

(52)

Then  $\{a_n\}$  be monot  
in  $\mathbb{R}$ . Then

$$\lim_{n \rightarrow \infty} a_n = \begin{cases} \sup\{a_k\} & \text{if } a_1 \leq a_2 \leq a_3 \leq \dots \\ \inf\{a_k\} & \text{if } a_1 \geq a_2 \geq \dots \end{cases}$$

Pf. W.l.o.g. assume  $a_1 \leq a_2 \leq \dots$

Take any  $b < \sup\{a_k\}$

Then  $\exists k_* < \infty$  s.t.

$b \leq a_{k_*}$ . For all

$n \geq k_*$ ,  $a_n \geq a_{k_*}$  so

$a_n > b$ . But also  $a_n \leq \sup\{a_k\}$