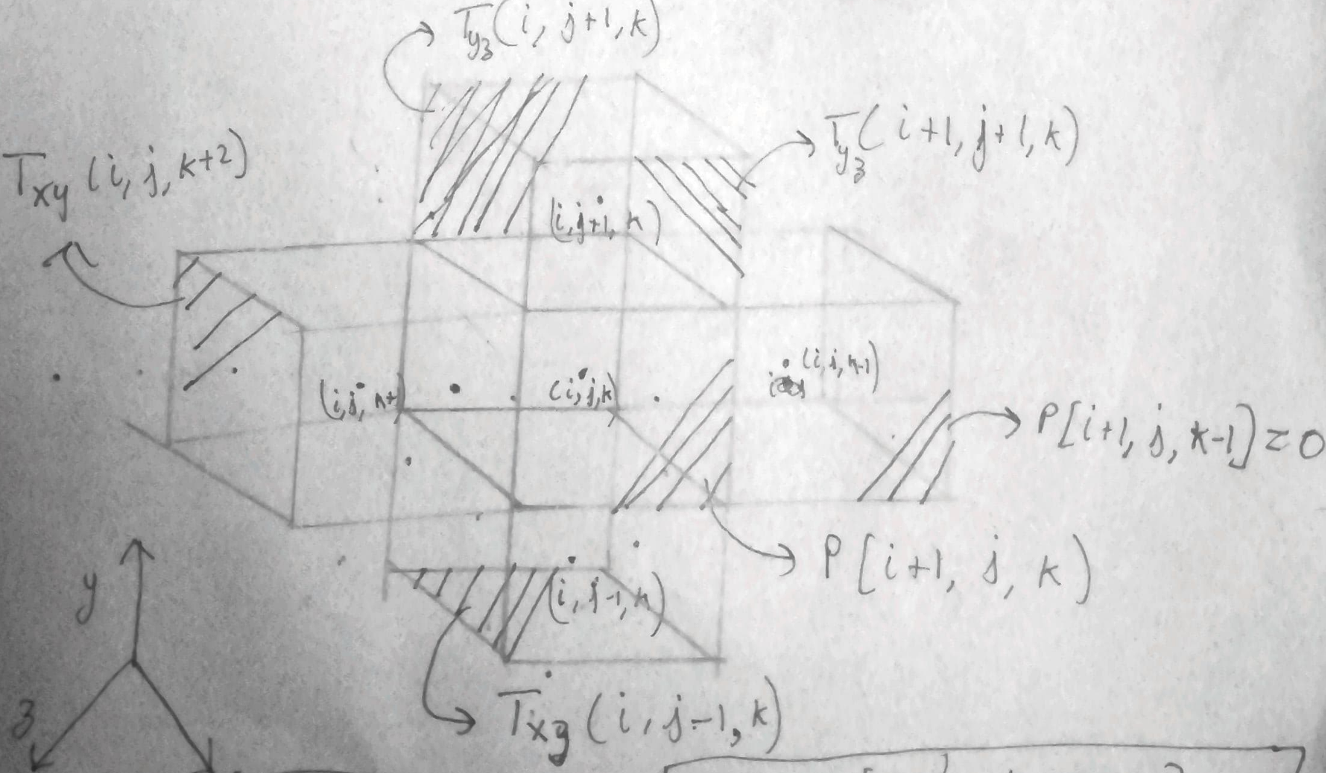
$a \to$ Elements along x
b " " y
c " " z

Hotfluid flows $+\vec{x}$
cold flows $-\vec{x}$



matrix $T_{yz} \to [a+1, b, c]$ Stores temperature at element FACE
or $[a, b+1, c]$
NOT at nodal

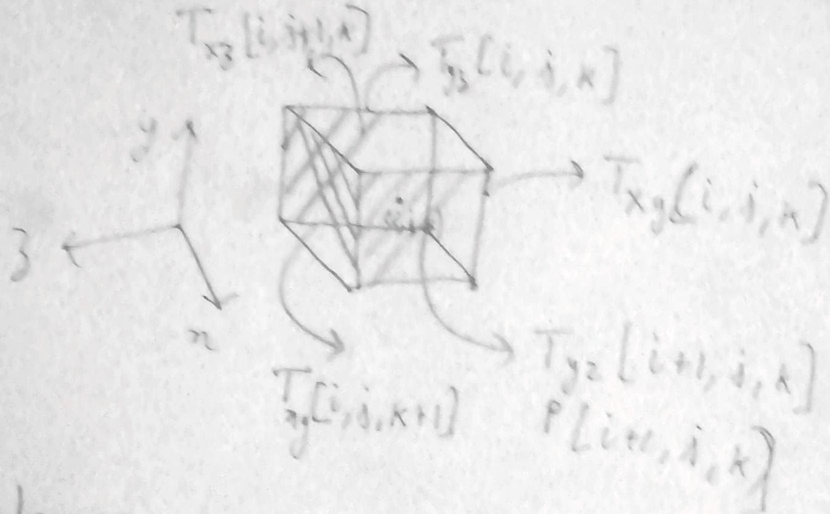
Similarly $P \to [a+1, b, c] \to$ Stores pressure at element face
↳ values at fluid-wall interface = 0



$T_{xy} \to [a, b, c+1]$
$T_{yz} \to [a+1, b, c]$
$T_{zx} \to [a, b+1, c]$
$P \to [a+1, b, c]$

=) ~~$T_{xy} \to [a+1, b, c]$~~
~~$T_{yz} \to [a, b+1, c]$~~
~~$T_{zx} \to [a+1, b+1, c+1]$~~
$P \to [a+1, b, c]$

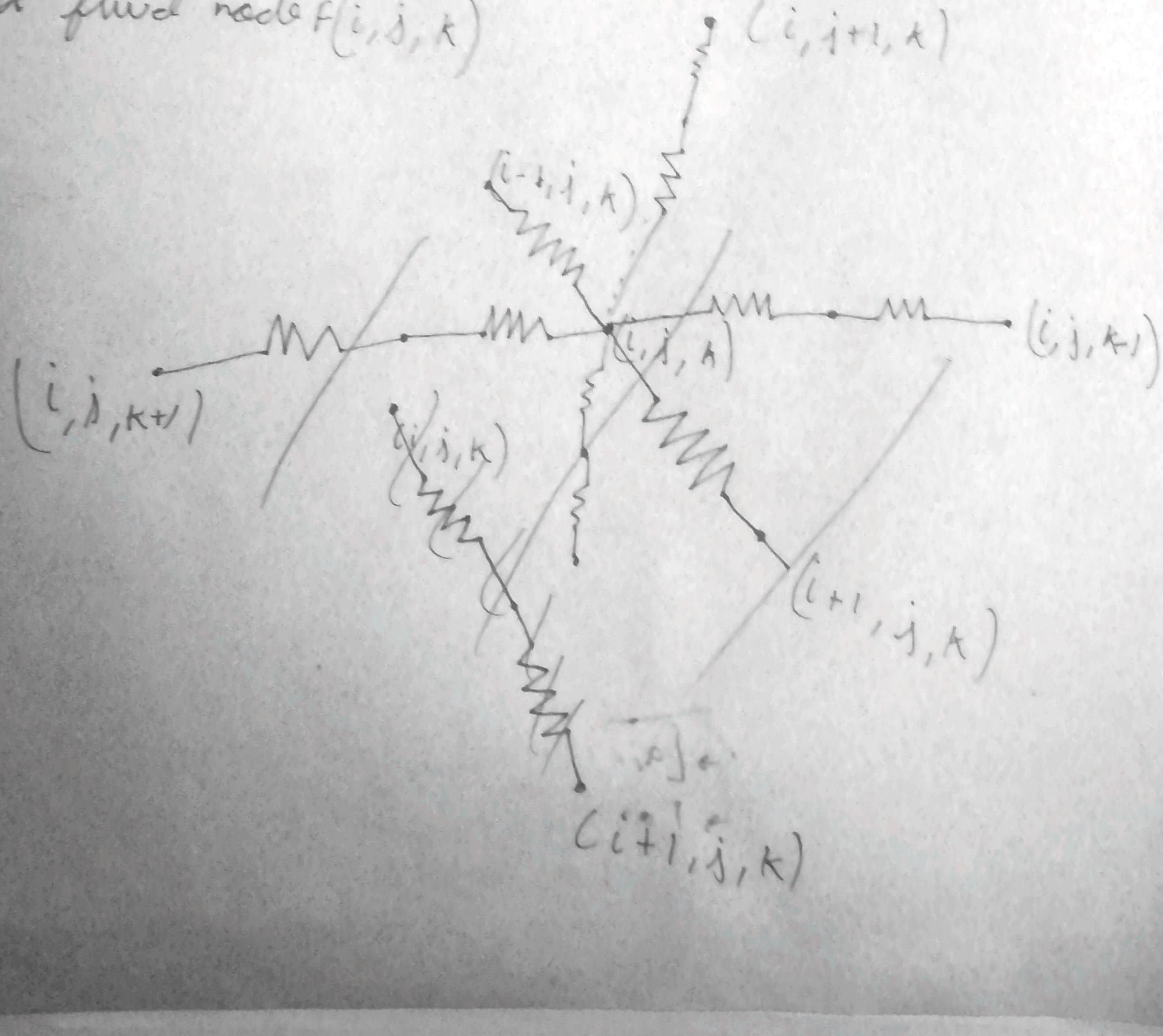
for a node $G(i, j, k)$ corresponding to element $E(i, j, k)$

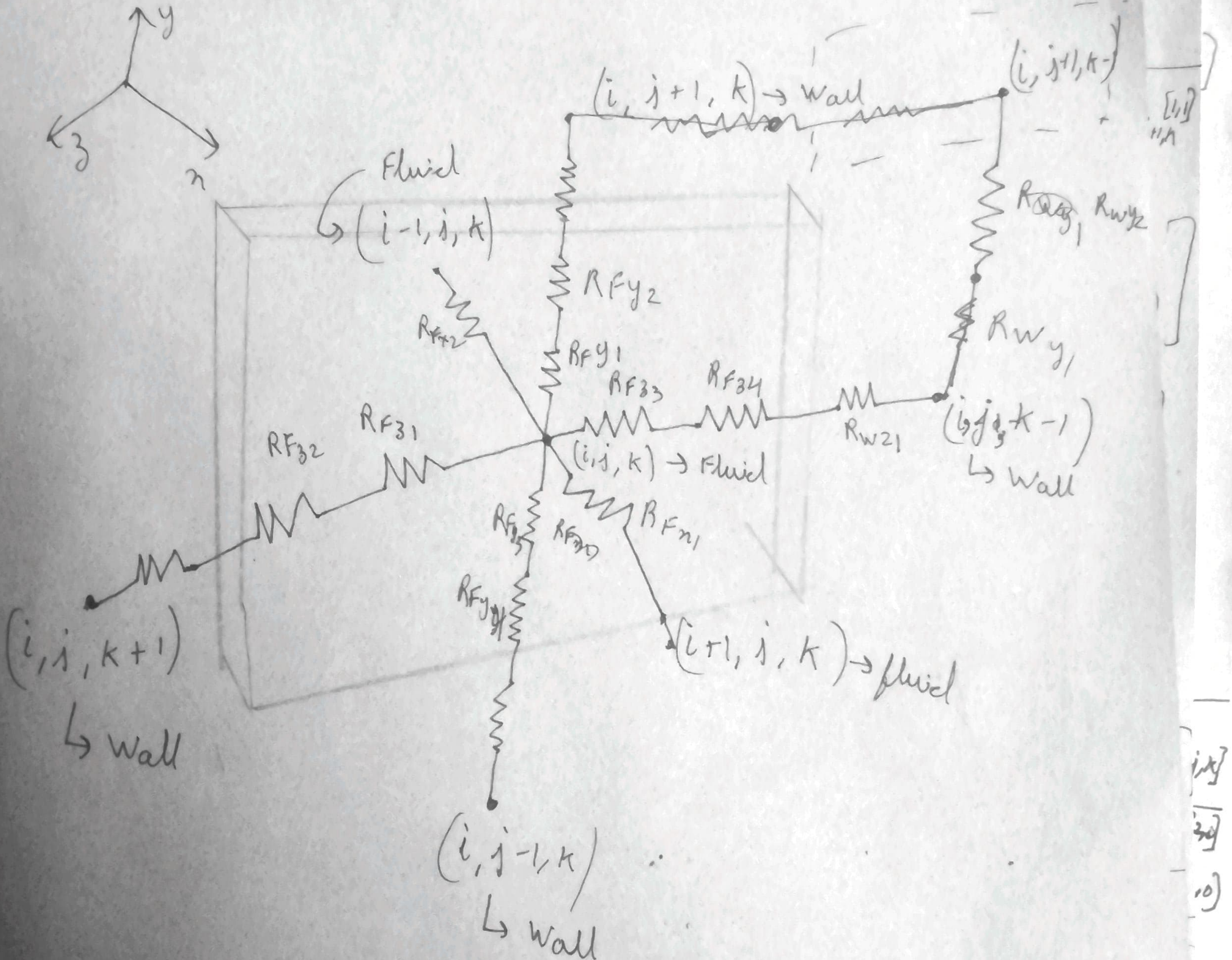


Energy Balance

→ P, T, h etc are stored / calculated at element faces
→ Energy balance at node.

For an fluid node $F(i, j, k)$





Resistance matrix R → [a, b c] → this is a matrix of matrices. i.e, each element is a matrix

→ wall node → fluid node

$$R \rightarrow \begin{bmatrix} \begin{bmatrix} R_{Wx_1} & R_{Wx_2} \\ R_{Wy_1} & R_{Wy_2} \\ R_{Wz_1} & R_{Wz_2} \end{bmatrix} & \begin{bmatrix} R_{Fx_1} & R_{Fx_2} & 0 & 0 & 0 \\ R_{Fy_1} & R_{Fy_2} & R_{Fy_3} & R_{Fy_4} & \\ R_{Fz_1} & R_{Fz_2} & R_{Fz_3} & R_{Fz_4} & \end{bmatrix} \\ & \end{bmatrix}$$

Energy balance on fluid node (i,j,k)

$$\frac{\dot{T}_g(i+1,j,k) - \dot{T}_g(i,j,k)}{R_{i,j,k}[0,0]} + \frac{-\dot{T}_g(i,j,k) + \dot{T}_g(i-1,j,k)}{R_{i,j,k}[0,1]}$$

$$+$$

$$\frac{\dot{T}_g(i,j,k+1) - \dot{T}_g(i,j,k)}{R_{i,j,k}[2,0] + R_{i,j,k}[2,1] + R_{i,j,k+1}[2,1]} + \frac{\dot{T}_g(i,j,k-1) - \dot{T}_g(i,j,k)}{R_{i,j,k}[2,2] + R_{i,j,k}[2,3] + R_{i,j,k-1}[2,0]}$$

$$+$$

$$\frac{\dot{T}_g(i,j+1,k) - \dot{T}_g(i,j,k)}{R(i,j,k)[1,0] + R_{i,j,k}[1,1] + R_{i,j+1,k}[1,1]} + \frac{\dot{T}_g(i,j-1,k) - \dot{T}_g(i,j,k)}{R(i,j,k)[1,\phi] + R(i,j,k)[1,2] + R_{i,j-1,k}[1,0]}$$

$$= 0$$

$$\Rightarrow T(i,j,k)\left[-\frac{1}{R_{i,j,k}[0,0]} + \frac{1}{R_{i,j,k}[0,1]} + \frac{1}{}\right.$$

3

$$T_g(i,j,k)\left[\frac{-1}{R_{i,j,k}(0,0)} + \frac{-1}{R_{i,j,k}(0,1)} + \frac{-1}{R_{i,j,k}[2,0]+R_{i,j,k}[2,1]+R_{i,j,k+1}[2,1]}\right.$$

$$-\frac{1}{R_{i,j,k}[2,2]+R_{i,j,k}[2,3]+R_{i,j,k-1}(2,0)} - \frac{1}{R_{ijk}(1,0)+R_{ijk}(1,1)+R_{i,j+1,k}[1,1]}$$

$$\left.-\frac{1}{R_{ijk}[1,3]+R_{ijk}[1,2]+R_{i,j-1,k}[1,0]}\right] + \bar{T}(i-1,j,k)\left(\frac{1}{R_{i,j,k}(0,1)}\right)$$

$$+\bar{T}(i+1,j,k)\left(\frac{1}{R_{i,j,k}(0,0)}\right) + T_g[i,j,k+1]\left[\frac{1}{R_{ijk}[2,0]+R_{i,j,k}[2,1]+R_{i,j,k+1}[2,1]}\right]$$

$$+T_g[i,j,k-1]\left[\frac{1}{R_{i,j,k}[2,2]+R_{ijk}[2,3]+R_{i,j,k-1}[2,0]}\right]$$

$$+T_g[i,j+1,k]\left[\frac{1}{R_{ijk}[1,0]+R_{ijk}[1,1]+R_{i,j+1,k}[1,1]}\right] + T_g[i,j-1,k]\left[\frac{1}{R_{ijk}[1,3]+R_{ijk}[1,2]+R_{i,j-1,k}[1,0]}\right]$$

$$= 0$$

At this junction, we define a ~~coef~~ coefficient Matrix A & ~~B~~ constants matrix ~~B~~ C and a vector B ~~C~~.

Such that $AB = C$

$$\underset{A}{\left[\quad\quad\right]}\underset{B}{\left[\quad\right]} = \underset{C}{\left[\quad\right]}$$

$$B = \begin{bmatrix} T_g[0,0,0] \\ \vdots \\ T_g[a-1,0,0] \\ T_g[0,1,0] \\ \vdots \\ T_g[a-1,b-1,0] \\ T_g[0,0,1] \\ \vdots \\ T_g[a-1,0,1] \\ \vdots \\ T_g[a-1,b-1,1] \\ T_g[0,0,2] \\ \vdots \\ T_g[a-1,b-1,c-1] \end{bmatrix}$$

B is a vector of length abc ⑧

eg: for $a = 2$, $b = 2$, $c = 2$

$$B = \begin{array}{c} \begin{bmatrix} T_g[0,0,0] \\ T_g[1,0,0] \\ T_g[0,1,0] \\ T_g[1,1,0] \\ T_g[0,0,1] \\ T_g[1,0,1] \\ T_g[0,1,1] \\ T_g[1,1,1] \end{bmatrix} \end{array} \begin{array}{c} \text{Index} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array}$$

$\Rightarrow T[i,j,k]$

the function '%' is the modulus function which gives ~~quotient~~ and remainder
eg: $10 \% 3 \rightarrow 1$ (~~3~~)

$\Rightarrow \cancel{T[i,j,k] \equiv B[\ldots}\ B(ab \cdot k$

$$T(i,j,k) \equiv B(a \cdot b \cdot k + \underbrace{a \cdot j + i}_{\text{alpha}})$$

for some Index α,

$$i = \alpha \% a$$

$$j = \left\lfloor \frac{\alpha - i}{a} \right\rfloor \% (b)$$

$$k = \frac{\alpha - aj - i}{ab}$$

from here onwards, index (i,j,k) refers to the index in B, corresponding to i,j,k

$\text{Index}(i,j,k) = abk + aj + i$

$\Rightarrow$ once we obtain the solutⁿ vector B, we run a loop

```
for alpha in range(0, length(B)):
    find i, j, k from alpha
    Tg[i, j, k] = B[alpha]
```

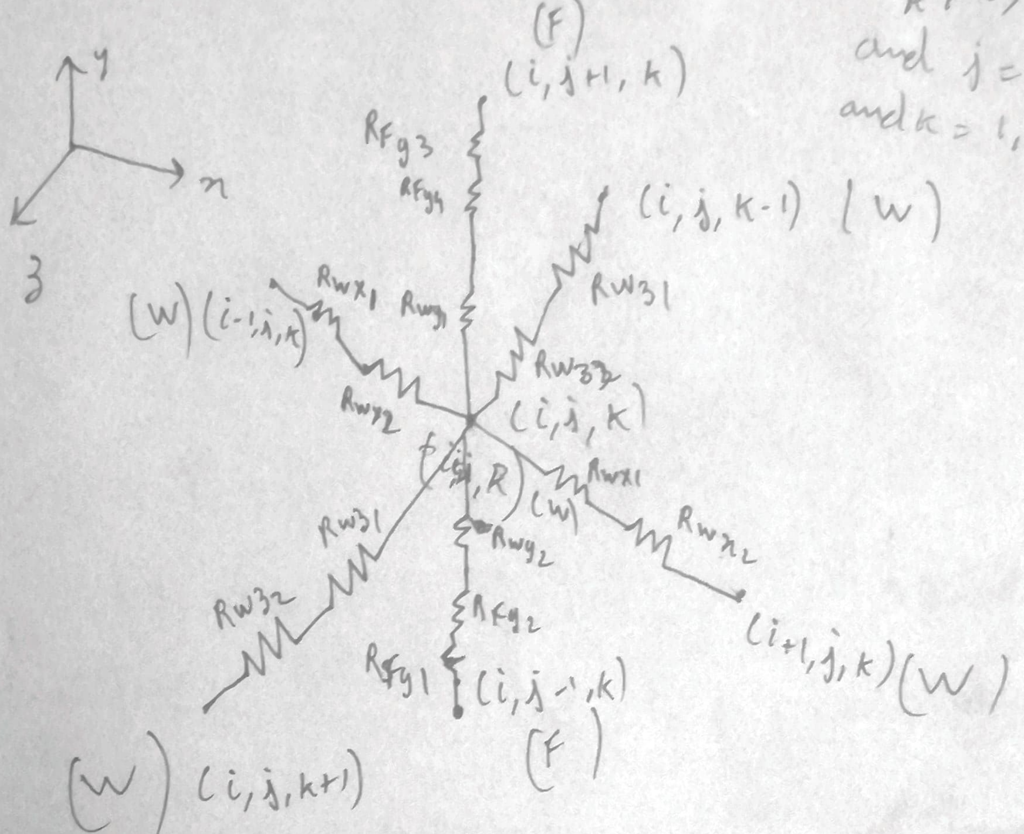
And updated T_g is thus obtained

(1,0) (0,1) (1,2) (2,1) → edge nodes
(1,1) → face node.

Energy balance for interior wall nodes Type 1 ($i \neq 0, a-1, j \neq 0, b-1, k \neq 0, c-1$)

and $j = 2, 4, 6 \ldots [c-3]$

and $k = 1, 3, 5, 7 \ldots$

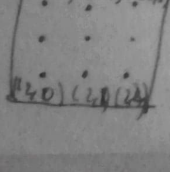


$$\frac{T_g(i+1,j,k)-T_g(i,j,k)}{R_{i,j,k}[0,0]+R_{i+1,j,k}[0,1]} + \frac{T_g[i-1,j,k]-T_g[i,j,k]}{R_{i,j,k}[0,1]+R_{i-1,j,k}[0,0]} + \frac{T_g[i,j,k-1]-T_g[i,j,k]}{R_{i,j,k}[2,1]\cancel{+R_{i,j,k}[2,0]}+R_{i,j,k-1}[2,0]}$$

$$+ \frac{T_g[i,j,k+1]-T_g[i,j,k]}{R_{ijk}[2,0]+R_{ijk+1}[2,1]} + \frac{T_g[i,j+1,k]-T_g[i,j,k]}{R_{i,j,k}[1,0]+R_{i,j+1,k}[1,2]+R_{i,j+1,k}[1,3]}$$

$$+ \frac{T_g[i,j-1,k]-T_g[i,j,k]}{R_{i,j,k}[1,1]+R_{i,j-1,k}[1,0]+R_{i,j-1,k}[1,1]} = 0$$

Terminology: face nodes, edge nodes & corner nodes

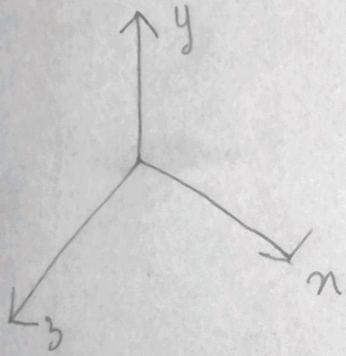


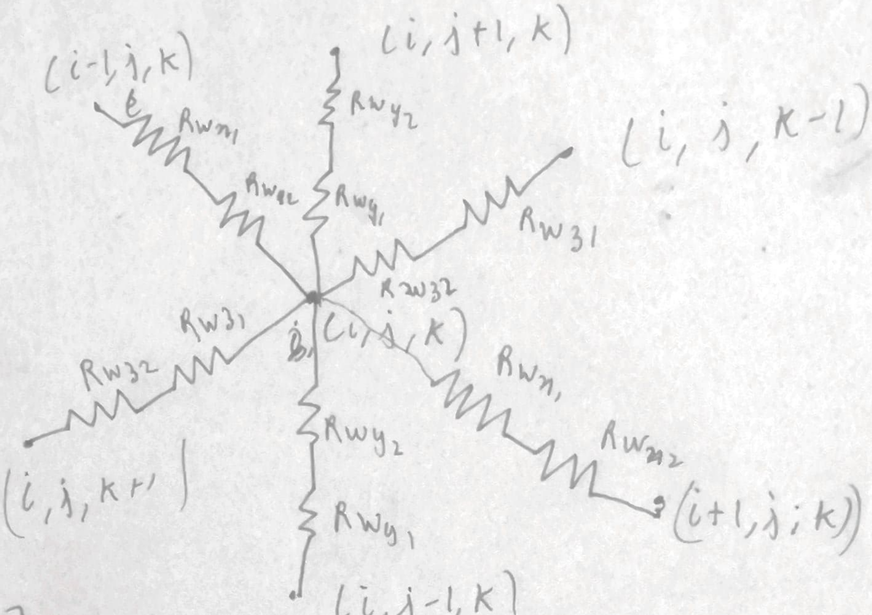
here (0,0), (0,2), (2,0) (2,2) → Corner nodes.
(1,0) (0,1) (1,2) (2,1) → edge nodes
(1,1) → face node.

Generating coeffi... t

Energy balance for type 2 Wall Node

$[i \neq 0, a-1 \mid j = 2, 4, 6 \cdots \mid k = 2, 4, 6, 8 \cdots]$





$$\frac{T_g[i+1,j,k] - T_g[i,j,k]}{R_{i,j,k}[0,0] + R_{i+1,j,k}[0,1]} + \frac{T_g[i-1,j,k] - T_g[i,j,k]}{R_{i,j,k}[0,1] + R_{i-1,j,k}[0,0]}$$

$$+ \frac{T_g[i,j+1,k] - T_g[i,j,k]}{R_{i,j,k}[1,0] + R_{i,j+1,k}[1,1]} + \frac{T_g[i,j-1,k] - T_g[i,j,k]}{R_{i,j,k}[1,1] + R_{i,j-1,k}[1,0]}$$

$$+ \frac{T_g[i,j,k+1] - T_g[i,j,k]}{R_{ijk}[2,0] + R_{i,j,k+1}[2,1]} + \frac{T_g[i,j,k-1] - T_g[i,j,k]}{R_{ijk}[2,1] + R_{ijk-1}[2,0]} = 0$$

$$T_g[i,j,k] \left[\frac{-1}{R_{i,j,k}[0,0] + R_{i+1,j,k}[0,1]} - \frac{1}{R_{i,j,k}[0,1] + R_{i-1,j,k}[0,0]} + \frac{-1}{R_{i,j,k}[1,0] + R_{i,j+1,k}[1,1]} - \frac{1}{R_{ijk}[1,1] + R_{ij-1,k}[1,0]} + \frac{-1}{R_{i,j,k}[2,0] + R_{i,j,k+1}[2,1]} + \frac{-1}{R_{i,j,k}[2,1] + R_{i,j,k-1}[2,0]} \right]$$

$$+ \frac{T_g[i+1,j,k]}{R_{i,j,k}[0,0] + R_{i+1,j,k}[0,1]} + \frac{T_g[i-1,j,k]}{R_{i,j,k}[0,1] + R_{i-1,j,k}[0,0]} + \frac{T_g[i,j+1,k]}{R_{i,j,k}[1,0] + R_{i,j+1,k}[1,1]} +$$

$$+ \frac{T_g[i,j-1,k]}{R_{i,j,k}[1,1] + R_{i,j-1,k}[1,0]} + \frac{T_g[i,j,k+1]}{R_{i,j,k}[2,0] + R_{i,j,k+1}[2,1]} + \frac{T_g[i,j,k-1]}{R_{i,j,k}[2,1] + R_{i,j,k-1}[2,0]} = 0$$

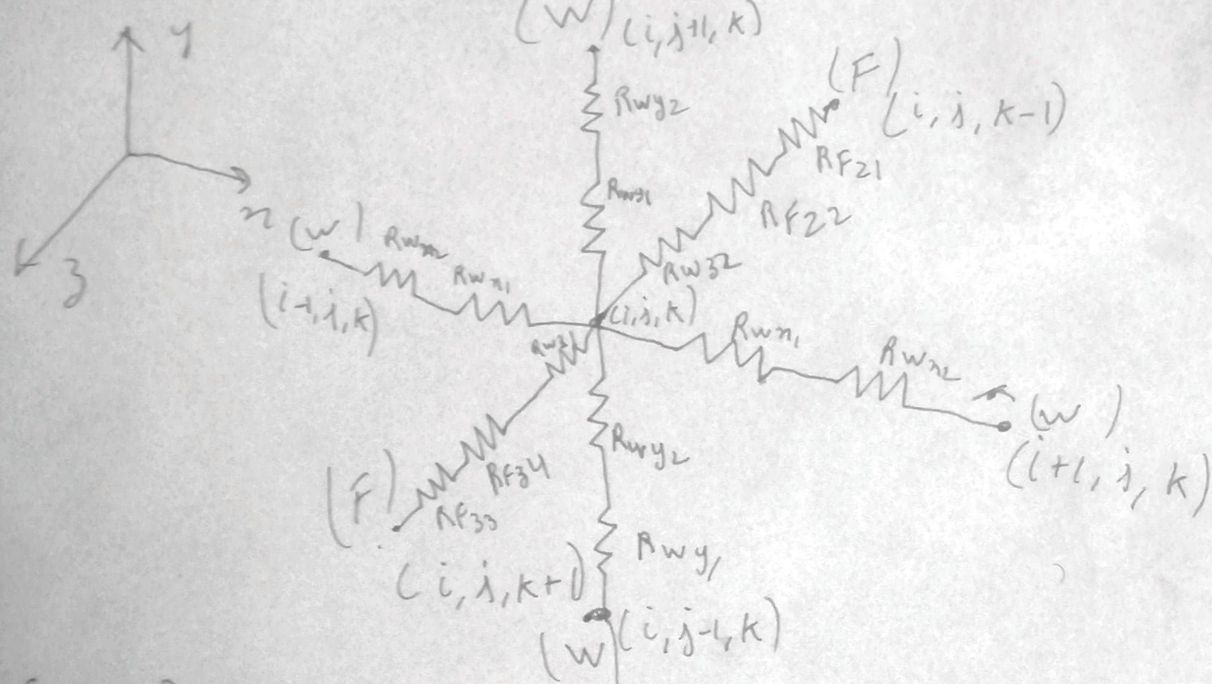
Generating coefficient

(5)

Energy balance for interior wall node $\rightarrow$ Type 3 ~~for~~ ($i \neq 0, a-1$, $j \neq 0, b-1$, $k \neq$ ~~$0, c-1$~~)

and $j = 1, 3, 5, 7 \ldots (-2)$

and $k = 2, 4, 6 \ldots$



$$\frac{T_g[i+1,j,k] - T_g[i,j,k]}{R_{i,j,k}[0,0] + R_{i+1,j,k}[0,1]} + \frac{T_g[i-1,j,k] - T_g[i,j,k]}{R_{i,j,k}[0,1] + R_{i-1,j,k}[0,0]}$$

$$+ \frac{T_g[i,j+1,k] - T_g[i,j,k]}{R_{i,j,k}[1,0] + R_{i,j+1,k}[1,1]} + \frac{T_g[i,j-1,k] - T_g[i,j,k]}{R_{i,j,k}[1,1] + R_{i,j-1,k}[1,0]}$$

$$+ \frac{T_g[i,j,k+1] - T_g[i,j,k]}{R_{i,j,k}[2,0] + R_{i,j,k+1}[2,2] + R_{i,j,k+1}[2,3]} + \frac{T_g[i,j,k-1] - T_g[i,j,k]}{R_{i,j,k}[2,1] + R_{i,j,k-1}[3,0] + R_{i,j,k-1}[2,1]}$$

$$= 0$$

$\Rightarrow$ ~~for j = 2, 4, 6 ⋯ C − 3~~

FOR TYPE 1

$$T_g[i,j,k]\left[-\frac{1}{R_{ijk}[0,0]+R_{i+1,j,k}[0,1]}-\frac{1}{R_{ijk}[0,1]+R_{i-1,j,k}[0,0]}-\frac{1}{R_{ijk}[2,1]+R_{ijk-1}[2,0]}-\frac{1}{R_{ijk}[2,0]+R_{ij,k+1}[2,1]}-\frac{1}{R_{ijk}[1,0]+R_{i,j+1,k}[1,2]+R_{i,j+1,k}[1,3]}-\frac{1}{R_{i,j,k}[1,1]+R_{i,j-1,k}[1,0]+R_{i,j-1,k}[1,1]}\right]$$
$$+T_g[i+1,j,k]\left[\frac{1}{R_{ijk}[0,0]+R_{i+1,j,k}[0,1]}\right]+T_g[i-1,j,k]\left[\frac{1}{R_{i,j,k}[0,1]+R_{i-1,j,k}[0,0]}\right]$$
$$+T_g[i,j+1,k]\left[\frac{1}{R_{i,j,k}[1,0]+R_{i,j+1,k}[1,2]+R_{i,j+1,k}[1,3]}\right]+T_g[i,j-1,k]\left[\frac{1}{R_{i,j,k}[1,1]+R_{i,j-1,k}[1,0]+R_{i,j-1,k}[1,1]}\right]$$
$$+T_g[i,j,k+1]\left[\frac{1}{R_{ijk}[2,0]+R_{i,j,k+1}[2,1]}\right]+T_g[i,j,k-1]\left[\frac{1}{R_{i,j,k}[2,1]+R_{i,j,k-1}[2,0]}\right]$$
$$=0$$

TYPE 3

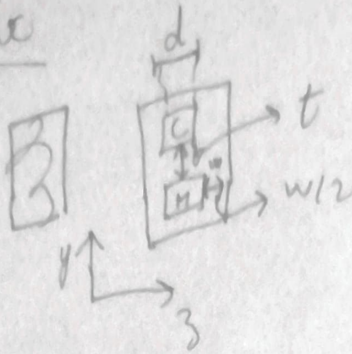
$$T_g[i,j,k]\left[-\frac{1}{R_{ijk}[0,0]+R_{i+1,j,k}[0,1]}-\frac{1}{R_{ijk}[0,1]+R_{i-1,j,k}[0,0]}-\frac{1}{R_{ijk}[1,0]+R_{i,j+1,k}[1,1]}-\frac{1}{R_{ijk}[1,1]+R_{i,j-1,k}[1,0]}-\frac{1}{R_{ijk}[2,0]+R_{ijk+1}[2,2]+R_{ijk+1}[2,3]}-\frac{1}{R_{ijk}[2,1]+R_{ijk+1}[2,0]+R_{i,j,k-1}[2,1]}\right]$$
$$+T_g[i+1,j,k]\left[\frac{1}{R_{ijk}[0,0]+R_{i+1,j,k}[0,1]}\right]+T_g[i-1,j,k]\left[\frac{1}{R_{ijk}[0,1]+R_{i-1,j,k}[0,0]}\right]+T_g[i,j+1,k]\left[\frac{1}{R_{ijk}[1,0]+R_{ij+1,k}[1,1]}\right]$$
$$+T_g[i,j-1,k]\left[\frac{-1}{R_{ijk}[1,1]+R_{i,j-1,k}[1,0]}\right]+T_g[i,j,k+1]\left[\frac{1}{R_{ijk}[2,0]+R_{ijk+1}[2,2]+R_{ijk+1}[2,3]}\right]$$
$$-T_g[i,j,k-1]\left[\frac{1}{R_{ijk}[2,1]+R_{i,j,k-1}[2,0]+R_{i,j,k-1}[2,1]}\right]=0$$

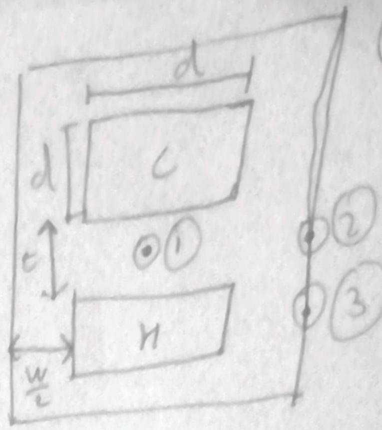
$\boxed{\text{for } j = 3, 5, 7 \cdots C-2}$

Generating coefficients .. .. ..

Resistance matrix

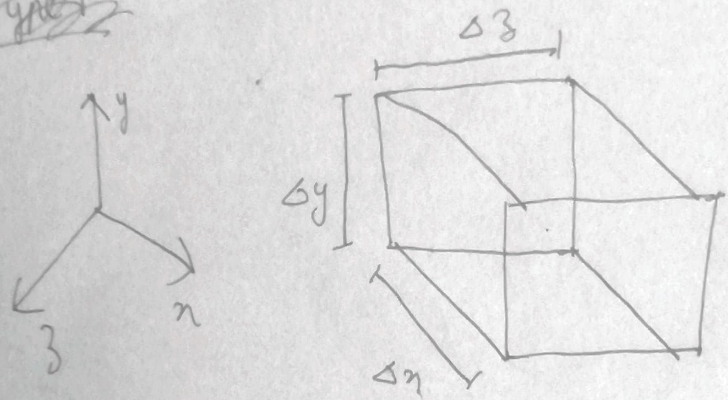
Solid node





Three types of wall nodes are shown in figure.

~~Type 1~~



~~Type~~

conduct^n resistance = $\frac{l}{KA}$

$$R_{w_{x_1}} = \frac{\Delta x}{2K\Delta y\Delta z} = R_{w_{x_2}}$$

$$R_{w_{y_1}} = \frac{\Delta y}{2K\Delta x\Delta z} = R_{w_{y_2}}$$

$$R_{w_{z_1}} = \frac{\Delta z}{2K\Delta x\Delta y} = R_{w_{z_2}}$$

Type 1

$\Delta x = \frac{L}{a}$

$\Delta y = t$

$\Delta z = d$

Type 2

$\Delta x = L/a$

$\Delta y = t$

$\Delta z = w$

Type 3

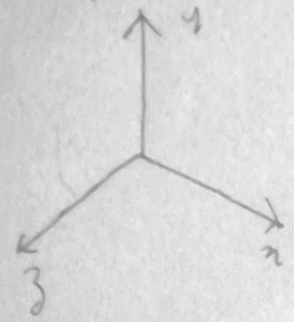
$\Delta x = L/a$

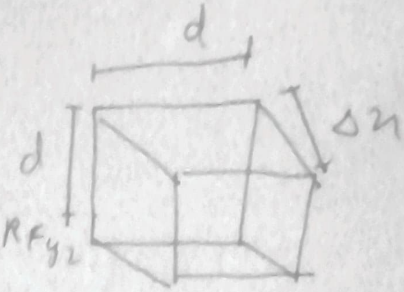
$\Delta y = d$

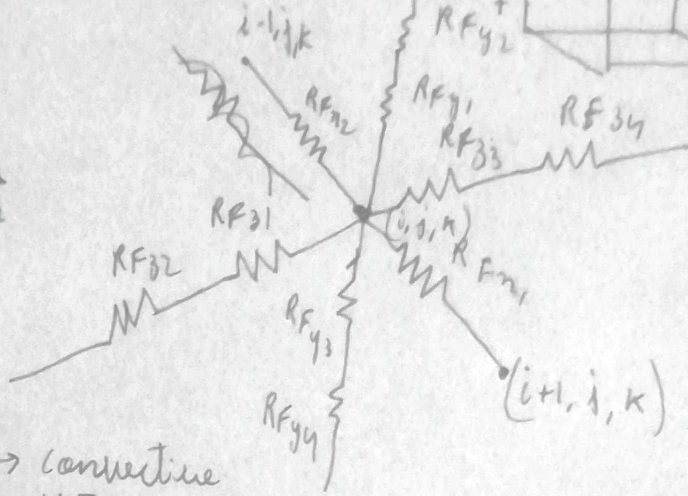
$\Delta z = w$

Resistance matrix → fluid node

R_{Fy_2}, R_F







h_g → spec enthalpy of node & ϕ → convective H.T.C

$R_{Fz_4}, R_{Fz_2}, R_{Fx_4}, R_{Fx_2}, R_{Fy_4}, R_{Fy_2} = \dfrac{1}{\phi\, d\, \Delta x} = \dfrac{a}{\phi\, d L}$

hot fluid flows in $\hat{x}$
cold " " " $-\hat{x}$

for a hot channel $(i = 1, 5, 9, 13 \ldots)$

* BC

if $i = a-1$,
$T[i+1, j, k] \equiv T_{y_3}[i+1, j, k]$
$h_g[i+1, j, k] = h[i+1, j, k]$

$$R_{Fx_1} = \frac{T_g[i+1, j, k] - T_g[i, j, k]}{\dot{m}_{channel}\left[h_g[i+1, j, k] - h_g[i, j, k]\right]}$$

$$R_{Fx_2} = \frac{T_g[i-1, j, k] - T_g[i, j, k]}{\dot{m}_{channel}\left[h_g[i-1, j, k] - h_g[i, j, k]\right]}$$

for a cold channel $(i = 3, 7, 11 \ldots$

* BC

if $i = 0$,
$T[i-1, j, k] \equiv T_{y_3}[i, j, k]$
$h_g[i-1, j, k] = h[i, j, k]$

$$R_{Fx_1} = \frac{T_g[i+1, j, k] - T_g[i, j, k]}{\dot{m}_{channel}\left[h_g[i, j, k] - h_g[i+1, j, k]\right]}$$

$$R_{Fx_2} = \frac{T_g[i-1, j, k] - T_g[i, j, k]}{\dot{m}_{channel}\left[h_g[i, j, k] - h_g[i-1, j, k]\right]}$$