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<u>Thm</u> $\{a_n\}$ conv in $\mathbb{R}$ iff $\{a_n\}$ is Cauchy.

<u>Pf</u>: ($\Rightarrow$) ~~Spse~~ $a_n \to a$ in $\mathbb{R}$.
Take any $\varepsilon > 0$. $\exists N_\varepsilon$ s.t. for $n \geq N_\varepsilon$, $|a_n - a| < \frac{\varepsilon}{2}$

For $j, k \geq N_\varepsilon$

$$\begin{aligned} |a_j - a_k| &= |a_j - a + a - a_k| \\ &\leq |a_j - a| + |a - a_k| \\ &< \varepsilon/2 + \varepsilon/2 \\ &= \varepsilon \end{aligned}$$