Strand 1: Statistics and Probability

The aim of the probability unit is two-fold: it provides certain understandings intrinsic to problem solving and it underpins the statistics unit. It is expected that the conduct of experiments (including simulations), both individually and in groups, will form the primary vehicle through which the knowledge, understanding and skills in probability are developed. References should be made to appropriate contexts and applications of probability.

It is envisaged that throughout the statistics course learners will be involved in identifying problems that can be explored by the use of appropriate data, designing investigations, collecting data, exploring and using patterns and relationships in data, solving problems, and communicating findings. This strand also involves interpreting statistical information, evaluating data-based arguments, and dealing with uncertainty and variation.

As they engage with this strand and make connections across other strands, learners develop and reinforce their synthesis and problem-solving skills.

At each syllabus level students should be able to

- explore patterns and formulate conjectures
- explain findings
- justify conclusions
- communicate mathematics verbally and in written form
- apply their knowledge and skills to solve problems in familiar and unfamiliar contexts
- analyse information presented verbally and translate it into mathematical form
- devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

Strand 1: Statistics and Probability – Foundation level

Topic	Description of topic *Students learn about*	Learning outcomes *Students should be able to*
1.1 Counting	Listing outcomes of experiments in a systematic way.	– list all possible outcomes of an experiment – apply the fundamental principle of counting
1.2 Concepts of probability	The probability of an event occurring: students progress from informal to formal descriptions of probability. Predicting and determining probabilities. Difference between experimental and theoretical probability.	– decide whether an everyday event is likely or unlikely to occur – recognise that probability is a measure on a scale of 0-1 of how likely an event is to occur – use the language of probability to discuss events, including those with equally likely outcomes – estimate probabilities from experimental data – recognise that, if an experiment is repeated, there will be different outcomes and that increasing the number of times an experiment is repeated generally leads to better estimates of probability – associate the probability of an event with its long-run, relative frequency
1.3 Outcomes of simple random processes	Finding the probability of equally likely outcomes.	– construct sample spaces for two independent events – apply the principle that, in the case of equally likely outcomes, the probability is given by the number of outcomes of interest divided by the total number of outcomes (examples using coins, dice, spinners, containers with different coloured objects, playing cards, sports results, etc.)
1.4 Statistical reasoning with an aim to becoming a statistically aware consumer	Situations where statistics are misused and learn to evaluate the reliability and quality of data and data sources. Different types of data.	– engage in discussions about the purpose of statistics and recognise misconceptions and misuses of statistics – discuss populations and samples – decide to what extent conclusions can be generalised – work with different types of data: • categorical: nominal or ordinal • numerical: discrete or continuous in order to clarify the problem at hand

Strand 1: Statistics and Probability – Ordinary level and Higher level

Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
1.1 Counting	– count the arrangements of n distinct objects ($n!$) – count the number of ways of arranging r objects from n distinct objects	– count the number of ways of selecting r objects from n distinct objects – compute binomial coefficients
1.2 Concepts of probability	– use set theory to discuss experiments, outcomes, sample spaces – discuss basic rules of probability (AND/OR, mutually exclusive) through the use of Venn diagrams – calculate expected value and understand that this does not need to be one of the outcomes – recognise the role of expected value in decision making and explore the issue of fair games	– extend their understanding of the basic rules of probability (AND/OR, mutually exclusive) through the use of formulae – Addition Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ – Multiplication Rule (Independent Events): $P(A \cap B) = P(A) \times P(B)$ – Multiplication Rule (General Case): $P(A \cap B) = P(A) \times P(B \mid A)$ – solve problems involving sampling, with or without replacement – appreciate that in general $P(A \mid B) \neq P(B \mid A)$ – examine the implications of $P(A \mid B) \neq P(B \mid A)$ in context
1.3 Outcomes of random processes	– find the probability that two independent events both occur – apply an understanding of Bernoulli trials* – solve problems involving up to 3 Bernoulli trials – calculate the probability that the 1^{st} success occurs on the n^{th} Bernoulli trial where n is specified	– solve problems involving calculating the probability of k successes in n repeated Bernoulli trials (normal approximation not required) – calculate the probability that the k^{th} success occurs on the n^{th} Bernoulli trial – use simulations to explore the variability of sample statistics from a known population, to construct sampling distributions and to draw conclusions about the sampling distribution of the mean – solve problems involving reading probabilities from the normal distribution tables
1.4 Statistical reasoning with an aim to becoming a statistically aware consumer	– discuss populations and samples – decide to what extent conclusions can be generalised – work with different types of bivariate data	

* A Bernoulli trial is an experiment whose outcome is random and can be either of two possibilities: “success” or “failure”.

Strand 1: Statistics and Probability – Foundation level

Topic	Description of topic *Students learn about*	Learning outcomes *Students should be able to*
1.5 Finding, collecting and organising data	The use of statistics to gather information from a selection of the population with the intention of making generalisations about the whole population. Formulating a statistics question based on data that vary, allowing for distinction between different types of data.	– clarify the problem at hand – formulate one (or more) questions that can be answered with data – explore different ways of collecting data – generate data, or source data from other sources including the internet – select a sample from a population (Simple Random Sample) – recognise the importance of representativeness so as to avoid biased samples – design a plan and collect data on the basis of above knowledge – summarise data in diagrammatic form, including data presented in spreadsheets
1.6 Representing data graphically and numerically	Methods of representing data. Students develop a sense that data can convey information and that organising data in different ways can help clarify what the data have to tell us. They see a data set as a whole and so are able to use proportions and measures of centre to describe the data.	**Graphical** – select appropriate methods to represent and describe the sample (univariate data only) – evaluate the effectiveness of different displays in representing the findings of a statistical investigation conducted by others – use pie charts, bar charts, line plots, histograms (equal intervals), stem and leaf plots to display data – use appropriate graphical displays to compare data sets **Numerical** – use a variety of summary statistics to describe the data: • central tendency mean, median, mode • variability – range

Strand 1: Statistics and Probability – Ordinary level and Higher level

Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
1.5 Finding, collecting and organising data	– select a sample (Simple Random Sample) – recognise the importance of representativeness so as to avoid biased samples – discuss different types of studies: sample surveys, observational studies and designed experiments – design a plan and collect data on the basis of above knowledge	– recognise the importance of randomisation and the role of the control group in studies – recognise biases, limitations and ethical issues of each type of study – select a sample (stratified, cluster, quota – no formulae required, just definitions of these) – design a plan and collect data on the basis of above knowledge
1.6 Representing data graphically and numerically	**Graphical** – describe the sample (both univariate and bivariate data) by selecting appropriate graphical or numerical methods – explore the distribution of data, including concepts of symmetry and skewness – compare data sets using appropriate displays including back-to-back stem and leaf plots – determine the relationship between variables using scatterplots – recognise that correlation is a value from -1 to +1 and that it measures the extent of the linear relationship between two variables – match correlation coefficient values to appropriate scatterplots – understand that correlation does not imply causality **Numerical** – recognise standard deviation and interquartile range as measures of variability – use a calculator to calculate standard deviation – find quartiles and the interquartile range – use the interquartile range appropriately when analysing data – recognise the existence of outliers	**Graphical** – analyse plots of the data to explain differences in measures of centre and spread – draw the line of best fit by eye – make predictions based on the line of best fit – calculate the correlation coefficient by calculator **Numerical** – recognise the effect of outliers – use percentiles to assign relative standing

Strand 1: Statistics and Probability – Foundation level

Topic	Description of topic *Students learn about*	Learning outcomes *Students should be able to*
1.7 Analysing, interpreting and drawing conclusions from data	Drawing conclusions from data; limitations of conclusions.	– interpret graphical summaries of data – relate the interpretation to the original question – recognise how sampling variability influences the use of sample information to make statements about the population – use appropriate tools to describe variability, drawing inferences about the population from the sample – interpret the analysis – relate the interpretation to the original question

Strand 1: Statistics and Probability – Ordinary level and Higher level

Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
1.7 Analysing, interpreting and drawing inferences from data	– recognise how sampling variability influences the use of sample information to make statements about the population – use appropriate tools to describe variability drawing inferences about the population from the sample – interpret the analysis and relate the interpretation to the original question – interpret a histogram in terms of distribution of data – make decisions based on the empirical rule – recognise the concept of a hypothesis test – calculate the margin of error ($\frac{1}{\sqrt{n}}$) for a population proportion* – conduct a hypothesis test on a population proportion using the margin of error	– build on the concept of margin of error and understand that increased confidence level implies wider intervals – construct 95% confidence intervals for the population mean from a large sample and for the population proportion, in both cases using z tables – use sampling distributions as the basis for informal inference – perform univariate large sample tests of the population mean (two-tailed z-test only) – use and interpret p-values

* The margin of error referred to here is the maximum value of the radius of the 95% confidence interval.

Strand 2: Geometry and Trigonometry

The synthetic geometry covered at Leaving Certificate is a continuation of that studied at junior cycle. It is based on the *Geometry for Post-primary School Mathematics*, including terms, definitions, axioms, propositions, theorems, converses and corollaries. The formal underpinning for the system of post-primary geometry is that described by Barry (2001) .[1]

At Ordinary and Higher level, knowledge of geometrical results from the corresponding syllabus level at Junior Certificate is assumed. It is also envisaged that, at all levels, learners will engage with a dynamic geometry software package.

In particular, at Foundation level and Ordinary level learners should first encounter the geometrical results below through investigation and discovery. Learners are asked to accept these results as true for the purpose of applying them to various contextualised and abstract problems. They should come to appreciate that certain features of shapes or diagrams appear to be independent of the particular examples chosen. These apparently constant features or results can be established in a formal manner through logical proof. Even at the investigative stage, ideas involved in mathematical proof can be developed. Learners should become familiar with the formal proofs of the specified theorems (some of which are examinable at Higher level). Learners will be assessed by means of problems that can be solved using the theory.

As they engage with this strand and make connections across other strands, learners develop and reinforce their synthesis and problem-solving skills.

At each syllabus level students should be able to

- explore patterns and formulate conjectures
- explain findings
- justify conclusions
- communicate mathematics verbally and in written form
- apply their knowledge and skills to solve problems in familiar and unfamiliar contexts
- analyse information presented verbally and translate it into mathematical form
- devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

1 P.D. Barry. *Geometry with Trigonometry*, Horwood, Chicester (2001)

Strand 2: Geometry and Trigonometry – Foundation level

Topic	Description of topic *Students learn about*	Learning outcomes *Students should be able to*
2.1 Synthetic geometry	Constructions and how to apply these in real-life situations. Dynamic geometry software. The instruments that are used to perform constructions with precision.	– revisit constructions 4,5,10,13 and 15 in real-life contexts – draw a circle of given radius – use the instruments: straight edge, compass, ruler, protractor and set square appropriately when drawing geometric diagrams
2.2 Co-ordinate geometry	Co-ordinating the plane. Linear relationships in real-life contexts and representing these relationships in tabular and graphical form. Equivalence of the slope of the graph and the rate of change of the relationship. Comparing linear relationships in real-life contexts, paying particular attention to the significance of the start value and the rate of change. The significance of the point of intersection of two linear relationships.	– select and use suitable strategies (graphic, numeric, mental) for finding solutions to real-life problems involving up to two linear relationships
2.3 Trigonometry	Right-angled triangles. Trigonometric ratios.	– apply the result of the theorem of Pythagoras to solve right-angled triangle problems of a simple nature involving heights and distances – use trigonometric ratios to solve real world problems involving angles
2.4 Transformation geometry, enlargements	Translations, central symmetry, axial symmetry and rotations. Enlargements.	– locate axes of symmetry in simple shapes – recognise images of points and objects under translation, central symmetry, axial symmetry and rotation – investigate enlargements and their effect on area, paying attention to • centre of enlargement • scale factor k where $0<k<1$, $k>1$ $k \in \mathbf{Q}$ – solve problems involving enlargements

Strand 2: Geometry and Trigonometry – Ordinary level and Higher level

<table>
<tr><th>Students learn about</th><th>Students working at OL should be able to</th><th>In addition, students working at HL should be able to</th></tr>
<tr><td>2.1 Synthetic geometry</td><td>– perform constructions 16-21 (see Geometry for Post-primary School Mathematics)
– use the following terms related to logic and deductive reasoning: theorem, proof, axiom, corollary, converse, implies
– investigate theorems 7, 8, 11, 12, 13, 16, 17, 18, 20, 21 and corollary 6 (see Geometry for Post-primary School Mathematics) and use them to solve problems</td><td>– perform construction 22 (see Geometry for Post-primary School Mathematics)
– use the following terms related to logic and deductive reasoning: is equivalent to, if and only if, proof by contradiction
– prove theorems 11,12,13, concerning ratios (see Geometry for Post-primary School Mathematics), which lay the proper foundation for the proof of the theorem of Pythagoras studied at junior cycle</td></tr>
<tr><td>2.2 Co-ordinate geometry</td><td>– use slopes to show that two lines are
• parallel
• perpendicular
– recognise the fact that the relationship $ax + by + c = 0$ is linear
– solve problems involving slopes of lines
– calculate the area of a triangle
– recognise that $(x\text{-}h)^2 + (y\text{-}k)^2 = r^2$ represents the relationship between the x and y co-ordinates of points on a circle with centre (h, k) and radius r
– solve problems involving a line and a circle with centre (0, 0)</td><td>– solve problems involving
• the perpendicular distance from a point to a line
• the angle between two lines
– divide a line segment internally in a given ratio $m: n$
– recognise that $x^2+y^2+2gx+2fy+c = 0$ represents the relationship between the x and y co-ordinates of points on a circle with centre $(-g,-f)$ and radius r where $r = \sqrt{(g^2+f^2-c)}$
– solve problems involving a line and a circle</td></tr>
<tr><td>2.3 Trigonometry</td><td>– use of the theorem of Pythagoras to solve problems (2D only)
– use trigonometry to calculate the area of a triangle
– solve problems using the sine and cosine rules (2D)
– define sin θ and cos θ for all values of θ
– define tan θ
– solve problems involving the area of a sector of a circle and the length of an arc
– work with trigonometric ratios in surd form</td><td>– use trigonometry to solve problems in 3D
– graph the trigonometric functions sine, cosine, tangent
– graph trigonometric functions of type
• $f(\boldsymbol{\theta})= a+b\text{Sin } c\boldsymbol{\theta}$
• $g(\boldsymbol{\theta}) = a+b\text{Cos } c\boldsymbol{\theta}$
for $a,b,c \in \mathbf{R}$
– solve trigonometric equations such as Sin $n\boldsymbol{\theta}=0$ and Cos $n\boldsymbol{\theta}= ½$ giving all solutions
– use the radian measure of angles
– derive the trigonometric formulae 1, 2, 3, 4, 5, 6, 7, 9 (see appendix)
– apply the trigonometric formulae 1-24 (see appendix)</td></tr>
<tr><td>2.4 Transformation geometry, enlargements</td><td>– investigate enlargements and their effect on area, paying attention to
• centre of enlargement
• scale factor k
where $0<k<1$, $k>1$ $k \in \mathbf{Q}$
– solve problems involving enlargements</td><td></td></tr>
</table>

Strand 3: Number

Strand 3 further develops the proficiency learners have gained through their study of strand 3 at junior cycle. Learners continue to make meaning of the operations of addition, subtraction, multiplication and division of whole and rational numbers and extend this sense-making to complex numbers.

They extend their work on proof and become more proficient at using algebraic notation and the laws of arithmetic and induction to show that something is always true. They utilise a number of tools: a sophisticated understanding of proportionality, rules of logarithms, rules of indices and 2D representations of 3D solids to solve single and multi-step problems in numerous contexts.

As they engage with this strand and make connections across other strands, learners develop and reinforce their synthesis and problem-solving skills.

At each syllabus level students should be able to

- explore patterns and formulate conjectures
- explain findings
- justify conclusions
- communicate mathematics verbally and in written form
- apply their knowledge and skills to solve problems in familiar and unfamiliar contexts
- analyse information presented verbally and translate it into mathematical form
- devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

Strand 3: Number – Foundation level

Topic	Description of topic *Students learn about*	Learning outcomes *Students should be able to*
3.1 Number systems **N:** the set of natural numbers, **N** = {1,2,3,4…} **Z:** the set of integers, including 0 **Q:** the set of rational numbers	Number: they develop a unified understanding of number, recognising fractions, decimals (that have a finite or a repeating decimal representation), and percentages as different representations of rational numbers. Addition, subtraction, multiplication, and division and extend their whole number understanding to rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. Explaining and interpreting the rules for addition, subtraction, multiplication and division with negative numbers by applying the properties of arithmetic, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero) Representing problems set in context, using diagrams to solve the problems so they can appreciate how the mathematical concepts are related to real life. Solve problems involving fractional amounts set in context.	– revisit the operations of addition, multiplication, subtraction and division in the following domains: • **N** of natural numbers • **Z** of integers • **Q** of rational numbers and use the number line to represent the order of these numbers – investigate models such as decomposition, skip counting, arranging items in arrays and accumulating groups of equal size to make sense of the operations of addition, subtraction, multiplication and division, in **N** where the answer is in **N** including their inverse operations – investigate the properties of arithmetic: commutative, associative and distributive laws and the relationships between them – appreciate the order of operations, including the use of brackets – investigate models, such as the number line, to illustrate the operations of addition, subtraction, multiplication and division in **Z** – generalise and articulate observations of arithmetic operations – investigate models to help think about the operations of addition, subtraction, multiplication and division of rational numbers

Strand 3: Number – Ordinary level and Higher level

Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
3.1 Number systems	– recognise irrational numbers and appreciate that **R** ≠ **Q** – work with irrational numbers – revisit the operations of addition, multiplication, subtraction and division in the following domains: • **N** of natural numbers • **Z** of integers • **Q** of rational numbers • **R** of real numbers and represent these numbers on a number line – investigate the operations of addition, multiplication, subtraction and division with complex numbers **C** in rectangular form a+ib – illustrate complex numbers on an Argand diagram – interpret the modulus as distance from the origin on an Argand diagram and calculate the complex conjugate – develop decimals as special equivalent fractions strengthening the connection between these numbers and fraction and place-value understanding – consolidate their understanding of factors, multiples, prime numbers in **N** – express numbers in terms of their prime factors – appreciate the order of operations, including brackets – express non-zero positive rational numbers in the form $a \times 10^n$, where $n \in \mathbf{Z}$ and $1 \le a < 10$ and perform arithmetic operations on numbers in this form	– geometrically construct √2 and √3 – prove that √2 is not rational – calculate conjugates of sums and products of complex numbers – verify and justify formulae from number patterns – investigate geometric sequences and series – prove by induction • simple identities such as the sum of the first n natural numbers and the sum of a finite geometric series • simple inequalities such as $n! > 2^n$, $2^n \ge n^2$ $(n \ge 4)$ $(1+x)^n \ge 1+nx$ $(x > -1)$ • factorisation results such as 3 is a factor of $4^n - 1$ – apply the rules for sums, products, quotients of limits – find by inspection the limits of sequences such as $\lim_{n\to\infty} \frac{n}{n+1}$; $\lim_{n\to\infty} r^n$, $\lvert r \rvert < 1$ – solve problems involving finite and infinite geometric series including applications such as recurring decimals and financial applications, e.g. deriving the formula for a mortgage repayment – derive the formula for the sum to infinity of geometric series by considering the limit of a sequence of partial sums

Strand 3: Number – Foundation level

Topic	Description of topic *Students learn about*	Learning outcomes *Students should be able to*
3.1 Number systems (continued)		– consolidate the idea that equality is a relationship in which two mathematical expressions hold the same value – analyse solution strategies to problems – calculate percentages – use the equivalence of fractions, decimals and percentages to compare proportions – consolidate their understanding and their learning of factors, multiples and prime numbers in **N** and the relationship between ratio and proportion – check a result by considering whether it is of the right order of magnitude and by working the problem backwards; round off a result – make and justify estimates and approximations of calculations – present numerical answers to the degree of accuracy specified – express non-zero positive rational numbers in the form $a \times 10^n$, where $n \in \mathbf{Z}$ and $1 \leq a < 10$
3.2 Indices	Representing numbers as squares, cubes, square roots, and reciprocals	– solve contextual problems involving numbers represented in the following ways: $\sqrt{a}, a^{\frac{1}{2}}, a^2, a^3, \frac{1}{a}$

Strand 3: Number – Ordinary level and Higher level

Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
3.1 Number systems (continued)	– appreciate that processes can generate sequences of numbers or objects – investigate patterns among these sequences – use patterns to continue the sequence – generalise and explain patterns and relationships in algebraic form – recognise whether a sequence is arithmetic, geometric or neither – find the sum to n terms of an arithmetic series	
3.2 Indices	– solve problems using the rules for indices (where $a, b \in \mathbf{R}$; $p, q \in \mathbf{Q}$; $a^p, a^q \in \mathbf{Q}$; $a, b \neq \mathbf{0}$): • $a^p a^q = a^{p+q}$ • $\frac{a^p}{a^q} = a^{p-q}$ • $a^0 = 1$ • $(a^p)^q = a^{pq}$ • $a^{\frac{1}{q}} = \sqrt[q]{a} \quad q \in \mathbf{Z},\ q \neq 0, a > 0$ • $a^{\frac{p}{q}} = \sqrt[q]{a^p} = (\sqrt[q]{a})^p \ p, q \in \mathbf{Z},\ q \neq 0, a > 0$ • $a^{-p} = \frac{1}{a^p}$ • $(ab)^p = a^p b^p$ • $(\frac{a}{b})^p = \frac{a^p}{b^p}$	– solve problems using the rules of logarithms • $\log_a(xy) = \log_a x + \log_a y$ • $\log_a(\frac{x}{y}) = \log_a x - \log_a y$ • $\log_a x^q = q\log_a x$ • $\log_a a = 1$ and $\log_a 1 = 0$ • $\log_a x = \frac{\log_b x}{\log_b a}$

Strand 3: Number – Foundation level

Topic	Description of topic *Students learn about*	Learning outcomes *Students should be able to*
3.3 Arithmetic	Solving everyday problems, including problems involving mobile phone tariffs, currency transactions, shopping, VAT, meter readings, and timetables. Making value for money calculations and judgments. Using ratio and proportion. Measure and time.	– solve problems that involve finding profit or loss, % profit or loss (on the cost price), discount, % discount, selling price, compound interest for not more than 3 years, income tax (standard rate only), net pay (including other deductions of specified amounts) – calculate, interpret and apply units of measure and time – solve problems that involve calculating average speed, distance and time
3.4 Length, area and volume	2D shapes and 3D solids, including nets of solids. Using nets to analyse figures and to distinguish between surface area and volume. Problems involving perimeter, surface area and volume. Modelling real-world situations and solving a variety of problems (including multi-step problems) involving surface areas, and volumes of cylinders and rectangular solids. The circle, and develop an understanding of the relationship between its circumference, diameter and π.	– investigate the nets of rectangular solids and cylinders – select and use suitable strategies to find length of the perimeter and the area of the following plane figures: disc, triangle, rectangle, square, and figures made from combinations of these – select and use suitable strategies to estimate the area of a combination of regular and irregular shapes – select and use suitable strategies to find the volume and surface area of rectangular solids, cylinders and spheres – draw and interpret scaled diagrams

Strand 3: Number – Ordinary level and Higher level

Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
3.3 Arithmetic	– check a result by considering whether it is of the right order of magnitude and by working the problem backwards; round off a result – accumulate error (by addition or subtraction only) – make and justify estimates and approximations of calculations; calculate percentage error and tolerance – calculate average rates of change (with respect to time) – solve problems that involve • calculating cost price, selling price, loss, discount, mark up (profit as a % of cost price), margin (profit as a % of selling price) • compound interest, depreciation (reducing balance method), income tax and net pay (including other deductions) • costing: materials, labour and wastage • metric system; change of units; everyday imperial units (conversion factors provided for imperial units) – make estimates of measures in the physical world around them	– use *present value* when solving problems involving loan repayments and investments
3.4 Length, area and volume	– investigate the nets of prisms, cylinders and cones – solve problems involving the length of the perimeter and the area of plane figures: disc, triangle, rectangle, square, parallelogram, trapezium, sectors of discs, and figures made from combinations of these – solve problems involving surface area and volume of the following solid figures: rectangular block, cylinder, right cone, triangular-based prism (right angle, isosceles and equilateral), sphere, hemisphere, and solids made from combinations of these – use the trapezoidal rule to approximate area	

Strand 4: Algebra

This strand builds on the relations-based approach of junior cycle where the five main objectives were

- to make use of letter symbols for numeric quantities
- to emphasise relationship based algebra
- to connect graphical and symbolic representations of algebraic concepts
- to use real life problems as vehicles to motivate the use of algebra and algebraic thinking
- to use appropriate graphing technologies (graphing calculators, computer software) throughout the strand activities.

Learners build on their proficiency in moving among equations, tables and graphs and become more adept at solving real-world problems.

As they engage with this strand and make connections across other strands, learners develop and reinforce their synthesis and problem-solving skills.

At each syllabus level students should be able to

- explore patterns and formulate conjectures
- explain findings
- justify conclusions
- communicate mathematics verbally and in written form
- apply their knowledge and skills to solve problems in familiar and unfamiliar contexts
- analyse information presented verbally and translate it into mathematical form
- devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

Strand 4: Algebra – Foundation level

Topic	Description of topic *Students learn about*	Learning outcomes *Students should be able to*
4.1 (a) Generating arithmetic expressions from repeating patterns	Patterns and the rules that govern them; students construct an understanding of a relationship as that which involves a set of inputs, a set of outputs and a correspondence from each input to each output.	– use tables to represent a repeating-pattern situation – generalise and explain patterns and relationships in words and numbers – write arithmetic expressions for particular terms in a sequence
4.1 (b) Representing situations with tables, diagrams and graphs	Relations derived from some kind of context – familiar, everyday situations, imaginary contexts or arrangements of tiles or blocks. Students look at various patterns and make predictions about what comes next.	– use tables, diagrams and graphs as tools for representing and analysing linear patterns and relationships – develop and use their own generalising strategies and ideas and consider those of others – present and interpret solutions, explaining and justifying methods, inferences and reasoning
4.1 (c) Finding formulae	Ways to express a general relationship arising from a pattern or context.	– find the underlying formula written in words from which the data is derived (linear relationships)
4.1 (d) Examining algebraic relationships	Features of a linear relationship and how these features appear in the different representations. Constant rate of change. Proportional relationships.	– show that relations have features that can be represented in a variety of ways – distinguish those features that are especially useful to identify and point out how those features appear in different representations: in tables, graphs, physical models, and formulae expressed in words – use the representations to reason about the situation from which the relationship is derived and communicate their thinking to others – discuss rate of change and the y-intercept; consider how these relate to the context from which the relationship is derived, and identify how they can appear in a table, in a graph and in a formula – decide if two linear relationships have a common value – recognise problems involving direct proportion and identify the necessary information to solve them
4.1 (e) Relations without formulae	Using graphs to represent phenomena quantitatively.	– explore graphs of motion – make sense of quantitative graphs and draw conclusions from them – make connections between the shape of a graph and the story of a phenomenon – describe both quantity and change of quantity on a graph
4.1 (f) Expressions	Evaluating expressions derived from real life contexts.	– evaluate expressions given the value of the variables

Strand 4: Algebra – Ordinary level and Higher level

Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
4.1 Expressions	– evaluate expressions given the value of the variables – expand and re-group expressions – factorise expressions of order 2 – add and subtract expressions of the form • $(ax+by+c) \pm \ldots \pm (dx+ey+f)$ • $(ax^2+bx+c) \pm \ldots \pm (dx^2+ex+f)$ where $a,b,c,d,e,f \in \mathbf{Z}$ • $\frac{a}{bx+c} \pm \frac{p}{qx+r}$ where $a,b,c,p,q,r \in \mathbf{Z}$ – use the associative and distributive properties to simplify expressions of the form • $a(bx \pm cy \pm d) \pm \ldots \pm e(fx \pm gy \pm h)$ where $a, b, c, d, e, f, g, h \in \mathbf{Z}$ • $(x \pm y)(w \pm z)$ – rearrange formulae	– perform the arithmetic operations of addition, subtraction, multiplication and division on polynomials and rational algebraic expressions paying attention to the use of brackets and surds – apply the binomial theorem

Strand 4: Algebra – Foundation level

Topic	Description of topic *Students learn about*	Learning outcomes *Students should be able to*
4.2 Solving equations	Solving linear equations set in context.	– select and use suitable strategies (graphic, numeric, mental) for finding solutions to equations of the form: $f(x) = g(x)$, with $f(x) = ax+b$, $g(x) = cx+d$, where $a, b, c, d \in \mathbf{Q}$ and interpret the results
4.3 Inequalities	Solving linear inequalities set in context.	– select and use suitable strategies (graphic, numeric, mental) for finding solutions to inequalities of the form: • $g(x) \leq k$, $g(x) \geq k$, • $g(x) < k$, $g(x) > k$, where $g(x) = ax + b$ and $a, b, k \in \mathbf{Q}$ and interpret the results

Strand 4: Algebra – Ordinary level and Higher level

Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
4.2 Solving equations	– select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to equations of the form: • $f(x) = g(x)$, with $f(x) = ax+b$, $g(x) = cx+d$ where $a, b, c, d \in \mathbf{Q}$ • $f(x) = g(x)$ with $f(x) = \frac{a}{bx+c} \pm \frac{p}{qx+r}$; $g(x) = \frac{e}{f}$ where $a, b, c, e, f, p, q, r \in \mathbf{Z}$ • $f(x) = k$ with $f(x) = ax^2 + bx + c$ (and not necessarily factorisable) where $a, b, c \in \mathbf{Q}$ and interpret the results – select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to • simultaneous linear equations with two unknowns and interpret the results • one linear equation and one equation of order 2 with two unknowns (restricted to the case where either the coefficient of x or the coefficient of y is $\pm$ 1 in the linear equation) and interpret the results – form quadratic equations given whole number roots	– select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to equations of the form: $f(x) = g(x)$ with $f(x) = \frac{ax+b}{ex+f} \pm \frac{cx+d}{qx+r}$; $g(x) = k$ where $a, b, c, d, e, f, q, r \in \mathbf{Z}$ – use the Factor Theorem for polynomials – select and use suitable strategies (graphic, numeric, algebraic and mental) for finding solutions to • cubic equations with at least one integer root • simultaneous linear equations with three unknowns • one linear equation and one equation of order 2 with two unknowns and interpret the results
4.3 Inequalities	– select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to inequalities of the form: • $g(x) \le k$, $g(x) \ge k$, • $g(x) < k$, $g(x) > k$, where $g(x) = ax + b$ and $a, b, k \in \mathbf{Q}$	– use notation $\mid x \mid$ – select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to inequalities of the form: • $g(x) \le k$, $g(x) \ge k$; • $g(x) < k$, $g(x) > k$, with $g(x) = ax^2+bx+c$ or $g(x) = \frac{ax+b}{cx+d}$ and $a, b, c, d, k \in \mathbf{Q}$, $x \in \mathbf{R}$ • $\mid x - a \mid < b$, $\mid x - a \mid > b$ and combinations of these, with $a, b, \in \mathbf{Q}$, $x \in \mathbf{R}$
4.4 Complex Numbers	See strand 3, section 3.1	– use the Conjugate Root Theorem to find the roots of polynomials – work with complex numbers in rectangular and polar form to solve quadratic and other equations including those in the form $z^n = a$, where $n \in \mathbf{Z}$ and $z = r(\text{Cos}\,\theta + i\text{Sin}\,\theta)$ – use De Moivre's Theorem – prove De Moivre's Theorem by induction for $n \in \mathbf{N}$ – use applications such as n^{th} roots of unity, $n \in \mathbf{N}$, and identities such as $\text{Cos}\,3\theta = 4\,\text{Cos}^3\,\theta - 3\,\text{Cos}\,\theta$

Strand 5: Functions

This strand builds on the learners' experience in junior cycle where they were formally introduced to the concept of a function as that which involves a set of inputs, a set of possible outputs and a rule that assigns one output to each input. The relationship between functions and algebra is further emphasised and learners continue to connect graphical and symbolic representations of functions. They are introduced to calculus as the study of how things change and use derivatives to solve various kinds of real-world problems. They learn how to go from the derivative of a function back to the function itself and use such methods to solve various geometric problems, such as computation of areas of specified regions.

As they engage with this strand and make connections across other strands, learners develop and reinforce their synthesis and problem-solving skills.

At each syllabus level students should be able to

- explore patterns and formulate conjectures
- explain findings
- justify conclusions
- communicate mathematics verbally and in written form
- apply their knowledge and skills to solve problems in familiar and unfamiliar contexts
- analyse information presented verbally and translate it into mathematical form
- devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

Strand 5: Functions – Foundation level

Topic	Description of topic *Students learn about*	Learning outcomes *Students should be able to*
5.1 Functions	Functions as a special type of relationship. Representing linear functions set in context graphically.	– recognise that a function assigns a unique output to a given input – graph functions of the form $ax+b$ where $a,b \in \mathbf{Q}$, $x \in \mathbf{R}$

Strand 5: Functions – Ordinary level and Higher level

Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
5.1 Functions	– recognise that a function assigns a unique output to a given input – form composite functions – graph functions of the form • $ax+b$ where $a,b \in \mathbf{Q}$, $x \in \mathbf{R}$ • ax^2+bx+c where a, b, $c \in \mathbf{Z}$, $x \in \mathbf{R}$ • ax^3+bx^2+cx+d where $a,b,c,d \in \mathbf{Z}$, $x \in \mathbf{R}$ • ab^x where $a \in \mathbf{N}$, b, $x \in \mathbf{R}$ – interpret equations of the form $f(x) = g(x)$ as a comparison of the above functions – use graphical methods to find approximate solutions to • $f(x) = 0$ • $f(x) = k$ • $f(x) = g(x)$ where $f(x)$ and $g(x)$ are of the above form, or where graphs of $f(x)$ and $g(x)$ are provided – investigate the concept of the limit of a function	– recognise surjective, injective and bijective functions – find the inverse of a bijective function – given a graph of a function sketch the graph of its inverse – express quadratic functions in complete square form – use the complete square form of a quadratic function to • find the roots and turning points • sketch the function – graph functions of the form • ax^2+bx+c where $a,b,c \in \mathbf{Q}$, $x \in \mathbf{R}$ • ab^x where a, $b \in \mathbf{R}$ • logarithmic • exponential • trigonometric – interpret equations of the form $f(x) = g(x)$ as a comparison of the above functions – informally explore limits and continuity of functions
5.2 Calculus	– find first and second derivatives of linear, quadratic and cubic functions by rule – associate derivatives with slopes and tangent lines – apply differentiation to • rates of change • maxima and minima • curve sketching	– differentiate linear and quadratic functions from first principles – differentiate the following functions • polynomial • exponential • trigonometric • rational powers • inverse functions • logarithms – find the derivatives of sums, differences, products, quotients and compositions of functions of the above form – apply the differentiation of above functions to solve problems – use differentiation to find the slope of a tangent to a circle – recognise integration as the reverse process of differentiation – use integration to find the average value of a function over an interval – integrate sums, differences and constant multiples of functions of the form • x^a where $a \in \mathbf{Q}$ • a^x where $a \in \mathbf{R}$, $a>0$ • Sin ax where $a \in \mathbf{R}$ • Cos ax where $a \in \mathbf{R}$ – determine areas of plane regions bounded by polynomial and exponential curves

Assessment

Assessment in Leaving Certificate Mathematics

Assessment for certification will be based on the aim, objectives and learning outcomes of the syllabus. Differentiation at the point of assessment will be achieved through examinations at three levels – Foundation level, Ordinary level, and Higher level. Ordinary level is a subset of Higher level; thus, learners at Higher level are expected to achieve the Ordinary level and Higher level learning outcomes. Differentiation will be achieved also through the language level in the examination questions, the stimulus material presented, and the amount of structured support given in the questions. It is accepted that, at Foundation level, learners engage with the mathematics at a concrete level.

Assessment components

At Ordinary level and Higher level there are two assessment components

- Mathematics Paper 1
- Mathematics Paper 2

Each paper will contain two sections – A and B.

- Section A will address core mathematics topics, with a focus on concepts and skills.
- Section B will include questions that are context-based applications of mathematics.

At Foundation level there is one assessment component, a written paper. Learners will be assessed by means of problems set in meaningful contexts.

General assessment criteria

A high level of achievement in Mathematics is characterised by a demonstration of a thorough knowledge and comprehensive understanding of mathematics as described by the learning outcomes associated with each strand. The learner is able to make deductions with insight even in unfamiliar contexts and can move confidently between different forms of representation. When investigating challenging problems, the learner recognises pattern structures, describes them as relationships or general rules, draws conclusions and provides justification or proof. The learner presents a concise, reasoned justification for the method and process and, where appropriate, considers the range of approaches which could have been used, including the use of technology.

A moderate level of achievement in Mathematics is characterised by a demonstration of a broad knowledge and good understanding of mathematics as described by the learning outcomes associated with each strand. The learner is able to make deductions with some insight even in unfamiliar contexts and can move between different forms of representation in most situations. When investigating problems of moderate complexity, the learner recognises pattern structures, describes them as relationships or general rules and draws conclusions consistent with findings. The learner successfully selects and applies skills and problem solving techniques. The learner presents a reasoned justification for the method and process and provides an evaluation of the significance and reliability of findings.

A low level of achievement in Mathematics is characterised by a demonstration of limited knowledge or understanding of mathematics as described by the learning outcomes associated with each strand. The learner recognises simple patterns or structures when investigating problems and applies basic problem solving techniques with some success. An attempt is made to justify the method used and to evaluate the reliability of findings.