Dispersive Quantum Interface for Nanofiber-Trapped Atoms: QND Measurement and Spin Squeezing

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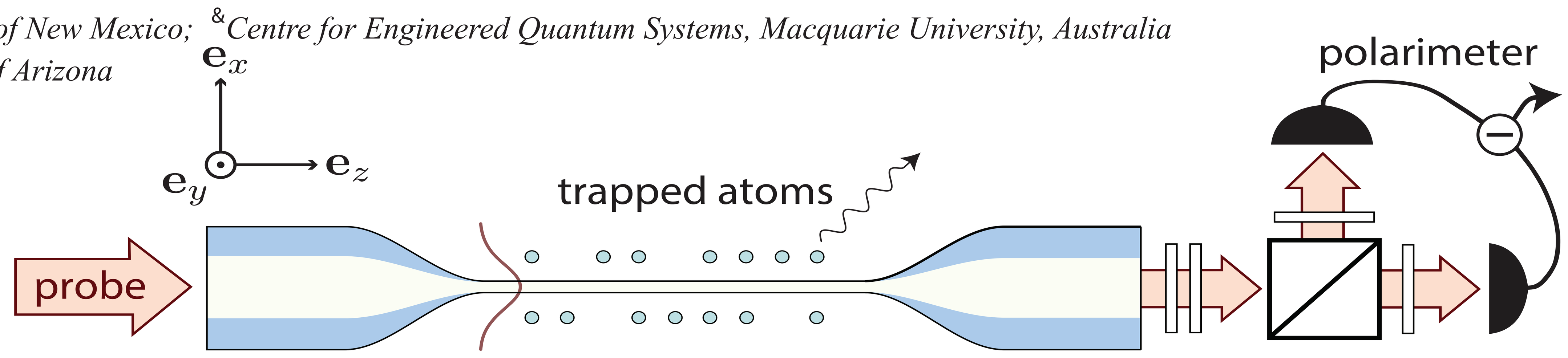


dispersive interaction: phase shift

An off-resonant probe of detuning Δ induces an atomic dipole

$$\mathbf{d}(\mathbf{r}') = \overleftrightarrow{\alpha}\cdot\mathbf{E}_0(\mathbf{r}')$$

the re-radiated field interferes with the probe, producing a **phase shift**

$$\delta\phi_{\{H,V\}} = \frac{\Gamma^{1D}_{\{H,V\}}}{\Delta}$$

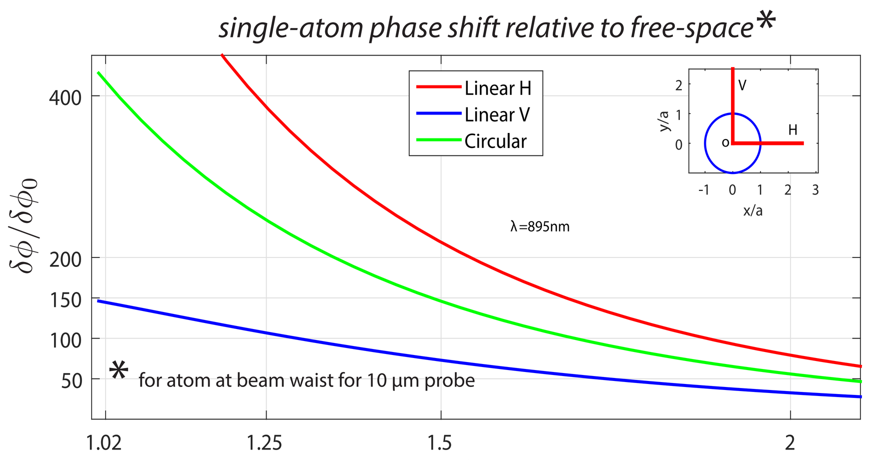
that depends on the coupling rate to the guided modes H and V:

$$\Gamma^{1D}_{\{H,V\}} = 2\pi\frac{\omega_{eg}}{v_g}|\mathbf{d}_{eg}\cdot\mathbf{u}_{\{H,V\}}(\mathbf{r}')|^2$$

Comparing to free space phase shift:

$$\frac{\delta\phi^{fiber}}{\delta\phi^{vac}} = \frac{A^{vac}_{in}}{A^{fiber}_{in}} = An_g|\mathbf{u}^{fiber}_{in}(r'_\perp)|^2$$

single-atom phase shift relative to free-space[*]



geometric birefringence

even a scalar polarizable particle produces position-dependent **birefringence** given by the differential phase shift

$$\delta\phi_H - \delta\phi_V = \sigma_0\frac{c}{v_g}\frac{\Gamma_0}{4\Delta}\left(|\mathbf{u}_H(\mathbf{r}')|^2 - |\mathbf{u}_V(\mathbf{r}')|^2\right)$$

quantum interface on the atomic clock states

off-resonant probe dispersively couples the **collective atomic pseudospin**

$$\left.\begin{aligned}|\uparrow\rangle &\equiv |F=4, m_f=0\rangle\\ |\downarrow\rangle &\equiv |F=3, m_f=0\rangle\end{aligned}\right\}\ \text{ground hyperfine clock states}$$

to the **polarization** (Stokes operator) of the guided modes

$$\hat{H}_{J_3} = \Delta\chi\hat{J}_3\hat{S}_0 + \chi_{J_3}\hat{J}_3\hat{S}_1$$

$$\hat{J}_3 = \frac{1}{2}\sum_{n=1}^{N_A}\left(|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|\right)^{(n)} \qquad \hat{S}_1 = \frac{1}{2}\left(\hat{a}^\dagger_H\hat{a}_H - \hat{a}^\dagger_V\hat{a}_V\right)$$

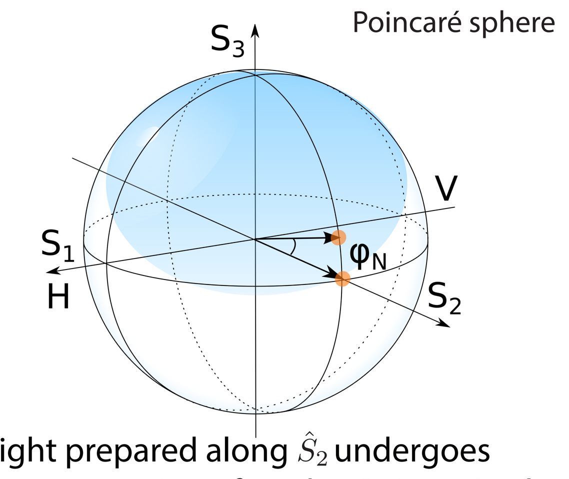
the coupling strengths:

$$\chi_{J_3} = (\chi_{H,\uparrow} - \chi_{H,\downarrow}) + (\chi_{V,\uparrow} - \chi_{V,\downarrow})$$
$$\Delta\chi = (\chi_{H,\uparrow} - \chi_{H,\downarrow}) - (\chi_{V,\uparrow} - \chi_{V,\downarrow})$$

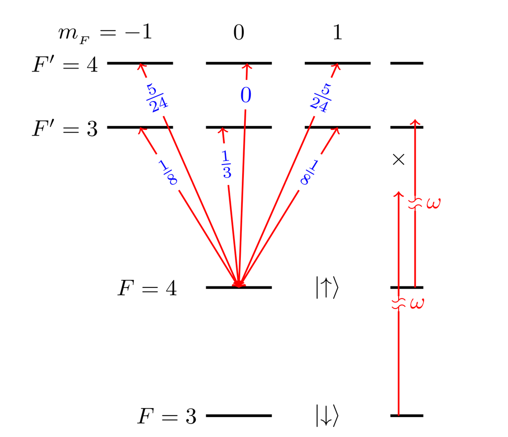
depend on the atomic position and its internal state

$$\chi_{H,\uparrow} = -\frac{2\pi\omega_0}{v_g}\langle\uparrow|\mathbf{u}^*_H(\mathbf{r}')\cdot\overleftrightarrow{\alpha}\cdot\mathbf{u}_H(\mathbf{r}')|\uparrow\rangle$$

atomic polarizability tensor

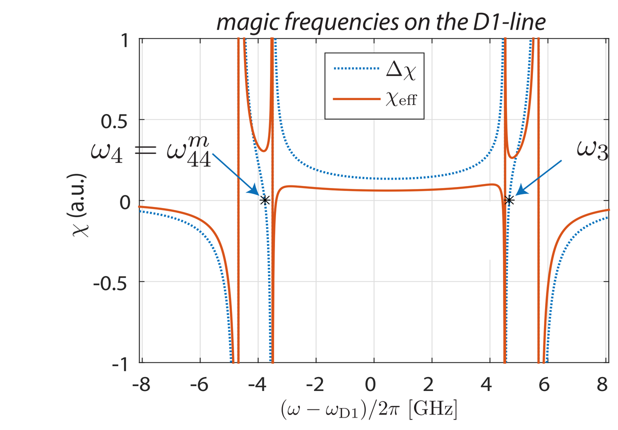


light prepared along $\hat{S}_2$ undergoes **birefringent** rotation of on the Poincaré sphere

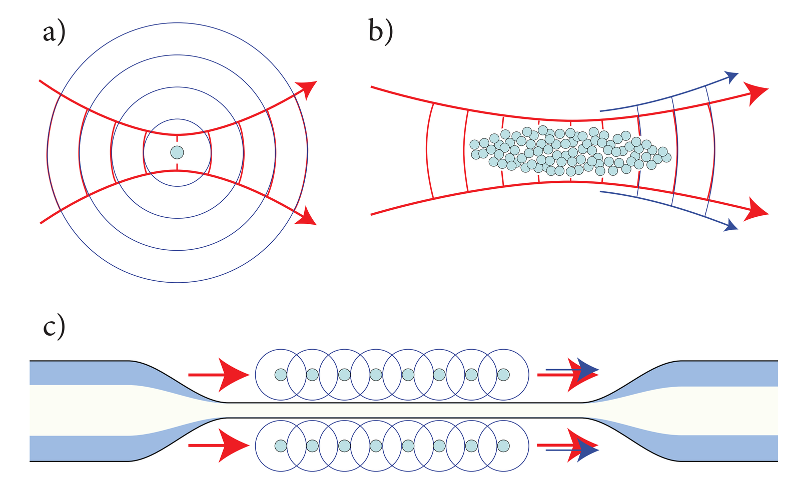


additional birefringence results from atomic-state-dependent coupling

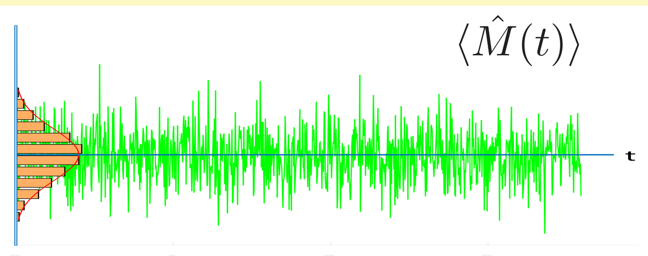
"magic" probe frequency to isolate entangling interaction



mode-matching in different atomic systems



shotnoise-limited atom number detection



$$\Delta M^2 = \underbrace{\frac{N_L}{4}}_{\text{shotnoise}} + \underbrace{\frac{N_L}{4}\kappa t\Delta J_z^2}_{\text{projection noise}}$$

$\kappa = |\chi_{J_3}|^2\dot{N}_L$
measurement strength

the resolution of an atom number measurement is set by the fluctuations in the measurement of $\hat{S}_3$. When the **atomic projection noise** is equal to the **shotnoise**:

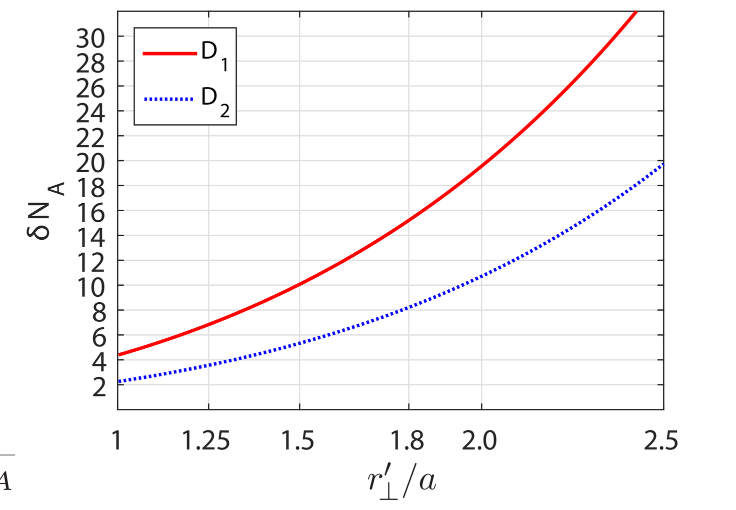
$$\Delta M^2_{\rm PN} = \Delta M^2_{\rm SN}$$

the minimum detectable atom number is

$$\delta N_A \sim \frac{1}{C^{(0)}_{j'}}\sqrt{\frac{A_N^2}{A_{in}\sigma_0}} \quad \frac{1}{OD/N_A}$$

Two effective mode areas:

$$A_{in} \equiv \frac{1}{n_g|\mathbf{u}_{in}(\mathbf{r}'_\perp)|^2},\quad A_N \equiv \frac{2}{n_g\left(|\mathbf{u}_H(\mathbf{r}'_\perp)|^2 - |\mathbf{u}_V(\mathbf{r}'_\perp)|^2\right)}$$



Heisenberg-Langevin equations and input-output relationships

$$\begin{cases}\frac{d}{dt}\hat{a}_\mu = -i\omega\hat{a}_\mu + i\sum_{e,g}g^*_{\mu,e,g}\hat{\sigma}_{ge}, \qquad \frac{d}{dt}\hat{a}_\nu = -i\omega\hat{a}_\nu + i\sum_{e,g}g^*_{\nu,e,g}\hat{\sigma}_{ge},\\ \frac{d}{dt}\hat{\sigma}_{ge} = -i\omega_{eg}\hat{\sigma}_{ge} + i\int_0^\infty d\omega\sum_{e',g'}(\delta_{ee'}\hat{\sigma}_{gg'} - \delta_{gg'}\hat{\sigma}_{e'e})\left[\sum_{b,p}g_{\mu,e',g'}\hat{a}_\mu + \sum_{m,p}\int_{-kn_2}^{kn_2}d\beta\, g_{\nu,e',g'}\hat{a}_\nu\right]\end{cases}$$

Input-output relationships of the guided mode photon creation operators

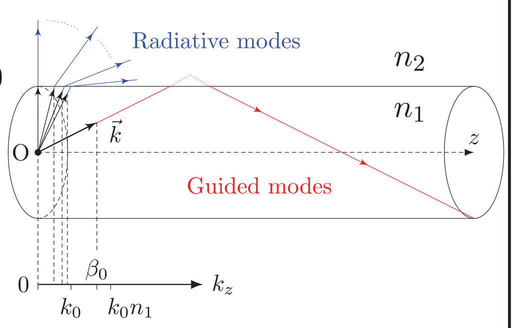
$$\hat{a}^{out}_\mu(\omega) = \hat{a}^{in}_\mu(\omega) - 2\pi i\sum_{b',p'}\sum_{e,g,g'}\hat{\sigma}_{gg'}\frac{g^*_{\mu,e,g}g_{\mu',e,g'}}{\omega-\omega_{eg}+i\Gamma_e/2}\hat{a}^{in}_{\mu'}(\omega)$$
$$-2\pi i\sum_{m,p}\sum_{e,g,g'}\int_{-kn_2}^{kn_2}d\beta\,\hat{\sigma}_{gg'}\frac{g^*_{\mu,e,g}g_{\nu',e,g'}}{\omega-\omega_{eg}+i\Gamma_e/2}\hat{a}^{in}_{\nu'}(\omega).$$

Phase-shift for a given guided mode

$$\delta\phi_{p,g} = -\sum_e\frac{\Gamma^p_e}{\Delta_{eg}}$$
$$\Gamma^{\ p}_e = \frac{2}{\hbar}\sum_g\langle g|\hat{\mathbf{d}}|e\rangle\cdot Im\left[\mathbf{G}^{(+)}_{pp}(\mathbf{r}',\mathbf{r}';\omega_{eg})\right]\cdot\langle e|\hat{\mathbf{d}}|g\rangle$$

guided modes of the nanofiber

$$\left(\nabla^2 + (k^2\varepsilon(r) - \beta^2)\right)\begin{pmatrix}\mathcal{E}(r,\phi,z)\\ \mathcal{B}(r,\phi,z)\end{pmatrix} = 0$$

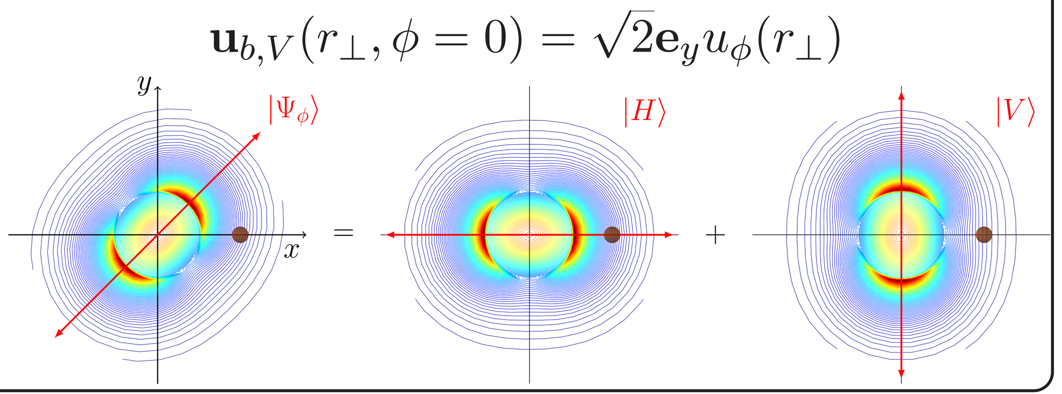


for our frequency band, the nanofiber admits a single guided mode with degeneracies in polarization and propagation direction

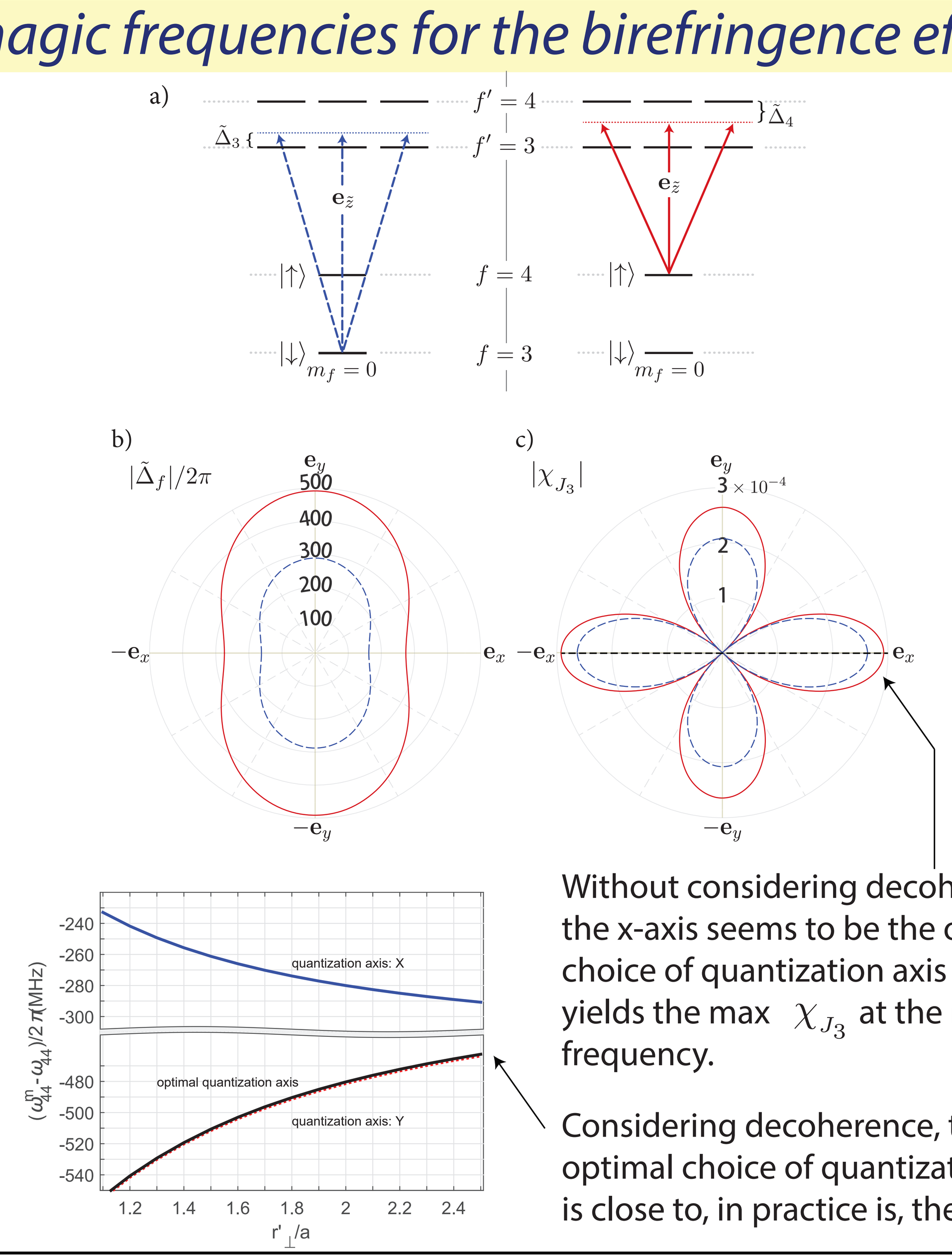
quasilinear guided modes

placing the atom at $\phi = 0$, the two "quasilinear" forward-propagating guided modes are

$$\mathbf{u}_{b,H}(r_\perp, \phi=0) = \sqrt{2}\left[\mathbf{e}_x u_r(r_\perp) + ib\mathbf{e}_z u_z(r_\perp)\right]$$
$$\mathbf{u}_{b,V}(r_\perp, \phi=0) = \sqrt{2}\mathbf{e}_y u_\phi(r_\perp)$$



magic frequencies for the birefringence effect



Without considering decoherence, the x-axis seems to be the optimal choice of quantization axis which yields the max χ_{J_3} at the magic frequency.

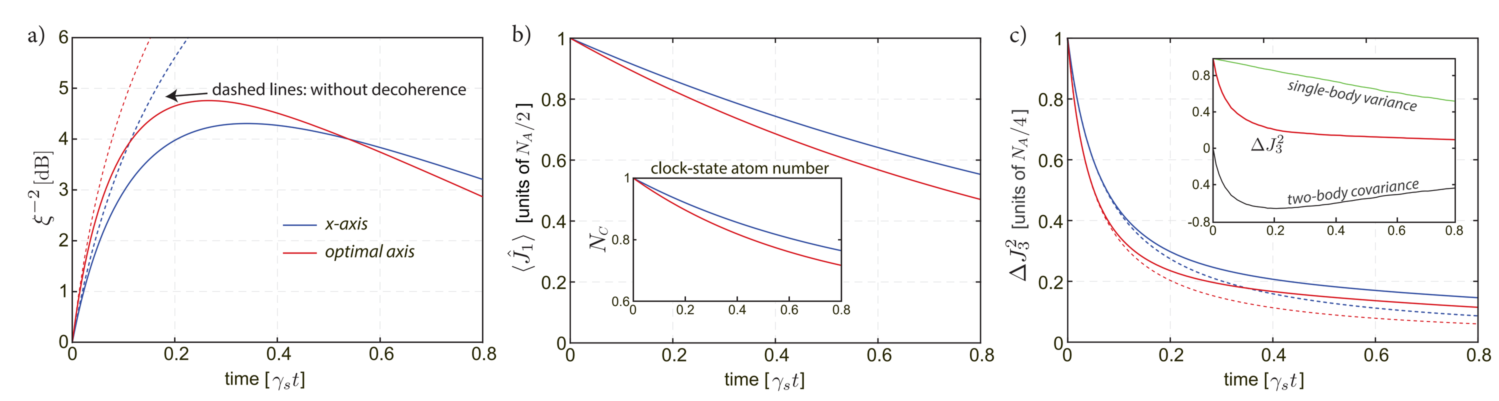
Considering decoherence, the optimal choice of quantization axis is close to, in practice is, the y-axis.

spin squeezing via QND measurement

The spin squeezing dynamics via quantum nondemolition (QND) measurement

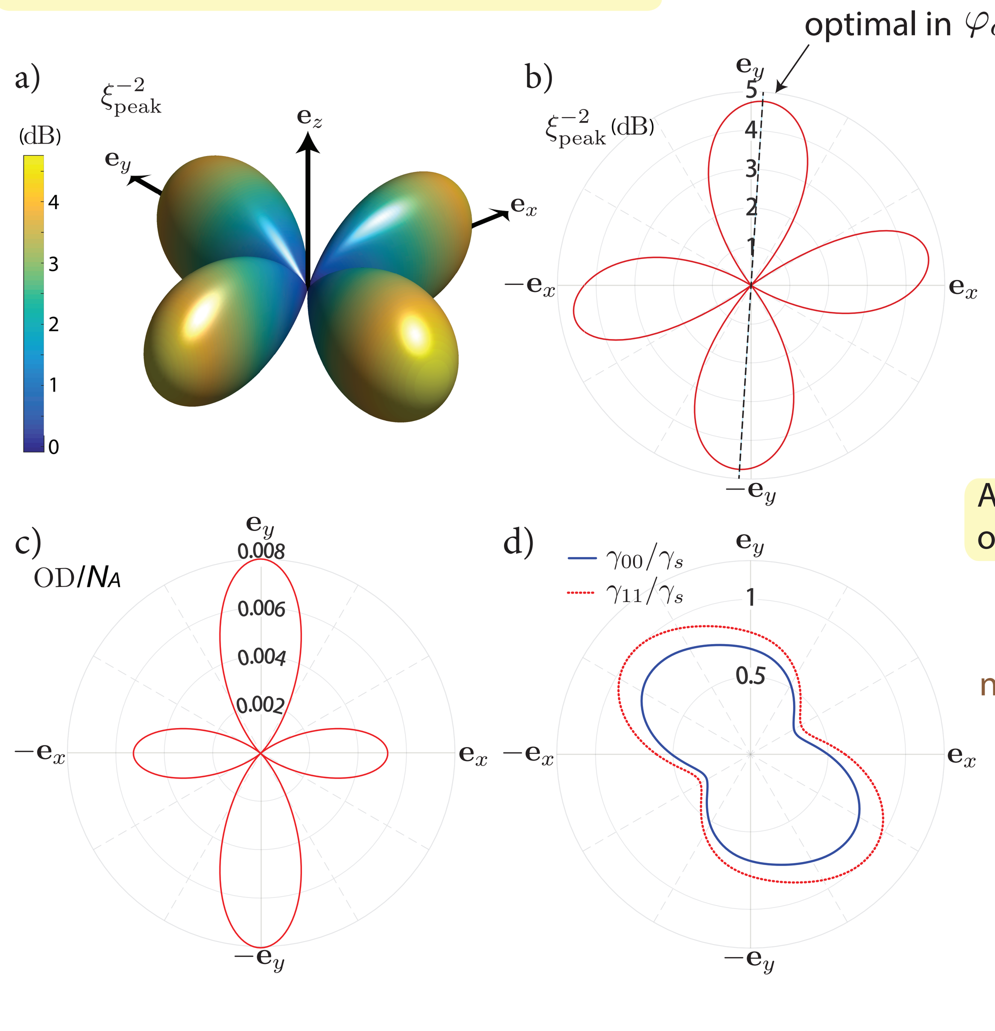
squeezing parameter:

$$\xi^2 = N_A\frac{\Delta J_3^2(t)}{|\langle\hat{J}_1(t)\rangle|^2}$$



(parameters: NA=2500, a=225nm, $r'_\perp$ =1.8a)

The optimal choice of **quantization axis**



Stochastic master equations:

$$d\hat{\rho} = s\sqrt{\frac{\kappa}{4}}\mathcal{H}[\hat{\rho}]dW + \frac{\kappa}{4}\mathcal{L}[\hat{\rho}]dt$$

collective measurement

$$+\sum_n\mathcal{D}_n[\hat{\rho}]dt$$

diffusive photon scattering

with $s = sign(\chi_{J_3})$.

Alternative expressions of **cooperativity** or **Purcell enhancement factor**

cavities: $C \equiv \frac{2g^2}{\kappa_c\Gamma} = \mathcal{F}\frac{\sigma_0}{A} = \mathcal{F}\frac{\Gamma_{1D}}{\Gamma_0}$

nanofiber: $C = N_A\frac{OD}{N_A} = N_A\frac{\Gamma_{1D}}{\Gamma_0}$

Other characteristic parameters

coupling strength $\chi_{J_3} = \frac{\sigma_0}{A_{J_3}}\frac{\Gamma}{2\Delta_{J_3}}$

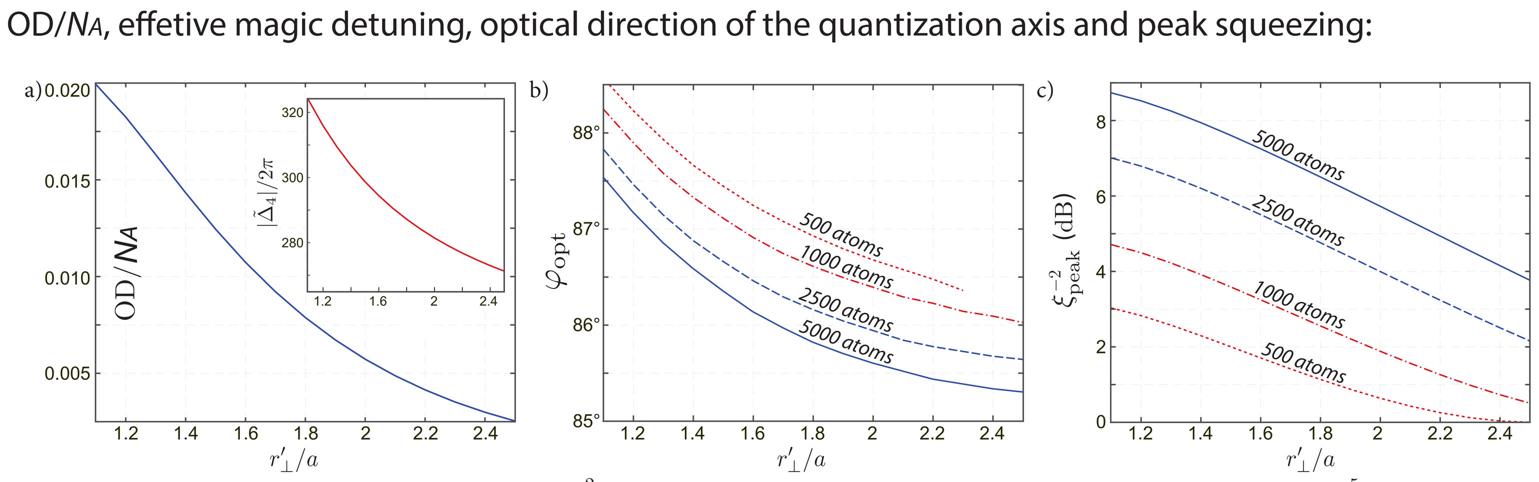
$$OD/N_A = \frac{\kappa}{\gamma_s} = \frac{\sigma_0 A_{in}}{A^2_{J_3}}$$

photon scattering rate $\gamma_s \equiv \frac{\Gamma\Omega^2}{4\Delta^2_{J_3}} = \frac{\sigma_0}{A_{in}}\frac{\Gamma^2}{4\Delta^2_{J_3}}\dot{N}_L$

The effective mode area for the birefringence interaction using the clock states is determined by

$$\frac{1}{A_{J_3}} = n_g\frac{|\mathbf{e}_{\tilde{z}}\cdot\mathbf{u}_H(\mathbf{r}'_\perp)|^2|\mathbf{u}_V(\mathbf{r}'_\perp)|^2 - |\mathbf{e}_{\tilde{z}}\cdot\mathbf{u}_V(\mathbf{r}'_\perp)|^2|\mathbf{u}_H(\mathbf{r}'_\perp)|^2}{|\mathbf{u}_H(\mathbf{r}'_\perp)|^2 + |\mathbf{u}_V(\mathbf{r}'_\perp)|^2}$$

OD/N_A, effetive magic detuning, optical direction of the quantization axis and peak squeezing:



Optical density per atom can be on the order of 10^{-2} using the nanofiber, while in free-space it is typically on the order of 10^{-5}.

Green's function for the nanofiber

The Green's function is a tensor that characterizes the **scattering** from a point source

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0(\mathbf{r}) + \overleftrightarrow{\mathbf{G}}(\mathbf{r},\mathbf{r}')\cdot\mathbf{d}_{eg}$$

Expression of the guided mode ***dyadic Green's function:***

$$\overleftrightarrow{\mathbf{G}}_g(\mathbf{r},\mathbf{r}';\omega_0) = 2\pi i\frac{\omega_0}{v_g}\sum_{b,p}\mathbf{u}_{b,p}(\mathbf{r}_\perp)\mathbf{u}^*_{b,p}(\mathbf{r}'_\perp)\, e^{ib\beta_0(z-z')}\Theta\big(b(z-z')\big)$$

sum over all propogation directions, b, and polarizations, p.

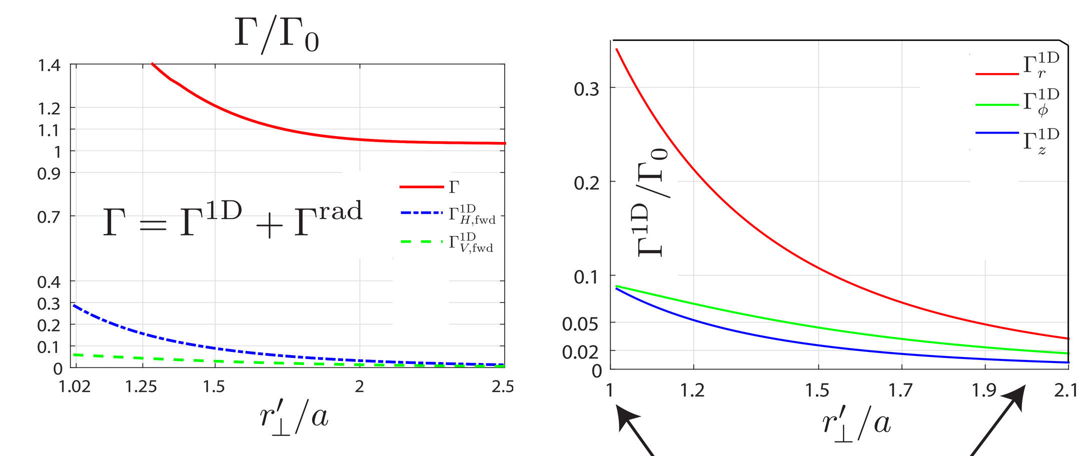
The imaginary part of the guided-mode Green's function tensor is

$$\mathrm{Im}\left[\overleftrightarrow{\mathbf{G}}_g(\mathbf{r}',\mathbf{r}')\right] = \pi\frac{\omega_{eg}}{v_g}\sum_{b,p}\mathbf{u}_{b,p}(\mathbf{r}'_\perp)\mathbf{u}^*_{b,p}(\mathbf{r}'_\perp)$$

modification of the atomic decay rate

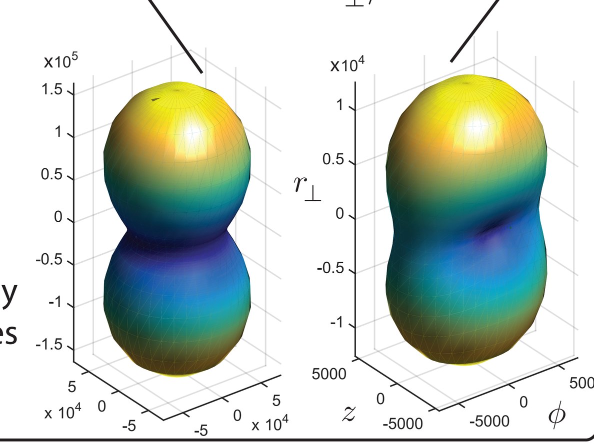
the **total decay rate** of an atom is modified by the presence of the dielectric nanofiber

$$\Gamma \propto \mathrm{Im}\left[\mathbf{d}_{eg}\cdot\overleftrightarrow{\mathbf{G}}^*(\mathbf{r}',\mathbf{r}')\cdot\mathbf{d}^*_{eg}\right]$$



the **decay rate into the guided modes** Γ^{1D} depends on the orientation of the dipole

emission surfaces show relative guided-mode decay rate as dipole orientation changes



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Fundings:

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