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1. Setup

Template

```
#include <bits/stdc++.h>
using namespace std;
#define forall(it,v) for (auto it = begin(v); it != end(v); it++)
#define forr(i,a,b) for(int i = (a); i < (b); i++)
#define forn(i,n) forr(i,0,n)
#define all(v) begin(v), end(v)
#define mp(a,b) make_pair(a,b)
#define pb push_back
#define fst first
#define snd second
#define endl '\n'
#define dprint(x) cerr << #x << " = " << (x) << endl
#define raya cerr << "=====" << endl
#define templT template <class T>
#define templAB template <class A, class B>
templAB ostream& operator << (ostream& o, pair<A,B>& p) { return o <<
    ↪ p.first << " " << p.second; }
templT ostream& operator << (ostream& o, vector<T>& v) { forall(it,v
    ↪ ) { o << *it << " "; } return o; }

int main () { ios::sync_with_stdio(0); cin.tie(0); cout.tie(0);

    return 0;
}
```

Makefile

```
CC = g++
CPPFLAGS = -Wall -g \
-fsanitize=undefined -fsanitize=bounds \
-std=c++17 -O0

comp.sh: Compilar $1 y mostrar primeras $2 lineas de error

clear
make -s $1 2>&1 | head -$2

run.sh: Correr $1 con el input $2

clear
make -s $1 && ./ $1 < $2
```

2. STL

2.1. <algorithm.h>

Funciones que modifican rangos

Función	Params	Ejemplo
copy	first last result	B.resize(A.size()); copy(all(A), B)
fill	first last val	memo.resize(MAXN); fill(all(memo), -1)
rotate	first middle last	rotate(begin(A), begin(A) + 3, end(A));

Búsqueda binaria en vector ordenado

```
templT int primer_igual (vector<T>& arr, T x) {
    auto it = lower_bound(all(arr), x);
    if (it == arr.end() || *it != x) return -1;
    return it - arr.begin();
}

templT int ultimo_igual (vector<T>& arr, T x) {
    if (arr.begin() == arr.end()) return -1;
    auto it = prev(upper_bound(all(arr), x));
    if (*it != x) return -1;
    return it - arr.begin();
}

templT int ultimo_menor (vector<T>& arr, T x) {
    if (arr.begin() == arr.end()) return -1;
    auto it = prev(lower_bound(all(arr), x));
    if (*it >=) return -1;
    return it - arr.begin();
}

templT int primer_mayor (vector<T>& arr, T x) {
    auto it = upper_bound(all(arr), x);
    if (it == arr.end()) return -1;
    return it - arr.begin();
}
```

Operaciones de conjuntos con vectores ordenados (lineal)

```
// Siempre hacer resize al final asi:

vector<int> A = { 5, 10, 15, 20, 25};
vector<int> B = {10, 20, 30, 40, 50};

vector<int> U(A.size() + B.size());

auto it = set_union(all(A), all(B), begin(U));

U.resize(it - U.begin());
```

Función	Descripción
set_union	Unión
set_intersection	Intersección
set_difference	Elementos que están en el primero y no en el segundo
set_symmetric_difference	Elementos que están en uno pero no los dos (como el xor)

2.2. Set

Indexed set y multiset

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

templT struct IndexedSet {
    tree<
        T, null_type, less<T>,
        rb_tree_tag, tree_order_statistics_node_update
    > s;
    void add (T x) { ms.insert(x); }
    int idx (T x) { return ms.order_of_key(x); }
    bool has (T x) { return ms.find(x) != ms.end(); }
    T ith (int i) { return *ms.find_by_order(i); }
};

templT struct IndexedMultiset {
    int t = 0; tree<
        pair<T, int>, null_type, less<pair<T, int>>,
        rb_tree_tag, tree_order_statistics_node_update
    > ms;
    void add (T x) { ms.insert(mp(x, t++)); }
    int nle (T x) { return ms.order_of_key(mp(x, -1)); }
    int nleq (T x) { return ms.order_of_key(mp(x, INT_MAX)); }
    int cnt (T x) { return nleq(x) - nle(x); }
    T ith (int i) { return (*ms.find_by_order(i)).fst; }
};
```

Intervalos consecutivos

```
struct IntervalosConsecutivos {
    set<int> I;
    map<int, int> L;
    IntervalosConsecutivos (int i, int j) {
        I.insert(i);
        I.insert(j);
        L[j - i]++;
    }
    void cortar (int k) {
        int i = *prev(I.lower_bound(k));
        int j = *(I.lower_bound(k));
        L[j - i]--;
```

```

        if (L[j - i] == 0) L.erase(j - i);
        L[k - i]++;
        L[j - k]++;
        L.insert(k);
    }
    int max_intervalo () {
        return (*L.rbegin()).fst;
    }
};

```

3. Range queries

Prefix/dff arrays

```

template<T> vector<T> diff_array (vector<T>& A) {
    vector<T> D(A.size());
    D[0] = A[0];
    forn(i, D.size() - 1) D[i+1] = A[i+1] - A[i];
    return D;
}

// Aplica +x en A[i] ... A[j]
template<T> void update_diff_array (vector<T>& D, int i, unsigned j, T x)
    ↪ {
    D[i] += x;
    if (j + 1 < D.size()) D[j+1] -= x;
}

template<T> vector<T> prefix_array (vector<T>& A) {
    vector<T> P(A.size());
    P[0] = A[0];
    forn(i, P.size() - 1) P[i+1] = P[i] + A[i+1];
    return P;
}

// Retorna A[i] + ... + A[j]
template<T> T query_prefix_array (vector<T>& P, int i, int j) {
    T res = P[j];
    if (i > 0) res -= P[i-1];
    return res;
}

```

Segment tree range query point set

```

template<T> struct SegmentTree {
    vector<T>& arr; int N;
    // Elegir operacion y neutro
    T id;
    T op (T a, T b) { return 0; }
    vector<T> t;

```

```

    void make () {
        t.resize(N << 1); forn(i,N) t[i+N] = arr[i];
        for (int i = N - 1; i; i--) t[i] = op(t[i<<1], t[i<<1|1]);
    }
    void set (int i, T v) {
        for(t[i += N] = v; i > 1; i >= 1) t[i>>1] = op(t[i], t[i^1])
            ↪ ;
    }
    T query (int l, int r) {
        T res = id;
        for (l += N, r += N; l < r; l >= 1, r >= 1) {
            if (l&1) res = op(res, t[l++]);
            if (r&1) res = op(res, t[--r]);
        } return res;
    }
};

// Usar asi:
vector<int> A = {...};

SegmentTree<int> segment_tree = {A, A.size(), 0};
segment_tree.make();

Sparse table

// Operacion asociativa IDEMPOTENTE

#define log2fl(x) (x ? 63 - __builtin_clzll(x) : -1)

template<T> struct SparseTable {
    vector<T>& arr; int N;
    vector<vector<T>>> st;
    // Modificar operacion
    T op (T a, T b) { return min(a,b); }
    void make () {
        st.resize(20, vector<T>(N));
        st[0] = arr; forn(w,19) forn(i,N - (1 << (w+1)) - 1)
            st[w+1][i] = op(st[w][i], st[w][i + (1 << w)]);
    }
    T query (int i, int j) {
        int w = log2fl(j - i + 1);
        return op(st[w][i], st[w][j - (1 << w) + 1]);
    }
};

// Usar asi:
vector<int> A = {...};

SparseTable<int> sparse_table = {A, A.size()};
sparse_table.make();

```

4. Grafos

Toposort

```
using AdjList = vector<vector<int>>;

vector<int> kahn_toposort (AdjList& G) {
    int N = G.size();
    vector<int> indegree(N), visit(N), res;
    forn(u, N) for (int v : G[u]) indegree[v]++;
    // Elegir criterio de priorizacion cambiando el orden en el que se
    ↪ sacan
    // (por defecto el menor)
    using Bag = priority_queue<int, vector<int>, greater<int>>;
    Bag bag;
    forn(u, N) if(indegree[u] == 0) bag.push(u);
    while (bag.size()) {
        int u = bag.top();
        bag.pop();
        res.push_back(u);
        for (int v : G[u]) {
            indegree[v]--;
            if (indegree[v] == 0) bag.push(v);
        }
    }
    return res;
}
```

Bipartite check

```
using AdjList = vector<vector<int>>;

bool es_bipartito (AdjList& G) {
    vector<int> color(G.size(), -1);
    color[0] = 0;
    queue<int> bag;
    for (bag.push(0); bag.size();) {
        int u = bag.front();
        bag.pop();
        for (int v : G[u]) {
            if (color[u] == color[v]) return false;
            if (color[v] == -1) {
                color[v] = 1 - color[u];
                bag.push(v);
            }
        }
    }
    return true;
}
```

5. Matemática

5.1. Aritmética

Techo de la división

```
#define ceildiv(a,b) ((a + b - 1) / b)
```

Piso de la raiz cuadrada

```
using ll = long long;

ll isqrt (ll x) {
    ll s = 0;
    for (ll k = 1 << 30; k; k >>= 1)
        if ((s+k) * (s+k) <= x) s += k;
    return s;
}
```

Piso del log2

```
#define log2fl(x) (x ? 63 - __builtin_clzll(x) : -1)
```

Aritmética en \mathbb{Z}_p

```
using ll = long long;

const ll mod = 1e9 + 7;

ll resta_mod (ll a, ll b) { return (a - b + mod) % mod; }

ll pow_mod (ll x, ll n) {
    ll res = 0;
    while (n) {
        if (n % 2) res = res * x % mod;
        n /= 2;
        x = x * x % mod;
    } return res;
}

ll div_mod (ll a, ll b) { return a * pow_mod(b, mod - 2) % mod; }
```

5.2. Teoria de numeros

Criba

```
struct Criba {
    bool c[1000001]; vector<int> p;
    Criba () {
        p.reserve(1<<16);
```

```

        for (int i = 2; i <= 1000000; i++) if (!c[i]) {
            p.pb(i);
            for (int j = 2; i*j <= 1000000; j++) c[i*j] = 1;
        }
    }
    bool isprime (int x) {
        for (int i = 0, d = p[i]; d*d <= x; d = p[++i])
            if (!(x % d)) return false;
        return x >= 2;
    }
};

```

Phollars Rho

```

#include <iostream>
#include <cstdio>
#include <cstdlib>
#include <cstring>
#include <vector>
#include <cassert>
#include <map>
#include <algorithm>
#include <cmath>
using namespace std;
#define dprint(v) cerr << #v"=" << v << endl //;)
#define forr(i,a,b) for(int i=(a); i<(b); i++)
#define forn(i,n) forr(i,0,n)
typedef long long ll;

ll gcd(ll a, ll b){return a?gcd(b %a, a):b;}

ll mulmod (ll a, ll b, ll c) { //returns (a*b)%c, and minimize
    ↪ overflow
    ll x = 0, y = a%c;
    while (b > 0){
        if (b % 2 == 1) x = (x+y) % c;
        y = (y*2) % c;
        b /= 2;
    }
    return x % c;
}

ll expmod (ll b, ll e, ll m){//0(log b)
    if(!e) return 1;
    ll q= expmod(b,e/2,m); q=mulmod(q,q,m);
    return e%2? mulmod(b,q,m) : q;
}

bool es_primo_prob (ll n, int a)
{
    if (n == a) return true;

```

```

    ll s = 0, d = n-1;
    while (d % 2 == 0) s++,d/=2;

    ll x = expmod(a,d,n);
    if ((x == 1) || (x+1 == n)) return true;

    forn (i, s-1){
        x = mulmod(x, x, n);
        if (x == 1) return false;
        if (x+1 == n) return true;
    }
    return false;
}

bool rabin (ll n){ //devuelve true si n es primo
    if (n == 1) return false;
    const int ar[] = {2,3,5,7,11,13,17,19,23};
    forn (j,9)
        if (!es_primo_prob(n,ar[j]))
            return false;

    return true;
}

ll rho(ll n){
    if( (n & 1) == 0 ) return 2;
    ll x = 2 , y = 2 , d = 1;
    ll c = rand() % n + 1;
    while( d == 1 ){
        x = (mulmod( x , x , n ) + c)%n;
        y = (mulmod( y , y , n ) + c)%n;
        y = (mulmod( y , y , n ) + c)%n;
        if( x - y >= 0 ) d = gcd( x - y , n );
        else d = gcd( y - x , n );
    }
    return d==n? rho(n):d;
}

map<ll,ll> prim;
void factRho (ll n){ //0 (lg n)^3. un solo numero
    if (n == 1) return;
    if (rabin(n)){
        prim[n]++;
        return;
    }
    ll factor = rho(n);
    factRho(factor);
    factRho(n/factor);
}

```

```
int main(){
    ll n;
    while(1){
        n=abs((ll)rand()*rand()*rand()%(ll(1e18))+50);
        dprint(n);
        if(!rabin(n)){
            //~ dprint(n);
            ll ans = rho (n);
            assert(!(n%ans) && ans>0);
        }
    }
    n = 2063512844981574047LL;
    while(cin >> n){
        dprint(n);
        if(rabin (n))
            cout << n << " es primo.\n";
        else{
            ll ans = rho (n);
            if (ans > n / ans) ans = n / ans;
            printf ("%lld = %lld * %lld\n", n, ans, n /
                ↪ ans);
        }
    }
    return 0;
}
```

5.3. Geometria

Template geometria

```
using flt = long double;
const flt EPS = 1e-9;
bool flt_leq (flt a, flt b) { return a < b + EPS; }
bool flt_eq (flt a, flt b) { return -EPS <= a - b && a - b <= EPS;
    ↪ }

using Sca = long long;
struct Vec { Sca x, y; };
Vec operator + (Vec a, Vec b) { return { a.x + b.x, a.y + b.y }; }
Vec operator - (Vec a, Vec b) { return { a.x - b.x, a.y - b.y }; }
Sca operator * (Vec a, Vec b) { return a.x * b.x + a.y * b.y; }
Sca operator ^ (Vec a, Vec b) { return a.x * b.y + a.y * b.x; }
bool operator < (Vec a, Vec b) { return (a.x != b.x) ? (a.x < b.x) :
    ↪ (a.y < b.y); }
ostream& operator << (ostream &o, Vec& p) { auto x = mp(p.x, p.y);
    ↪ return o << x; }

Sca norma2 (Vec p) { return p.x * p.x + p.y * p.y; }
```

6. Estructuras locas

6.1. Disjoint set union

```
struct DSU {
    vector<int> p, w; int nc;
    DSU (int n) {
        nc = n, p.resize(n), w.resize(n);
        forn(i,n) p[i] = i, w[i] = 1;
    }
    int get (int x) { return p[x] == x ? x : p[x] = get(p[x]); }
    void join (int x, int y) {
        x = get(x), y = get(y);
        if (x == y) return;
        if (w[x] > w[y]) swap(x,y);
        p[x] = y, w[y] += w[x];
    }
    bool existe_camino (int x, int y) { return get(x) == get(y); }
};
```

6.2. Binary trie

```
struct BinaryTrieVertex { vector<int> next = {-1, -1}; };

using BinaryTrie = vector<BinaryTrieVertex>;

void binary_trie_add (BinaryTrie& trie, int x) {
    int v = 0;
    for (int i = 31; i >= 0; i--) {
        bool b = (x & (1 << i)) > 0;
        if (trie[v].next[b] == -1) {
            trie[v].next[b] = trie.size();
            trie.emplace_back();
        }
        v = trie[v].next[b];
    }
}

int binary_trie_max_xor (BinaryTrie& trie, int x) {
    int v = 0, res = 0;
    for (int i = 31; i >= 0; i--) {
        bool b = (x & (1 << i)) > 0;
        if (trie[v].next[!b] != -1) {
            v = trie[v].next[!b];
            if (!b) res |= (1 << i);
        }
        else {
            v = trie[v].next[ b];
            if ( b) res |= (1 << i);
        }
    }
}
```

```
    } return res;
}
```

```
// Inicializar asi:
BinaryTrie trie(1);
```

7. Sin categorizar

Búsqueda binaria sobre un predicado

```
using ll = long long;
```

```
// Si existe, el primer i donde pred(i) == true
// Si es todo false, devuelve d
```

```
ll bsearch (ll i, ll j, bool (*pred)(ll), ll d) {
    while (!(i + 1 == j)) {
        ll m = i + ((j - i) >> 1);
        pred(m) ? j = m : i = m;
    }
    if (pred(i)) return i;
    if (pred(j)) return j;
    return d;
}
```

Enumerar subconjuntos de un conjunto con bitmask

```
// Imprimir representaciones en binario de todos los numeros "[0,
    ↪ ..., 2^N-1]"
for(mask, (1 << N)) {
    for(i, N) cout << "01"[(mask & (1 << i)) > 0] << "\0\n"[i == N
    ↪ -1];
}
```

```
// Iterar por los bits de cada subconjunto
for(mask, (1 << N)) {
    for(i, N) {
        bool on = (mask & (1 << i)) > 0;
        if (on) { ... }
        else { ... }
    }
}
```

Hashing Rabin Karp

```
using ll = long long;
```

```
const ll primo = 27, MAX_PRIME_POW = 1e6;
```

```
ll prime_pow[MAX_PRIME_POW];
void get_prime_pow () {
    prime_pow[0] = 1;
```

```
    forn(i, MAX_PRIME_POW) prime_pow[i+1] = prime_pow[i] * primo %
    ↪ mod;
}

vector<ll> get_rolling_hash (string& s) {
    vector<ll> rh(s.size() + 1);
    rh[0] = 0;
    // Ojo: es 'A' o 'a' ???
    forn(i, s.size()) rh[i+1] = (rh[i] * primo % mod + s[i] - 'A') %
    ↪ mod;
    return rh;
}
```

```
ll hash_range_query (vector<ll>& rh, int i, int j) {
    j++;
    return (rh[j] - (rh[i] * prime_pow[j - i] % mod) + mod) % mod;
}
```

8. Brainstorming

- Graficar como puntos/grafos
- Pensarlo al revez
- ¿Que propiedades debe cumplir una solución?
- Si existe una solución, ¿existe otra más simple?
- ¿Hay elecciones independientes?
- ¿El proceso es parecido a un algoritmo conocido?
- Si es busca calcular $f(x)$ para todo x , calcular cuánto contribuye x a $f(y)$ para los otros y
- Definiciones e identidades: ¿que significa que (por ejemplo) un array sea palindromo?