Given N tasks and K candidate groups, our method minimize the following empirical risk:

$$L^{\text{task}}(\theta, S) = L(\theta) \odot Z(S) = \sum_{i=1}^{N} \sum_{k=1}^{K} z_{ik} L_{ik},$$
(1)

or the relaxed empirical risk by the Concrete distribution:

$$L^{\text{relaxed task}}(\theta, S) = L(\theta) \odot \widetilde{Z}(S) = \sum_{i=1}^{N} \sum_{k=1}^{K} \tilde{z}_{ik} L_{ik}, \tag{2}$$

where \odot is the element-wise product, $L = \{L_{ik}\} \in \mathbb{R}^{N \times K}$ is the loss matrix, $S = \{s_{ik}\} \in \mathbb{R}^{N \times K}$ is the set of parameters of the categorical distributions, θ is the model weights. $Z = \{z_{ik}\} \in \mathbb{R}^{N \times K}$ and $\widetilde{Z} = \{\widetilde{z}_{ik}\} \in \mathbb{R}^{N \times K}$ are the discrete and continuous relaxed sampling, respectively:

$$z_{ik} \sim \text{Categorical}(s_{ik})$$
 (3)

$$\tilde{z}_{ik} = \frac{\exp((s_{ik} + g_{ik})/\tau)}{\sum_{m=1}^{K} \exp((s_{im} + g_{im})/\tau)}$$
(4)

Theorem .1. The convergence of Eq. (10) or (12) is no worse than the naive multi-task learning (MTL) baseline where all the tasks are categorized into one group.

Proof. For the naive MTL baseline:

$$\exists k \text{ and } \forall i, \quad z_{ik}/\tilde{z}_{ik} = 1,$$

otherwise, $z_{ik}/\tilde{z}_{ik} = 0,$ (5)

Equation (5) indicates that for the naive MTL baseline, in the $N \times K$ group assignment matrix Z or \widetilde{Z} , there exists a column with all **1**'s, and all **0**'s elsewhere. Given Eqs. (10) and (3) for Z, or Eqs. (12) and (4) for \widetilde{Z} , it is straightforward to identify that such a circumstance is a special case of our method. By minimizing either of our empirical risks, the convergence is no worse than the naive MTL baseline.

Theorem .2. The convergence of Eq. (10) or (12) is no worse than the single-task learning (STL) baseline given K = N.

Proof. For the STL baseline:

$$\forall i, \quad z_{ii}/\tilde{z}_{ii} = 1,$$
 if $k \neq i, \quad z_{ik}/\tilde{z}_{ik} = 0,$ (6)

Equation (6) indicates that when K = N, for the STL baseline, the diagonal of the $N \times K$ group assignment matrix Z or \widetilde{Z} are all 1's, and all 0's elsewhere. Given K = N, Eqs. (10) and (3) for Z, or Eqs. (12) and (4) for \widetilde{Z} , it is straightforward to identify that such a circumstance is a special case of our method. By minimizing either of our empirical risks with K = N, the convergence is no worse than the STL baseline.

We also note that when setting K=N, it is not necessary to categorize the input tasks into N groups in our method. Instead, our method automatically learns to categorize the input tasks into **less than** N groups that minimize the empirical risk. This is guaranteed by our categorical distribution which allows certain groups to contain $\mathbf{0}$ task, which is empirically verified in our Table 7.