For office use only	Team Control Number	For office use only
T1 ________________	**XXXXX**	F1 ________________
T2 ________________		F2 ________________
T3 ________________	Problem Chosen	F3 ________________
T4 ________________	**C**	F4 ________________

2019 Mathematical Contest in Modeling (MCM) Summary Sheet
(Attach a copy of this page to each copy of your solution paper.)

An MCM Paper Made by 666666

Summary

Energy contributes to the development of human society and improvement of scientific technology. In the Untied States, there are abundant energy resource in these states. In this paper, we are asked by the four governors of states to analyze and determine the energy profile of these states and give suggestions about their interstate energy compact.

To illustrate the energy profile of four states, we establish our model from four aspects containing total energy consumption, energy consumption by different sectors and energy architecture. The energy profiles for each state is obtained by doing data reprocess and selecting the useful information based on our standard of description.

Fitting model is applied to demonstrate how the energy profile of each state varies with increasing time. We first draw the figure corresponding to the evolution of energy profile, and then determine which kind of fitting function we could utilize by evaluating determinant coefficient. Finally, cubic,fourth and fifth order polynomial models are applied to describe the evolution of energy profile with state-of-the-art result.

For determination of best energy profile, IRES evaluation system is proposed in this paper. This determination methodology utilizes six index to describe renewable energy condition for each state, and the weight of each index is determined by Entropy Weight Methodology. Finally, best energy profile is selected by TOPSIS Methodology. The rank results show that California has best energy profile due to maximum score, followed by Texas, Arizona and New Mexico.

As for prediction of energy profile in 2025 and 2050, we consider the data as time series and firstly use differential operation to enable its stability. Then ARIMA series model is applied to predict the energy profile. The results are shown in this paper.

To determine the renewable energy usage target, an optimization model to maximize the score of clean energy indicators (decision variables) obtained from the evaluation system (optimization objectives) is established. The goal of target is to obtain maximum scores, which means increase the usage of renewable energy based on certain constraints. Based on the results, we propose several actions for governors to implement.

Keywords: Energy Profile Model;Fitting Model;IRES Evaluation System;ARIMA Series Model;Energy Target

Contents

1 Introduction

1.1 Problem Background

Energy production and consumption have been considered as a big part of any economy. Owing to the various geographies and industries of different states affecting energy usage and production in the United States, Western Interstate Energy Compact (WIEC) was established including twelve western states in 1970, which aimed to encourage the cooperation between different states on a specific policy issue, a set of standards, national matter and so on.

1.2 Restatement of Problems

To form a realistic new energy compact focused on increasing usage of cleaner, renewable energy sources for four states- California (CA), Arizona (AZ), New Mexico (NM), and Texas (TX), our team should give the goals for their interstate energy compact by performing data analysis and modeling. The specific problems are listed as follows:

- Based on the data, we should create an energy profile including the analysis of different resources for the four states. Then a model should be developed to characterize how the energy profile has evolved during $1960 - 2009$ and we should determine the "best" profile for using the cleaner, renewable energy in 2009. Moreover, we should predict the energy profile of each state.
- We should determine renewable energy usage targets contained in the new four-state energy compact for 2025 and 2050 based on the comparison between the four states. And give actions the four states might take to meet these targets.
- A one-page memo should be developed for the governors which summarizes our work before.

1.3 Our work

Our contribution in this paper can be concluded in five parts:

1. We build energy profile model to illustrate the energy consumption, energy consumed by different sectors and energy architecture in four states.
2. We utilize the fitting model to describe the evolution of the energy profile in each state, specially we analyze the similarities and differences in renewable energy consumption among four states.

3. An IRES evaluation system is proposed to determine the best energy profile in these states.

4. method has been developed to predict the energy profile of each state

5. We utilize the optimization model to determine the renewable energy usage targets for 2025 and 2050, and actions needed are listed in the paper.

2 Preparation of the Models

2.1 Assumptions

Owing to the lack of necessary data and limitation of our knowledge, we make the following assumptions to help us perform modeling. These assumptions are the premise for our subsequent analysis.

- We treat total energy consumed by the electric power sector as the consumption of electricity.
- The policy of each state will not change in the future.
- All kinds of energy they produce are consumed by themselves each year. In this case, we can utilize the energy consumption to be one of the index in the energy profile.
- The natural environment and climate of these states will not change.

2.2 Notations

Here are the notations and their meanings listing in **Table 1**, which are frequently used in our paper:

Table 1: Notations

Symbol	Definition	Unit
E_j	Information Entropy	bit
P	Power	Watt
NGTCB	Natural gas total consumption	Billion Btu
PATCB	All petroleum products total consumption	Billion Btu
POTCB	Other petroleum products total consumption	Billion Btu
TEEIB	Total energy consumed by the electric power sector	Billion Btu
RETCB	Renewable energy total consumption	Billion Btu
REPRB	Renewable energy production	Billion Btu
TEPRB	Total energy production	Billion Btu
TETCB	Total energy consumption	Billion Btu

2.3 Data Reprocess

For data-analysis problem, there are usually some incomplete and abnormal information in the large amount of data sets, which may seriously affect the efficiency of modeling and the accuracy of conclusions. Therefore, it is quite important to preprocess the data.

- We use Matlab tool to check whether there is some data without interpretation or values, and based on our standard of model we delete these incomplete data.
- We classify these data into four separate groups due to simplify the data set and select useful data to establish model.
- We identify these data by summing some value of several sectors to check whether it equals to the given total value or not.

3 Model for Energy Profile of States

To create energy profile model for four states, we firstly analyze the energy consumption and expenditures of four states from a overall view and then based on the data, the total energy used by different sectors has been established to demonstrate the similarities and differences of energy usage between these states. Moreover, owing to the classification of energy source, we select five most common energy resources to form the energy architecture for four states.

3.1 Overview of States

Owing to the data provided by tasks, we take an overview of the energy consumption as well as the expenditures for each state by adding the total consumption of energy for each year in $1960 - 2009$. Specifically we also figure out the total consumption for renewable energy and calculate the average total consumption for each year by the formula:

$$\bar{P} = \frac{\sum_{i=1}^{50} P_i}{N} \tag{1}$$

where $N = 50$ and all the results are shown in **Table 2**.

Table 2: Overview of Energy Profile for States

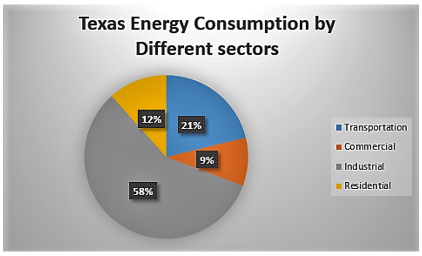
	Arizona	**California**	**New Mexico**	**Texas**
TETCB(BTU)	44004068.3	327814990.7	26311825.4	446522585.4
Average TETCB(BTU)	880081.4	6556299.8	526236.5	8930451.7
TETCV(Million dollars)	299099.7	2013081.6	123902.8	2078435.8
RETCB(BTU)	4612877.7	30186360.3	511883.8	5252428.3

Based on the **Table 2** and the information we have searched online[2], we give the energy profile in overview states as followings:

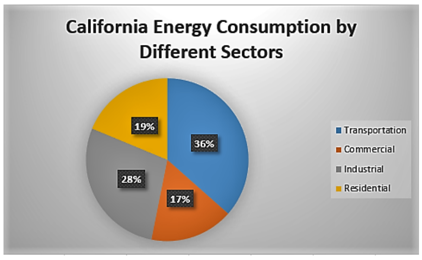
- Texas leads the nation in energy consumption over the fifty years among four states because of the second-largest population and the second-largest economy in United States ,and it also show a great potential in renewable energy sources due to the significant number of sunny days and wind generated electricity.
- As for California, the most populated state in the nation, has the largest economy, and is second only to Texas in total energy consumption. Furthermore, the state has abundant resources specially in the renewable energy including solar, geothermal and hydroelectric power resources.
- Arizona has the second-greatest consumption in renewable energy because of the greatest wind potential and abundant sunshine, but the total energy consumption is low compared with Arizona and Texas.
- New Mexico is the fifth-largest state by area, but it is the sixth-least densely populated so that all the characterizations are the lowest among four states. However, this state has a wealth of fossil fuel and mineral sources.

3.2 Energy Consumption by Different Sectors

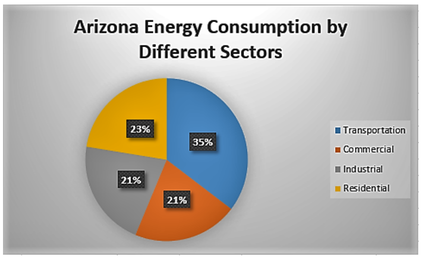
The consumption by different sectors is also considered as a part of energy profile in our work. According to the data, we calculate the average total energy consumption by one sector in 50 years and the percentages they take. We do our work by individual analysing four states.



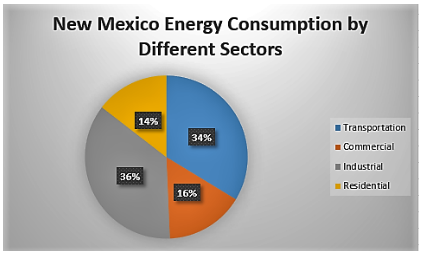
(a) Texas



(b) California



(c) Arizona



(d) New Mexico

Figure 1: Energy Consumption by Different Sectors

The results for all the four states are shown in **Figure 1**. From the figure, we find that industrial sector consumed the biggest part energy in Texas and New Mexico which the values are 58% and 35% respectively. The results can be understood because of the abundant crude oil, natural gas and coal in both Texas and New Mexico. Moreover, 36% and 36% percentage of energy is consumed by the transportation sector in California and Arizona states respectively. California is the state near the sea, in this case the energy is needed for the transportation on the sea including course ship. As for Arizona, it is famous for the mountain ranges feature which means there is huge demand for the strength power transportation. The energy consumption by commercial sector and residential is relatively low in these four states.

3.3 Energy Consumption of Specific Source

In this part, according to the classification principle of energy sources, we divide the given huge number of data into five common energy groups containing coal, petroleum, electricity, natural gas and renewable energy source. And we take the total consumption part into consideration, the corresponding short words and units as followings:

Table 3: Notations for Energy Architecture

Source	Description	Unit
Coal		
CLTCB	Coal total consumption	Billion Btu
Natural gas		
NGTCB	Natural gas total consumption	Billion Btu
Petroleum		
PATCB	All petroleum products total consumption	Billion Btu
POTCB	Other petroleum products total consumption	Billion Btu
Electricity		
TEEIB	Total energy consumed by the electric power sector	Billion Btu
Renewable		
RETCB	Renewable energy total consumption	Billion Btu

Based on the data, we filter the useful data and construct the energy architecture models for four states and the figure is shown in **Figure 2**:

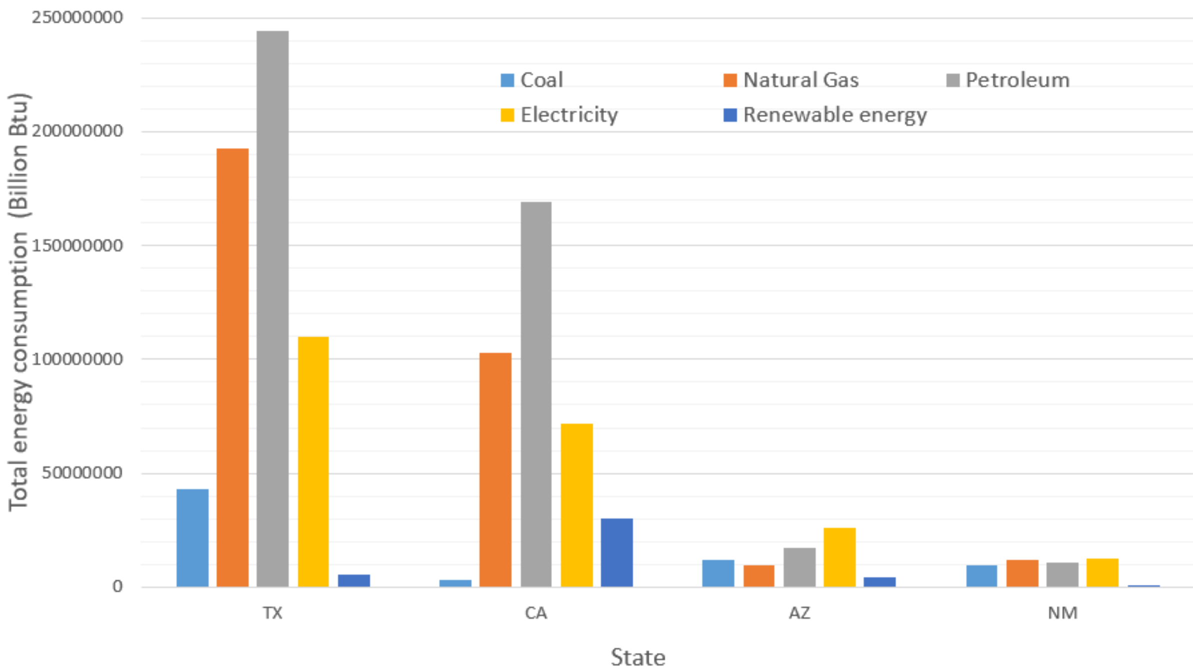


Figure 2: Energy Architecture in Four States

Based on the **Figure 2**, we can conclude that:

- Texas leads the energy consumption in each sources expect renewable energy and petroleum, coal and electricity contributes the huge part of the energy architecture of Texas, which confirm that there is a huge number of factories in this state.

- California shows the great strength in renewable energy consumption and other energy source consumptions formed energy architecture are very high. However, the consumption level for coal is relatively small.
- The consumption difference among these five energy resources is relatively small in Arizona, which means energy architecture for this state is balanced compared with the other states.
- New Mexico has almost same energy architecture with Arizona, and consumptions for coal, natural gas, petroleum and electricity are at the same level. However, the usage of renewable energy is extremely low in this state.

4 Model for Characterization Trend of Energy Profile

In this section, we utilize data fitting model to obtain the characterization of the trend of energy profile. To evaluate our model, determinant coefficient is applied in our model.

4.1 Data Fitting

Data fitting is a data processing method for approximating or analogizing the functional relations among coordinates represented by discrete points on a plane with continuous curves. It is also considered as a method of approximating discrete data by analytic expressions. In scientific experiments or social activities, there is a set of data pairs: (x_i, y_i) where $(i = 1, 2, ...M)$, and x_i is different from each other. It is hoped that a kind of analytic expression $y = f(x, c)$ corresponding to the law of background material of data, can be used to reflect the dependence between quantity x and y, which means to approximate or fit the known data "optimally" in a certain sense. $f(x, c)$ is considered as a fitting model, in which $\mathbf{c} = (c_1, c_2, ...c_n)$ are some undetermined parameters. In our model, we use higher order polynomial model to fit our data and apply determinant coefficient to evaluate the fitting model. The function of determinant coefficient is below:

$$R^2 = 1 - \frac{\sum(Y_{actual} - Y_{predict})^2}{\sum(Y_{actual} - Y_{mean})^2} \tag{2}$$

where $Y_{actual}, Y_{predict}$ and Y_{mean} represent real value, prediction value and mean value of real value respectively. The denominator is understood as the discreteness of the original data and the molecule is the error between the predicted data and the original data. The division of the denominator and the molecule can eliminate the influence of the discreteness of the original data. If the coefficient is closer to 1, the better the model fits the data, otherwise indicating the model is not apropos.

4.2 Evolution of Energy Profile for States

In this part, based on the structure of energy profile we mentioned before, we firstly draw the diagram of the energy consumption change during these years, and then determine which form of fitting model we will use by evaluating the determinant coefficient. In the end, we select the best fitting model as the representation of the characterization trend of energy profile. The flow-process diagram is shown **Figure 3**:

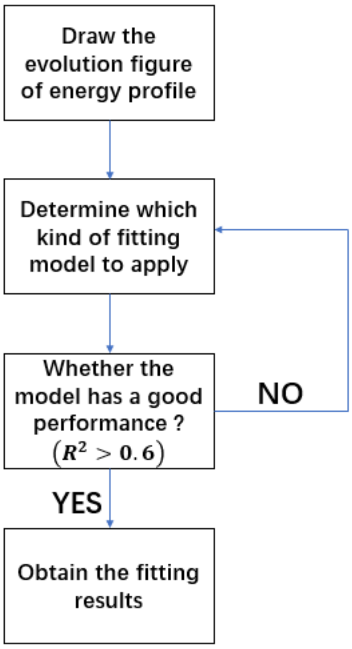


Figure 3: Flow-process Diagram of Fitting Model

4.2.1 Total Energy Consumption Evolution

We draw the diagram to illustrate how total energy consumption varies with the increasing of time. The results for all four states are in **Figure 4**:

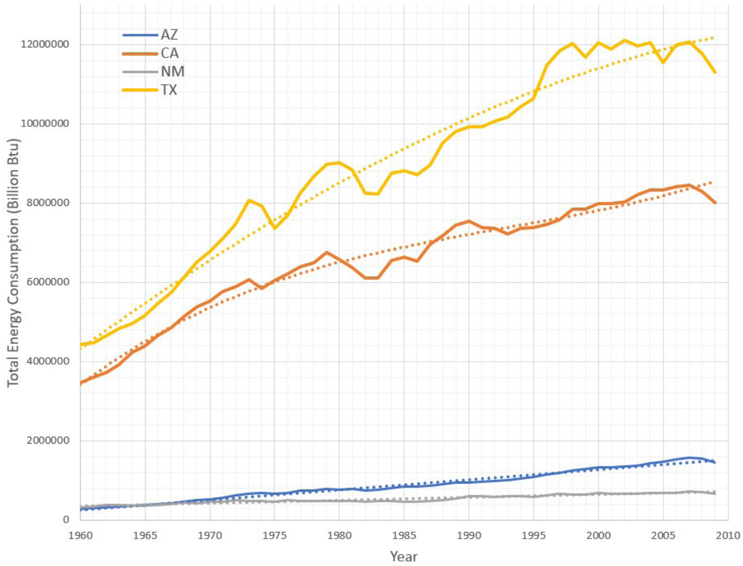


Figure 4: Total Energy Consumption Varies with Time

Based on our understanding and empirical results, we select cubic model shown below to fit our diagram:

$$y = c_0 + c_1 \cdot x + c_2 \cdot x^2 + c_3 \cdot x^3 \tag{3}$$

where $\mathbf{c} = (c_0, c_1, c_2, c_3)$ are some parameters. Based on this model, we utilize the Excel tool to fit the diagram and obtain the final equation which could best describe the characterization trend of total consumption. The equations are shown below:

$$\begin{cases} Y_1 = -7.3482 \cdot x^3 + 42017 \cdot x^2 - 8 \times 10^7 \cdot x + 5 \times 10^{10} \\ Y_2 = -58.051 \cdot x^3 - 347058 \cdot x^2 + 7 \times 10^8 \cdot x - 5 \times 10^{11} \\ Y_3 = 8.2783 \cdot x^3 - 49135 \cdot x^2 + 1 \times 10^8 \cdot x - 6 \times 10^{10} \\ Y_4 = 1.3523 \cdot x^3 - 8024.6 \cdot x^2 + 2 \times 10^7 \cdot x - 10^{10} \end{cases} \tag{4}$$

where Y_1, Y_2, Y_3 and Y_4 represent total consumption of Texas, California, Arizona and New Mexico respectively. The corresponding determinant coefficients are shown in **Table 4**:

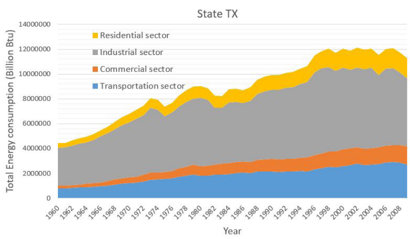
Table 4: Determinant Coefficients of States

States	Value of R^2
Texas	0.9647
California	0.9738
Arizona	0.9874
New Mexico	0.9244

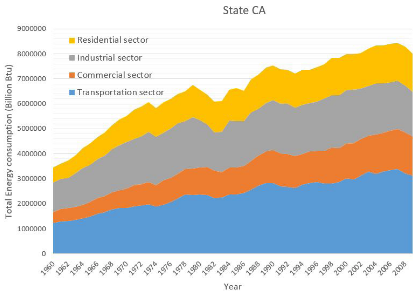
The performance of our fitting model is outstanding because all determinant coefficients are close to 1, in this case we can utilize it to describe the evolution of energy consumption for these states.

4.2.2 Energy Consumption Evolution by Sectors

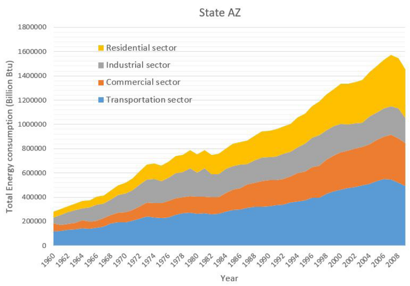
As for the energy consumption by different sectors, we draw the diagram to illustrate the evolution of this feature. The results are shown in **Figure 5**.



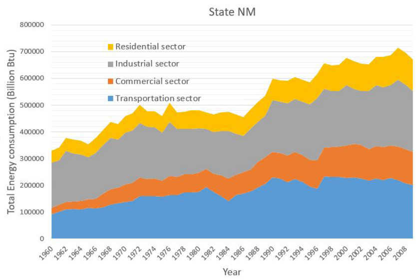
(a) Texas



(b) California



(c) Arizona



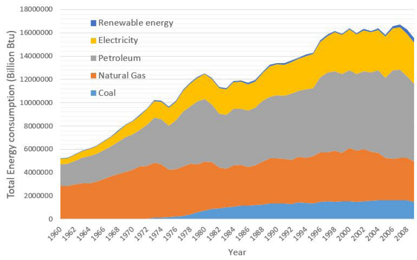
(d) New Mexico

Figure 5: Evolution Energy Consumption by Different Sectors

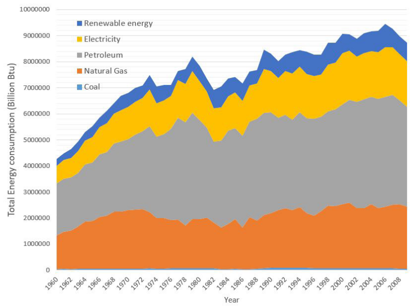
Owing to the stability of geography and climate condition, the energy consumption by different sectors does not have a huge change in $1960 - 2009$ for the four states. All the consumption by different sectors and maintain the statistical characteristics during these years.

4.2.3 Energy Structure Evolution

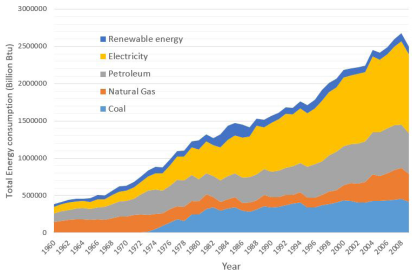
The energy structure have a huge change due to the development of technology, and **Figure 6** illustrates how the energy consumption of specific sources varies with increasing of time. Specially, in **Figure 7** we put much emphasis on the how renewable energy varies with increasing of time.



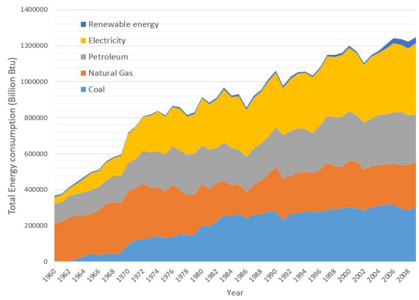
(a) Texas



(b) California

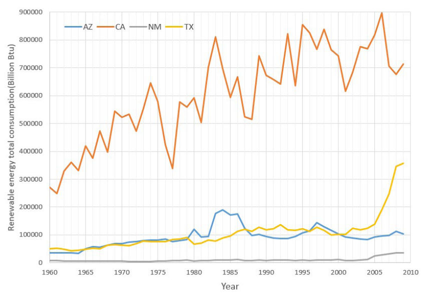


(c) Arizona

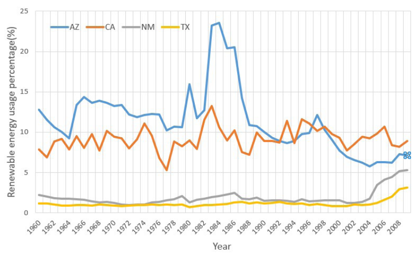


(d) New Mexico

Figure 6: Evolution Energy Consumption by Different Sectors



(a) Total Consumption



(b) Usage Index

Figure 7: Renewable Energy Evolution

According to the results in **Figure 6** and **Figure 7**, we can conclude that electricity has become a crucial energy source for four states and Petroleum also contributes to the development of these states. Moreover, the renewable energy has a dramatic increase during the 50 years in four states, specially in **California**, the usage of renewable energy contributes a big part in the energy consumption due to the biggest number of population and largest economy. In most of California's more densely populated areas, the climate is dry and relatively mild[6][7]. More than two-fifths of state households report that they do not use air conditioning, and about one-seventh do not use space heating[8]. **Texas** has a huge potential to use renewable energy because of the second largest land area and a significant number of sunny days. Temperatures in Texas average in the 90s during the summer in the most densely populated parts of Texas, and energy use for cooling is high[9]. Renewable energy in **Arizona** provided about 13% of Arizona's net electricity generation. More than one-tenth of that generation was from distributed (customer-sited, small-scale) facilities. Currently, almost half of the state's total renewable net generation and more than half of the state's utility-scale renewable net generation comes from hydroelectric power[10]. **New Mexico** has substantial renewable resources, particularly from wind and solar energy, but also from hydroelectric, biomass, and geothermal energ[11]. There is significant wind energy potential on the high plains in the eastern half of the state[12].

4.2.4 Evolution of Expenditures

In this part, we also utilize our fitting model to describe the evolution of expenditures of the total energy, which reflects the economy property of these states. We also first draw the evolution process in **Figure 8**. Based on our understanding

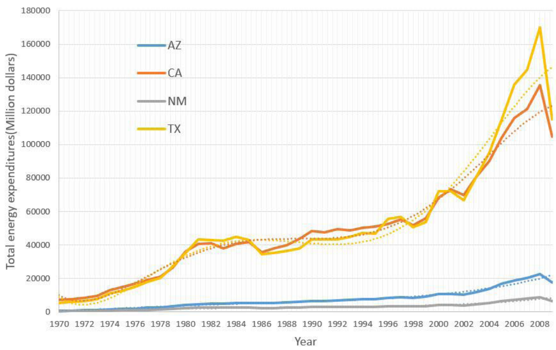


Figure 8: Expenditures Vary with Time

and empirical results, we select fourth and fifth order polynomial model shown below to fit our diagram:

$$\begin{cases} y = c_0 + c_1 \cdot x^1 + c_2 \cdot x^2 + c_3 \cdot x^3 + c_4 \cdot x^4 \\ y = c_0 + c_1 \cdot x^1 + c_2 \cdot x^2 + c_3 \cdot x^3 + c_4 \cdot x^4 + c_5 \cdot x^5 \end{cases} \tag{5}$$

where $c = (c_0, c_1, c_2, c_3, c_4, c_5)$ are some parameters. Based on this model, we utilize the Excel tool to fit the diagram and obtain the final equations shown below:

$$\begin{cases} Y_1 = -0.0413 \cdot x^5 + 411.23 \cdot x^4 - 2 \times 10^6 \cdot x^3 + 3 \times 10^9 \cdot x^2 - 3 \times 10^{12} \cdot x + 10^{15} \\ Y_2 = -0.032 \cdot x^5 + 318.77 \cdot x^4 - 1 \times 10^6 \cdot x^3 + 3 \times 10^9 \cdot x^2 - 3 \times 10^{12} \cdot x + 10^{15} \\ Y_3 = 0.0204 \cdot x^4 - 161.74 \cdot x^3 + 480202 \cdot x^2 - 6 \times 10^8 \cdot x + 3 \times 10^{11} \\ Y_4 = 0.0098 \cdot x^4 - 77.303 \cdot x^3 + 229508 \cdot x^2 - 3 \times 10^8 \cdot x + 10^{11} \end{cases} \tag{6}$$

where Y_1, Y_2, Y_3 and Y_4 represent expenditures of Texas, California, Arizona and New Mexico respectively. The corresponding determinant coefficients are shown in **Table 5**:

Table 5: Determinant Coefficients of States

States	Value of R^2
Texas	0.9433
California	0.9695
Arizona	0.9636
New Mexico	0.9391

The performance of our fitting model is outstanding because all determinant coefficients are close to 1, in this case we can utilize it to describe the evolution of energy expenditures for these states.

5 IRES Evaluation System

To evolute the usage of leaner, renewable energy in 2009, an evaluation system has been proposed in this paper called Index Renewable Energy Standard(IRES). In this section, there are three parts including IRES Definition, IRES evaluation process and results of IRES evaluation system to illustrate the methodology we used to determine the best profile among these states.

5.1 IRES Definition

IRES evaluation system determines the best energy profile by TOPSIS methodology in terms of the index number. The index numbers we defined are all related to the renewable energy, which means these numbers can be considered as the evaluation standard after giving the corresponding weight and can be applied to each year. Based on the index numbers, TOPSIS method is utilized to rank four states and select the best energy profile of a state.

5.2 IRES Evaluation Process

The determination process of IRES is shown in **Figure 9**:

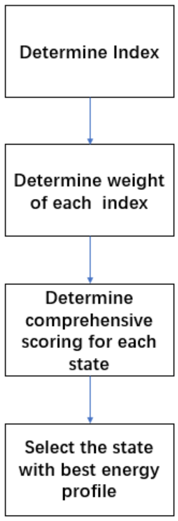


Figure 9: Process for determination IRES Evaluation Process

5.2.1 Determination of Index

For the first step, there are six indexes related to renewable energy we defined as follows:

- Proportion index of renewable energy usage RUP_i, represents what percentage renewable energy usage takes of total energy consumption. The determinant function is:

$$RUP_i = \frac{RETCB_i}{TETCB_i} \times 100 \tag{7}$$

- Total consumption index of renewable energy usage RUT_i, represents the total renewable energy consumption level for each state. The determinant function is:

$$RUT_i = \frac{RETCB_i}{\left(\sum_i RETCB_i\right)/4} \tag{8}$$

- Proportion index of renewable energy production RPP_i, represents what percentage renewable energy production takes of total energy production. The determinant function is:

$$RPP_i = \frac{REPRB_i}{TEPRB_i} \times 100\% \tag{9}$$

- Total consumption index of renewable energy production RPT_i, the total renewable energy production level for each state. The determinant function is:

$$RPT_i = \frac{REPRB_i}{\left(\sum_i REPRB_i\right)/4} \tag{10}$$

- Renewable energy production increase rate RPI_i, represents the increasing rate of renewable energy for each state. The determinant function is:

$$RPI_i = \frac{(REPRB_i)_T - (REPRB_i)_{T-1}}{(\sum_i ((REPRB_i)_T - (REPRB_i)_{T-1}))/4} \quad (11)$$

- Energy usage efficiency index EUE_i, represents the energy usage efficiency by total energy consumed per dollar of real gross domestic product. The determinant function is:

$$EUE_i = \frac{1}{TETGR_i} \quad (12)$$

5.2.2 Determination of Weight

Based on the indexes we defined, Entropy Weight Methodology is utilized to determine weight for each index number. Entropy was first introduced into information theory by Shannon, and has been widely used in engineering technology, social economy and other fields.

The basic idea of the Entropy Weight Methodology is to determine the objective weight according to the variability of the index. Generally speaking, the smaller the information entropy E_j of an index, the greater the variation degree of the index, which means the more information it provides, the greater the role it plays in the comprehensive evaluation, and the greater its weight. On the contrary, the larger the information entropy E_j of an index is, the smaller the degree of variation of the index, which means the less information it provides, and the smaller the role it plays in the comprehensive evaluation, the smaller its weight is.

Entropy Weight Methodology has the following three steps:
STEP1: Data standardization. Supposed that there are k indexes representing as $\mathbf{X} = (X_1, X_2, ..., X_k)$, in which $\mathbf{X_i} = x_1, x_2, ..., x_n$. Data standardization is to obtain $\mathbf{Y} = (Y_1, Y_2, ..., Y_k)$ by the following equation:

$$Y_{ij} = \frac{x_{ij} - \min x_i}{\max x_i - \min x_i} \quad (13)$$

STEP2: Determination the entropy. According to the information theory, the entropy for a set of information is defined as follows:

$$E_j = -\ln \frac{1}{n} \sum_{i=1}^{n} p_{ij} \ln p_{ij} \quad (14)$$

where p_{ij} is defined as:

$$p_{ij} = \frac{y_{ij}}{\sum_i y_{ij}} \quad (15)$$

STEP3: Determination the weight. Based on the entropy we calculate, the weight for each index can be obtained by the formula:

$$W_j = \frac{1 - E_j}{k - \sum_j E_j}(j = 1, 2, \ldots, k) \tag{16}$$

5.2.3 TOPSIS Methodology

After obtaining the weight of each index number, we can rank the four states to select the best profile by applying TOPSIS Methodology[4]. The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) is a multi-criteria decision analysis method[3], which is based on the concept that the chosen alternative should have the shortest geometric distance from the positive ideal solution (PIS)[5] and the longest geometric distance from the negative ideal solution (NIS)[5].

The TOPSIS process is carried out as follows[3]:
STEP1: Create an evaluation matrix consisting of m alternatives and n criteria, with the intersection of each alternative and criteria given as x_{ij}, we therefore have a matrix $(x_{ij})_{m\times n}$
STEP2: The matrix $(x_{ij})_{m\times n}$ is then normalised to form the matrix $R = (r_{ij})_{m\times n}$, using the normalisation method:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{k=1}^{m} x_{kj}^2}}, i = 1, 2, ..., m, j = 1, 2, ..., n \tag{17}$$

STEP3: Calculate the weighted normalised decision matrix:

$$t_{ij} = r_{ij} \cdot w_j, i = 1, 2, ..., m, j = 1, 2, ..., n$$

Where $w_j = W_j / \sum_{k=1}^{n} W_k, j = 1, 2, ..., n$ so that $\sum_{i=1}^{n} w_i = 1$, and W_j is the original weight given to the indicator $v_j, j = 1, 2, ..., n$
STEP4: Determine the worst alternative(A_w) and the best alternative (A_b) by the formula:

$$\begin{cases} A_w = \{\langle max(t_{ij}|i = 1, 2, ..., m)|j \in J_-\rangle, \langle min(t_{ij}|i = 1, 2, ..., m)|j \in J_+\rangle\} \equiv \{t_{wj}|j = 1, 2, ..., n\} \\ A_b = \{\langle min(t_{ij}|i = 1, 2, ..., m)|j \in J_-\rangle, \langle max(t_{ij}|i = 1, 2, ..., m)|j \in J_+\rangle\} \equiv \{t_{bj}|j = 1, 2, ..., n\} \end{cases} \tag{18}$$

where, $J_+ = \{j = 1, 2, ..., n|j\}$ associated with the criteria having a positive impact, and $J_- = \{j = 1, 2, ..., n|j\}$ associated with the criteria having a negative impact.
STEP5: Calculate the L2-distance between the target alternative i and the worst condition A_w by:

$$d_{iw} = \sqrt{\sum_{j=1}^{n}(t_{ij} - t_{wj})^2}, i = 1, 2, ..., m \tag{19}$$

and the distance between the alternative i and the best condition A_b calculated by:

$$d_{ib} = \sqrt{\sum_{j=1}^{n}(t_{ij} - t_{bj})^2}, i = 1, 2, ..., m \tag{20}$$

where d_{iw} and d_{ib} are L2-norm distances from the target alternative i to the worst and best conditions, respectively.
STEP6: Calculate the similarity to the worst condition:

$$s_{iw} = d_{iw}/(d_{iw} + d_{ib}), 0 \leq s_{iw} \leq 1, i = 1, 2, ..., m. \tag{21}$$

$s_{iw} = 1$ if and only if the alternative solution has the best condition; and $s_{iw} = 0$ if and only if the alternative solution has the worst condition.
STEP7: Rank the alternatives according to $s_{iw}(i = 1, 2, ..., m)$.

5.3 Results of IRES Evaluation System

We utilize the IRES Evaluation System to find the best energy profile.

Firstly, we calculate the indexes value for these states and obtain the results shown in **Table 6**:

Table 6: Index Number of States

	RUP	RUT	RPP	RPT	RPI	EUE
Arizona	7.116298	0.34256	15.51179	0.33388	-1.20059	0.158737
California	8.902668	2.359035	24.37568	2.393941	4.602812	0.216963
New Mexico	5.317964	0.117952	1.400585	0.127357	-0.17538	0.103048
Texas	3.156784	1.180453	2.548864	1.144821	0.773153	0.094397

Secondly, we perform Entropy Weight Methodology to obtain the weight of each index shown in **Table 7**:

Table 7: Weight of Index

	RUP	RUT	RPP	RPT	RPI	EUE
W	0.185587	0.164094	0.159232	0.162181	0.167838	0.161068

Finally, TOPSIS Methodology is applied to obtain the best energy profile, the result shown in **Table 8**:

Table 8: Rank of States

Rank	State	Comprehensive Evaluation Index
1	CA	1
2	TX	0.327622
3	AZ	0.273282
4	NM	0.138054

In this case, we figure out California state has best energy profile.

6 Model for Prediction of Energy Profile

To predict the energy profile of each state for 2025 and 2050, we consider this evolution as time-series process, and firstly determine whether it is a stable time-series process. Based on the determination, ARIMA series model is applied to predict the energy profile.

6.1 Time-Series Analysis

Time-series analysis is the theory and method of establishing mathematical model by curve fitting and parameter estimation based on the time series data observed systematically. In this task, we can consider the data provided as time-series. And then Daniel-test is applied to figure out whether the time-series is stable or not, the process of Daniel-test is: for significant level α, the Spearman rank correlation coefficient q_s is calculated from time series by formula:

$$q_s = 1 - \frac{6}{n(n^2-1)} \cdot \sum_{1}^{n} (t - R_t)^2 \tag{22}$$

where R_t is the rank of time series samples, n is the number of time series sample. If $q_s > 0$, we consider the time series has an increasing trend, otherwise it has a decreasing trend. A statistic parameter is constructed by the **Equation 23**. Series can be regarded as stationary sequence if $T > t_{\alpha/2}$.

$$T = \frac{q_s\sqrt{n-2}}{\sqrt{1-{q_s}^2}} \tag{23}$$

If the time series is not stable, differential operation is applied to enable the time series to become a stable one. In this task, First-order difference operation is applied:

$$\nabla X_t = X_t - X_{t-1} \tag{24}$$

where ∇ represents first order backward difference operation, X_t and X_{t-1} represent the samples of time series. Owing to this operation, we convert the unstable time series into stable time series.

6.2 ARIMA Series Model

After obtaining stable time series, ARIMA model is applied to predict the following series. ARIMA (Autoregressive Integrated Moving Average model), differential integrated moving average autoregressive model, also known as integrated moving average autoregressive model (moving can also be called sliding), is one of the time series prediction analysis methods. The model can be describe as follows:

$$(1-\sum_{i=1}^{p}\Phi_i L^i)(1-L)^d X_t = (1+\sum_{i=1}^{q}\Theta_i L^i)\varepsilon_t \tag{25}$$

where p is the number of autoregressive terms, q is the number of moving average terms, and d is the number of difference (order) that makes it a stationary sequence. Our task is to determine the value of p, q and d, thus we can perform prediction. q and d can be calculated by AIC criterion.

6.3 Prediction Results

We utilize the ARIMA model to predict TETCB of Arizona as an example, the rest of prediction has same process. Firstly, we draw the figure to illustrate how TETCP varies with increasing time, the result shown below:

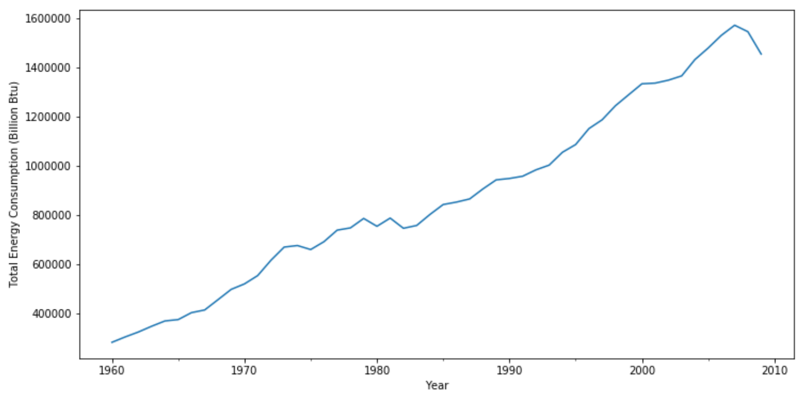


Figure 10: TETCB Evolution

From the figure show that the state's energy consumption is increasing year by year, which is not a stationary series. We use Daniel test to verify the correctness of our judgment and the significance level. For significance level 0.05, we calculate $q_s = 0.9962$, then $T = 78.8132$, $t_{0.025}(48) = 2.0106$, which means the time series is not stable. Thus, we do first order difference operation on it, the result is shown below:

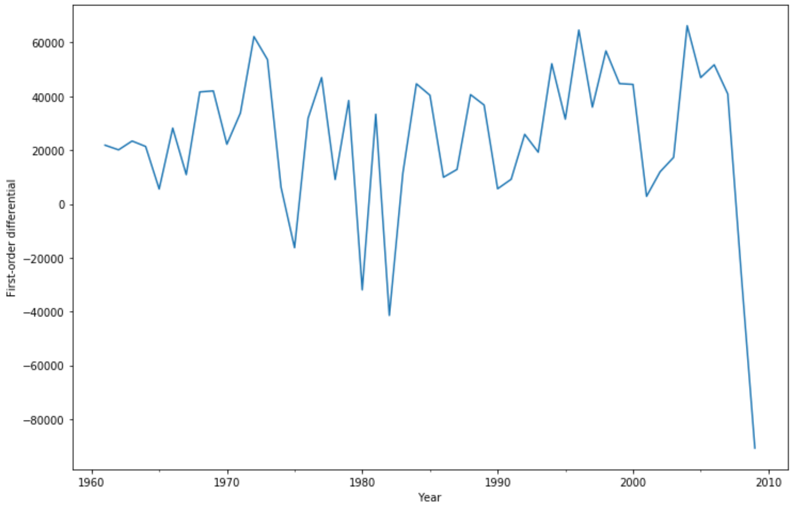


Figure 11: First Order Difference Sequence

Based on the result, we do Daniel test and as for significance level 0.05, $T = 0.3601$ and $t_{0.025}(48) = 2.0117$, which can be considered as a stable time series. Therefore, parameter d is set to 1.

Then we use the AIC criterion to determine the other two parameters in the model. We calculate the autocorrelation coefficient and partial autocorrelation of the difference sequence, as shown in **Figure 12**. From the graph, it can be seen that the autocorrelation coefficient has a greater correlation with its lag value of 1, and partial autocorrelation has a greater correlation with its lag value of 0. Therefore we can take $p = 1, q = 0$.

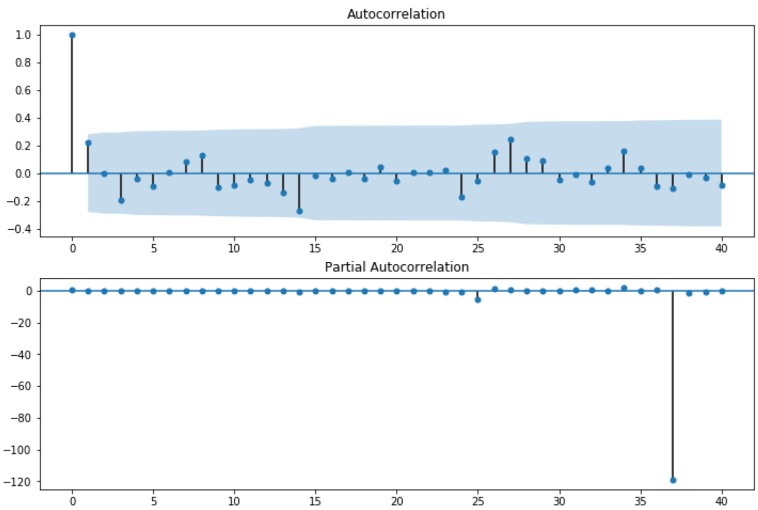


Figure 12: First-order Difference Sequence Correlation

To ensure the validity of the model, we calculated the residual of the model. From **Figure 14**, the correlation of the residual is low and the average distribution is zero. The result in **Figure 13** shows that the orderly distribution of residual (blue dot) follows the linear trend of sampling with N (0,1) standard normal distribution. Residues can well obey normal distribution. That is, the actual data can better meet the requirements of the model.

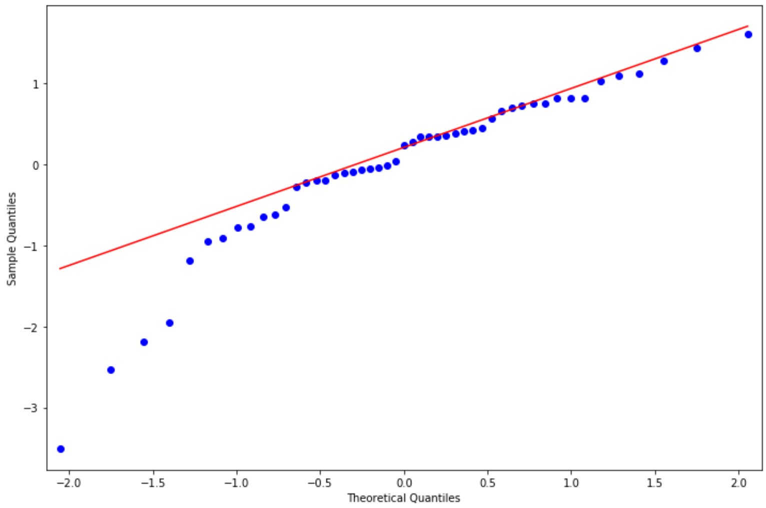


Figure 13: QQ Graph

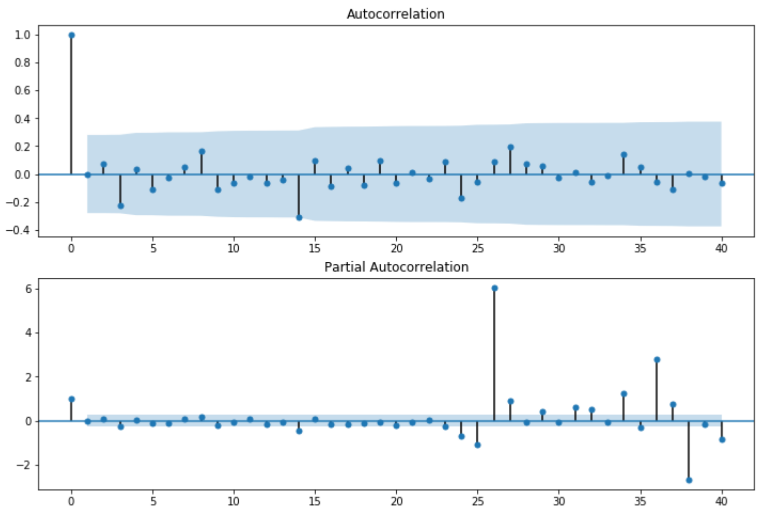


Figure 14: Model Residuals

We also determine ARIMA parameters by synchronization for other data and make prediction. The prediction structure is shown in **Table 10**.

Table 9: The Prediction Results of Total Energy Consumption

	2025	2050
AZ	1.7783×10^6	2.3476×10^6
NM	7.7876×10^5	9.3920×10^5
CA	2.3476×10^6	1.1209×10^7
TX	1.5091×10^7	1.8911×10^7

7 Model for Determination of Energy Target

To determine renewable energy usage targets for 2025 and 2050, we want to establish an optimization model to maximize the score of clean energy indicators (decision variables) obtained from the evaluation system (optimization objectives), while the indicators are constrained by the predicted value and cost (constraints). Then the optimization results are compared with the data in 2009 (now), and finally we determine the goal. Based on the results, three actions are proposed for four states to meet their goals.

7.1 Determination of Target

7.1.1 Determination of Individual State Target

According to the IRES Evaluation system, we firstly set the decision variable as follows:

$$\begin{cases} x_1 = RUP \\ x_2 = RPP \\ x_3 = RPI \\ x_4 = EUE \end{cases} \tag{26}$$

where RUP, RPP, RPI and EUE are defined in **Equation 7**, **Equation 9**, **Equation 11**and **Equation 12**. Our goal is to maximize the scores obtained from the IRES Evaluation system:

$$Score = WX = w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 x_4 \tag{27}$$

The constraints are considered as follows:

- In order to reduce energy consumption per unit GDP and reduce energy costs, the growth rate of clean energy consumption should not be higher than that of GDP.
- Clean energy production index should not exceed the production limit of clean energy. For the proportion of clean energy production, its production

limit should not exceed the predicted value. For the growth rate of clean energy, its production limit should not exceed the maximum value in previous years'data. The ratio index of clean energy production is predicted to be RPP_t, the maximum of the growth rate index of clean energy is $RPI_{\max}$.

- Restricted by the level of scientific and technological development, the energy use efficiency index should not exceed the predicted value.

Then based on these analysis, we can determine our model by **Equation 28**:

$$\max Score$$
$$s.t. \begin{cases} \frac{x_1 \cdot TCB_t - RUP_{09} \times TCB_{09}}{RUP_{09} \times TCB_{09}} \leq \frac{G_t - G_{09}}{G_{09}} \\ x_2 \leq RPP_t \\ x_3 \leq RPI_{max} \\ x_4 \leq EUE_t \\ x_1, x_2, x_3, x_4 \geq 0 \end{cases} \tag{28}$$

Finally, we obtain the targets that states should achieve in 2025 and 2050. The results are shown in **Table 10** and **Table 11**:

Table 10: Targets in 2020

	RUP	RPP	RPI	EUE
AZ	13.00196	21.56	85.399	0.2344
CA	20.82997	32.59	70.555	0.3051
NM	10.01343	2.759	94.932	0.1492
TX	5.618522	7.881	55.092	0.1258

Table 11: Target in 2050

	RUP	RPP	RPI	EUE
AZ	16.93616	35.67	85.399	0.2778
CA	20.73644	40.21	70.555	0.3608
NM	12.35062	4.052	94.932	0.1742
TX	7.379702	15.17	55.092	0.1521

7.1.2 Determination of Four-state Energy Compact

To determine the target of four-state energy compact, we establish average growth index defined as follows:

$$I_t = \frac{X_t - X_{09}}{|X_{09}|} \times 100\% \tag{29}$$

where X_t, X_{2009} represent index and index in 2009 respectively. Moreover consider the differences among four states, we define the final target by taking the mean of growth index as a unified cooperative target:

$$CG_{X-t} = \frac{X_{AZ-t} + X_{CA-t} + X_{NM-t} + X_{TX-t}}{4} \tag{30}$$

We solve the model and obtain the target as follows:

Table 12: Target for Four-state Energy Compact

	CG_{RUP-t}	CG_{RPP-t}	CG_{RPI-t}	CG_{EUE-t}
2025	95.74%	94.72%	1975.26%	41.59%
2050	134.23%	219.85%	1975.26%	67.87%

7.2 Proposed Actions

Based on the targets we obtain, we give following actions as suggestions for governors of four states:

1. Improve the cleaner, renewable energy usage

- Tax shifting has been widely discussed and endorsed by economists. It involves lowering income taxes while raising levies on environmentally destructive activities, in order to create a more responsive market.
- Subsidies for the use of the cleaner, renewable energy are effective as many technologies and industries emerged through government subsidy schemes.
- Set up funds in four states that aim to support renewable energy construction demonstration projects.

2. Improve the Energy usage efficiency

- Make more investment in scientific research of improving the same standard amount of different energy calorific value utilization degree.
- Develop the intensive economic growth mode, which mainly relies on the scientific and technological progress and improving the quality of workers to increase the quantity and quality of products and promote economic growth.

3. Interstate energy share

- Each state should focus on developing their own advantageous energy resource, take advantage of their superiority in produce cleaner energy and share their redundant cleaner energy and useful technologies with other states.

- CA performs the best and can develop as its current trend without exerting more additional actions. CA can help other three states cooperate in the way of providing more renewable export to other three states to stimulate the renewable energy consumption there.

4. Effective Superintendence

- Establish certain quantitative index and carry out statistical survey periodically to make sure the direction of energy usage is approaching goals.

8 Strengths and Weaknesses

8.1 Strengths

- **Clear and comprehensible energy profile:**Firstly, we give the overview of energy consumption and expenditure of each state. Then, we analyze the energy structure with five main energy sources and the consumption by four sectors in the way of giving plentiful and intuitional diagrams to give specific details.
- **Quantitative and comprehensive Characterization model:**When analyzing the evolution of energy profile in each state, we use data fitting model to Characterize the Total Energy Consumption Evolution and the Evolution of Expenditures of Energy Profile in a quantitative and precise way. We take the factor of economy, population, climate and geography into consideration in analyzing similarities and difference between the four states.
- **Fair and objective evaluation criteria model:**The IRES Evaluation System combines Entropy Weight Methodology and TOPSIS Methodology. The indexes we choose are comprehensive to describe the use of cleaner, renewable energy sources. TOPSIS is a multi-criteria decision analysis method, which allows trade-offs between criteria, where a poor result in one criterion can be negated by a good result in another criterion, to give a fair and objective evaluation result.
- **Accurate prediction:**ARIMA is employed to predict the energy profile. Firstly, we transfer given series into stationary sequence by differential operation and employ Daniel-test to exam itsąŕ stationary. The others parameters are determined by AIC rule. The efficiency of our model is proved by QQ graph. Based on built model, the prediction is made for energy consumption in 2025 and 2050.
- **Quantified and Rational Goals:**To determine renewable energy usage targets, We establish the optimization model to quantify targets. Then we set rational goals for the four states energy compact considering the actual situation of each state.

8.2 Weaknesses

- **Insufficient use of data:**There are 605 variables to describe the energy consumption condition and we only use a dozen of them for analyzing. We may need to make full use of all the data to obtain more information.
- **Subjectivity:**In the evaluation criteria model, the choose of evaluation index may be subjective. In the optimization model, the determination of some parameters may be subjective. We need to look for other methods and give more references to support them.

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MEMORANDUM

To: Governors of CA, AZ, NM, and TX

From: Team ♯66666

Date: Jan 12th, 2019

Subject: Energy Profiles Characterization, Prediction and Future Goals Honorable Governors of CA, AZ, NM, and TX

Dear governors, we are honored to inform you our achievement after performing data analysis and modeling.

Firstly, we analyze the energy consumption and expenditures of four states from a overall view and then based on the data, the total energy used by different sectors has been established to demonstrate the similarities and differences of energy usage between these states. Moreover, owing to the classification of energy source, we select five most common energy resources to form the energy architecture for four states.

Secondly, we utilize data fitting model to obtain the characterization of the trend of energy profile. To evaluate our model, determinant coefficient is applied in our model. We also put much emphasis on the renewable energy usage part, the results show that there is a huge potential to utilize the cleaner renewable resource.

Thirdly, an IRES Evaluation System is proposed to evaluation the energy profile of states. According to our results, California has the best energy profile.

Finally, based on our prediction model we predict the energy profile of each state in 2025 and 2050. Based on the results we set the goals and actions should be taken for you as follows:

- Subsidies for the use of the cleaner, renewable energy are effective as many technologies and industries emerged through government subsidy schemes.
- Develop the intensive economic growth mode, which mainly relies on the scientific and technological progress and improving the quality of workers to increase the quantity and quality of products and promote economic growth.
- Each state should focus on developing their own advantageous energy resource, take advantage of their superiority in produce cleaner energy and share their redundant cleaner energy and useful technologies with other states.
- CA performs the best and can develop as its current trend without exerting more additional actions. CA can help other three states cooperate in the way of providing more renewable export to other three states to stimulate the renewable energy consumption there.