Extension of CTR

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1 Model

Assume there are K topics $\beta := \beta_{1:K}$ and that these are the same for users and items.

1. For each user i,
   (a) Draw topic proportions $\theta_i^u \sim \text{Dirichlet}(\alpha)$.
   (b) Draw item latent offset $\epsilon_i^u \sim \text{N}(0, \lambda_u^{-1} I_k)$ and set the item latent vector as $\boldsymbol{u_i = \epsilon_i^u + \theta_i^u}$.
   (c) For each word w_{in}^u.
      i. Draw topic assignment $z_{im}^u \sim \text{Mult}(\theta^u)$.
      ii. Draw word $w_{im}^u \sim \text{Mult}(\beta_{z_{im}^u})$.
2. For each item j,
   (a) Draw topic proportions $\theta_j^v \sim \text{Dirichlet}(\alpha)$.
   (b) Draw item latent offset $\epsilon_j^v \sim \text{N}(0, \lambda_v^{-1} I_k)$ and set the item latent vector as $v_j = \epsilon_j^v + \theta_j^v$.
   (c) For each word w_{jn}^v.
      i. Draw topic assignment $z_{jn}^v \sim \text{Mult}(\theta)$.
      ii. Draw word $w_{jn}^v \sim \text{Mult}(\beta_{z_{jn}})$.
3. For each user-item pair (i, j), draw the rating $r_{ij} \sim \text{N}(u_i^T v_j, c_{ij}^{-1})$.

2 Inference

This is an EM-style algorithm to learn MAP estimates. The goal is to maximize the complete log likelihood

$$
\begin{aligned}
L = &-\frac{\lambda_u}{2}\sum_i \boldsymbol{(u_i - \theta_i^u)^T(u_i - \theta_i^u)} - \frac{\lambda_v}{2}\sum_j (v_j - \theta_j^v)^T(v_j - \theta_j^v) \\
&+ \boldsymbol{\sum_i \sum_m log(\sum_k \theta_{ik}^u \beta_{k,w_{im}^u})} + \sum_j \sum_n log(\sum_k \theta_{jk}^v \beta_{k,w_{jn}^v}) - \sum_{i,j} \frac{c_{i,j}}{2}(r_{ij} - u_i^T v_j)^2.
\end{aligned}
$$