Analysis of the Simulation Data from Saddle-Node Bifurcation

```
clearvars;
addpath(genpath('../'))
```

Generating simulation data

```
rng(1)
%{
lambda_0 = -3;
x_0 = 2.1038;
par.T_max = 6;
par.dt = 1e-5;
sigma = 40;
[x_out, t_out, lambda_out] = generating
figure;
plot(t_out,x_out)


sample_id = abs(t_out-3.257)<0.02;
x_out_sample = x_out(sample_id);
t_out_sample = t_out(sample_id);
```

```
figure;
plot(t_out_sample,x_out_sample)

N_cell = length(x_out_sample);
sub_sample_id = 1:2:N_cell;
N_cell = length(sub_sample_id);
data = x_out_sample(sub_sample_id )';
true_labs = t_out_sample (sub_sample_ic



%save

save('../Data/saddlenode_new.mat','x_ou
%}
```

Load the Data and Estimate Number of Clusters by EPI

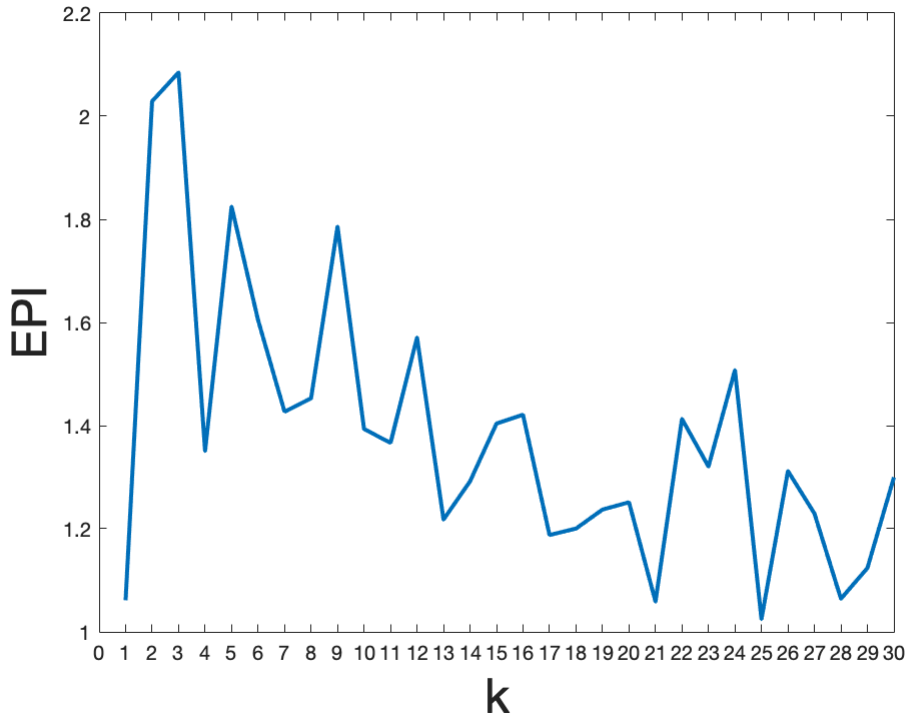
From EPI plot, we intend to seek for the index where peak occurs, which serves

as the candidate for the choice of cluster numbers

```
load saddlenode.mat
out = EstClusterNum(data,[]);
```

```
Computed P-values 500 of 2000 datapoint
Computed P-values 1000 of 2000 datapoir
Computed P-values 1500 of 2000 datapoir
Computed P-values 2000 of 2000 datapoir
Mean value of sigma: 0.62417
Minimum value of sigma: 0.54639
Maximum value of sigma: 0.75299
```

```
plot(out.ratio(1:30), 'linewidth',2.0)
xlabel('k', 'FontSize', 24);
ylabel('EPI', 'FontSize', 24);
xticks(0:30);
```



Dynamical Analysis and Output

```
tic;
par.choice_distance = 'euclid';
par.K_cluster = 2; %selected based on E
par.trials = 20; % number of random tra
% the main function of MuTrans
Output = DynamicalAnalysis (data, par);
```

```
Computed P-values 500 of 2000 datapoint
Computed P-values 1000 of 2000 datapoir
Computed P-values 1500 of 2000 datapoir
Computed P-values 2000 of 2000 datapoir
```

```
Mean value of sigma: 0.62417
Minimum value of sigma: 0.54639
Maximum value of sigma: 0.75299
J_new = 2.0186
J_new = 1.9458
J_new = 1.7978
J_new = 1.2289
J_new = 1.2248
J_new = 1.2228
J_new = 1.2227
J_new = 1.2227
J_new = 1.2726
J_new = 1.2064
J_new = 1.2029
J_new = 1.2022
J_new = 1.2014
J_new = 1.2012
J_new = 1.2012
J_new = 1.2039
J_new = 1.2013
J_new = 1.2012
```

```
J_new = 1.2012
J_new = 1.9314
J_new = 1.3082
J_new = 1.2736
J_new = 1.2414
J_new = 1.2271
J_new = 1.2238
J_new = 1.2228
J_new = 1.2227
J_new = 1.2227
J_new = 1.7034
J_new = 1.5456
J_new = 1.3847
J_new = 1.2039
J_new = 1.2013
J_new = 1.2012
J_new = 1.2012
J_new = 2.0398
J_new = 1.9975
J_new = 1.9374
J_new = 1.8711
```

```
J_new = 1.8338
J_new = 1.7806
J_new = 1.7073
J_new = 1.6795
J_new = 1.6351
J_new = 1.5776
J_new = 1.4679
J_new = 1.2018
J_new = 1.2013
J_new = 1.2012
J_new = 1.2012
J_new = 1.7439
J_new = 1.2251
J_new = 1.2064
J_new = 1.2029
J_new = 1.2022
J_new = 1.2014
J_new = 1.2012
J_new = 1.2012
J_new = 1.4074
J_new = 1.2198
```

```
J_new = 1.2118
J_new = 1.2047
J_new = 1.2022
J_new = 1.2012
J_new = 1.2012
J_new = 1.2566
J_new = 1.2138
J_new = 1.2031
J_new = 1.2022
J_new = 1.2014
J_new = 1.2012
J_new = 1.2012
J_new = 1.9787
J_new = 1.9039
J_new = 1.8986
J_new = 1.8913
J_new = 1.8793
J_new = 1.8363
J_new = 1.6907
J_new = 1.6654
J_new = 1.6276
```

```
J_new = 1.5695
J_new = 1.4416
J_new = 1.2014
J_new = 1.2012
J_new = 1.2012
J_new = 1.4496
J_new = 1.2050
J_new = 1.2024
J_new = 1.2016
J_new = 1.2012
J_new = 1.2012
J_new = 1.9824
J_new = 1.9375
J_new = 1.9336
J_new = 1.9305
J_new = 1.9276
J_new = 1.9196
J_new = 1.9075
J_new = 1.8997
J_new = 1.8934
J_new = 1.8835
```

```
J_new = 1.8584
J_new = 1.6937
J_new = 1.6728
J_new = 1.6341
J_new = 1.5790
J_new = 1.4702
J_new = 1.2014
J_new = 1.2012
J_new = 1.2012
J_new = 1.6831
J_new = 1.2031
J_new = 1.2022
J_new = 1.2014
J_new = 1.2012
J_new = 1.2012
J_new = 1.2270
J_new = 1.2235
J_new = 1.2227
J_new = 1.2227
J_new = 1.7183
J_new = 1.2072
```

```
J_new = 1.2031
J_new = 1.2022
J_new = 1.2014
J_new = 1.2012
J_new = 1.2012
J_new = 1.2220
J_new = 1.2057
J_new = 1.2024
J_new = 1.2016
J_new = 1.2012
J_new = 1.2012
J_new = 1.9753
J_new = 1.9521
J_new = 1.9035
J_new = 1.8986
J_new = 1.8833
J_new = 1.8606
J_new = 1.8512
J_new = 1.8470
J_new = 1.8458
J_new = 1.8448
```

```
J_new = 1.8441
J_new = 1.8414
J_new = 1.8297
J_new = 1.7931
J_new = 1.6386
J_new = 1.2334
J_new = 1.2262
J_new = 1.2235
J_new = 1.2227
J_new = 1.2227
J_new = 1.7555
J_new = 1.2307
J_new = 1.2275
J_new = 1.2241
J_new = 1.2234
J_new = 1.2209
J_new = 1.2157
J_new = 1.2083
J_new = 1.2039
J_new = 1.2013
J_new = 1.2012
```

```
J_new = 1.2012
J_new = 1.2070
J_new = 1.2026
J_new = 1.2021
J_new = 1.2014
J_new = 1.2012
J_new = 1.2012
J_new = 1.2866
J_new = 1.2247
J_new = 1.2232
J_new = 1.2227
J_new = 1.2227
J_new = 1.2227
E_best = 0.0375

 Iteration  Func-count       f(x)
     0           1          1.20181
     1          11          1.17728
     2          13          1.17708
     3          14          1.17143
     4          15          1.16935
```

```
     5              16              1.16811
     6              17              1.16574
     7              18              1.16484
     8              19              1.16393
     9              20              1.16347
    10              21              1.16293
    11              22              1.16238
    12              23              1.16147
    13              24              1.16084
    14              25              1.16049
    15              26              1.16029
    16              27              1.15977
    17              28              1.15947
    18              29               1.1593
    19              31              1.15915

Iteration  Func-count       f(x)
    20              32              1.15903
    21              33              1.15891
    22              34              1.15881
    23              35              1.15873
```

24	36	1.15864
25	38	1.15857
26	39	1.15848
27	41	1.15832
28	42	1.15818
29	43	1.15804
30	44	1.15792
31	45	1.15782
32	46	1.15773
33	47	1.15767
34	48	1.15757
35	49	1.15751
36	50	1.15743
37	51	1.15735
38	52	1.15729
39	53	1.15722

Iteration	Func-count	f(x)
40	54	1.15718
41	55	1.1571
42	56	1.15704

```
    43              57              1.15698
    44              58              1.15688
    45              59               1.1568
    46              60              1.15674
    47              61              1.15666
    48              62               1.1566
    49              63              1.15657
    50              64              1.15655
    51              65              1.15654
    52              66              1.15652
    53              67              1.15652
    54              68              1.15651
    55              69              1.15651
    56              70               1.1565
    57              71               1.1565
    58              72               1.1565
    59              73               1.1565

 Iteration  Func-count       f(x)
    60              74              1.15649
    61              75              1.15649
```

62	76	1.15649
63	77	1.15649
64	78	1.15649
65	79	1.15648
66	80	1.15648
67	81	1.15648
68	82	1.15648
69	83	1.15648
70	84	1.15648
71	85	1.15648
72	86	1.15647
73	87	1.15647
74	88	1.15647
75	89	1.15647
76	90	1.15647
77	91	1.15647
78	92	1.15647
79	93	1.15647

Iteration	Func-count	f(x)
80	94	1.15647

81	95	1.15647
82	96	1.15647
83	97	1.15647
84	98	1.15647
85	99	1.15647
86	100	1.15647
87	101	1.15646
88	102	1.15646
89	103	1.15646
90	104	1.15646
91	105	1.15646
92	106	1.15646
93	107	1.15646
94	108	1.15646
95	109	1.15646
96	110	1.15646
97	111	1.15646
98	112	1.15646
99	113	1.15646

Iteration	Func-count	f(x)

```
   100          114          1.15646
   101          115          1.15646
   102          116          1.15646
   103          117          1.15646
   104          118          1.15646
   105          119          1.15646
   106          120          1.15646
   107          121          1.15646
   108          122          1.15646
   109          123          1.15646
   110          124          1.15645
   111          125          1.15645
   112          126          1.15645
   113          127          1.15645
   114          128          1.15645
   115          129          1.15645

Optimization completed: The first-order
than options.OptimalityTolerance = 1.00

Elapsed time is 77.902009 seconds.
```

```
toc;
```

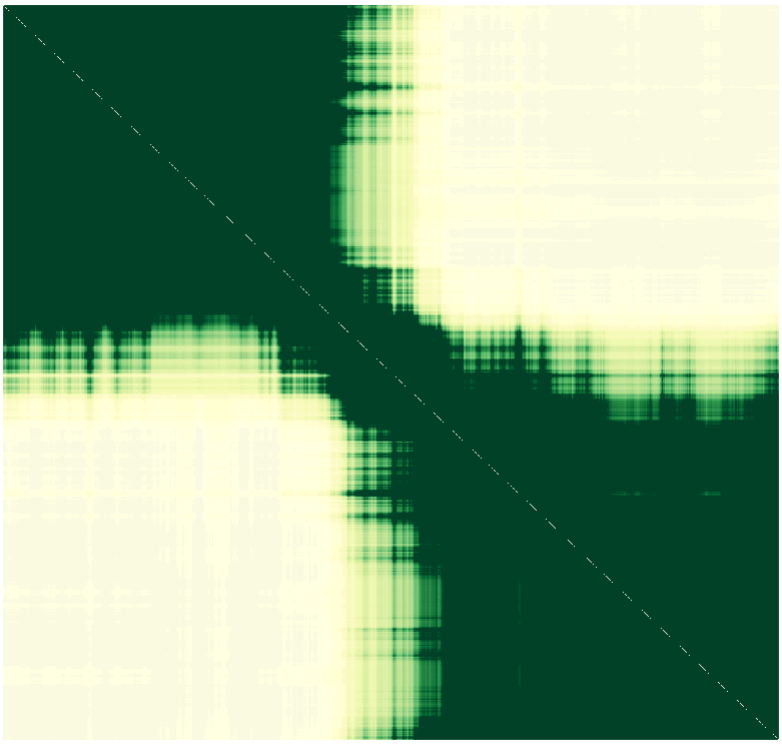
```
Elapsed time is 77.917476 seconds.
```

```
class_order = Output.class_order;
rho_class = Output.rho_class;
perm_class = Output.perm_class;
P_perm = Output.P_perm;
P_hat = Output.P_hat;
P_appr_perm = Output.P_appr_perm;
P_rho = Output.P_rho;
labs_perm = Output.labs_perm;
data_perm = Output.data_perm;
mu_hat = Output.mu_hat;
k = Output.k;
H = Output.H;
```

Plot the Cell-Cell Scale rwTPM

```
max_P = 0.2* max(max(P_rho));
c_lim = [0 max_P];
cmp = 'ylgn';
```

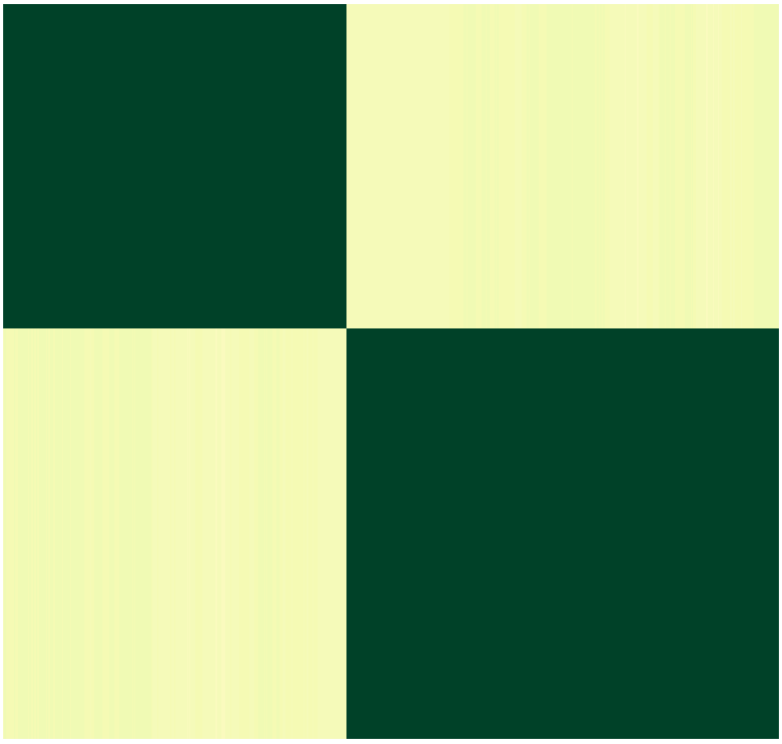
```
figure('rend','painters','pos',[10 10 5
colormap(brewermap([],cmp))
imagesc(P_perm);
axis off
set(gca,'xtick',[],'ytick',[]);
caxis(c_lim)
```



```
%colorbar;
```

Plot the Cluster-Cluster Scale rwTPM

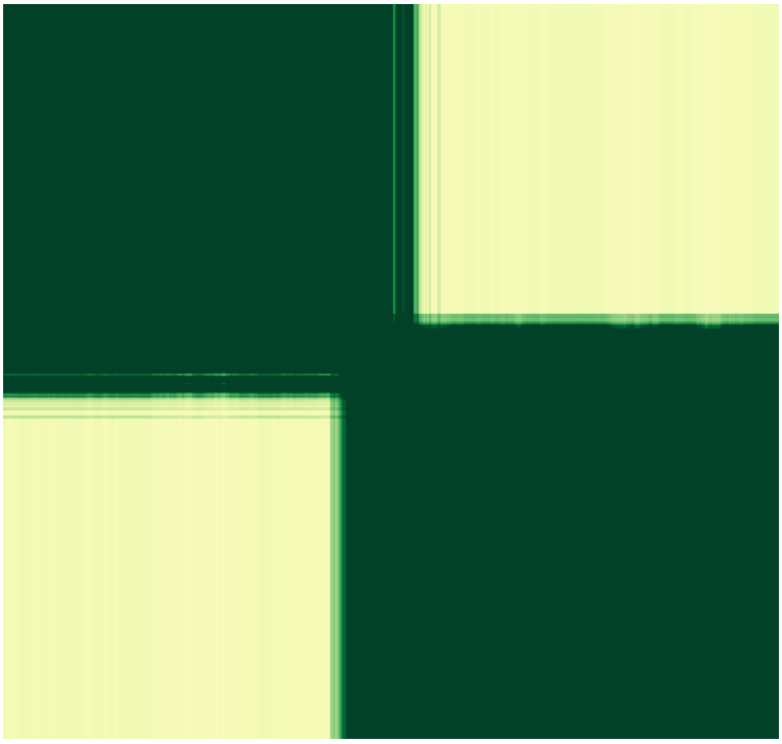
```
figure('rend','painters','pos',[10 10 5
colormap(brewermap([],cmp))
imagesc(P_appr_perm);
axis off
set(gca,'xtick',[],'ytick',[]);
caxis(c_lim)
```



```
%colorbar;
```

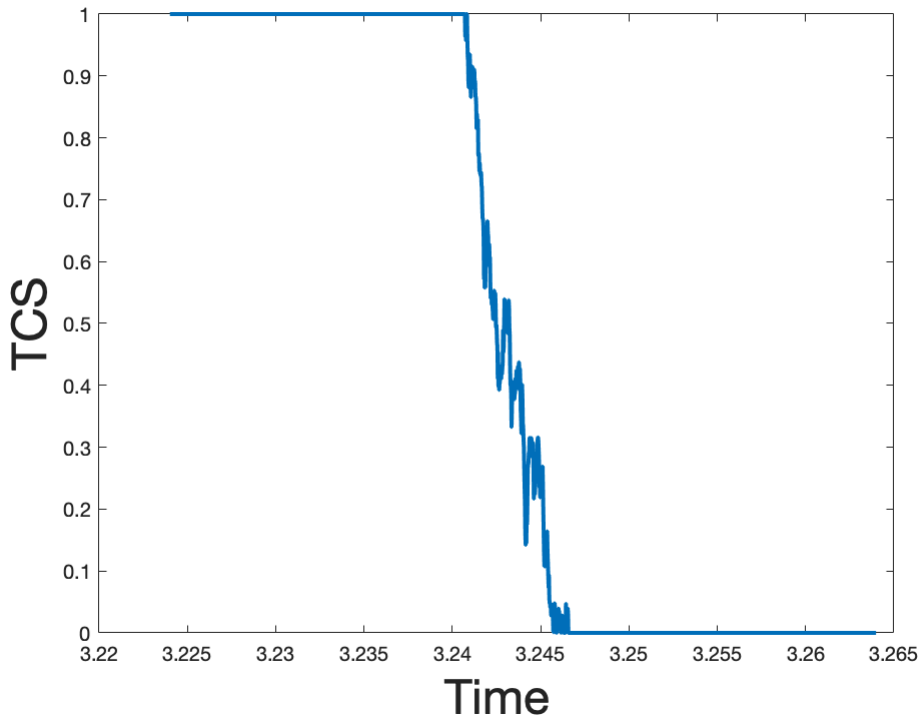
Plot the Cell-Cluster Scale rwTPM

```
figure('rend','painters','pos',[10 10 5
colormap(brewermap([],cmp))
imagesc(P_rho);
axis off
set(gca,'xtick',[],'ytick',[]);
caxis(c_lim)
%colorbar;
box off
```



Plot Transition Cell Score (TCS)

```
figure;
plot(labs_perm, rho_class(:,1),'LineWid
xlabel('Time', 'FontSize', 24);
ylabel('TCS', 'FontSize', 24);
```



Plot Actual Simulation Trajectory (Gene Expression)

```
figure;
plot(labs_perm, data_perm,'LineWidth',2
```

```
xlabel('Time', 'FontSize', 24);
ylabel('Gene Expression', 'FontSize', 2
```