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P1: Suppose  $r^* < 1$

Take any  $r^* < \lambda < 1$

$\exists N$ : for  $n \geq N$ ,  $\left| \frac{a_{n+1}}{a_n} \right| \leq \lambda$

$$\therefore \sum_{n=N+1}^{\infty} |a_n|$$

$$= \sum_{k=1}^{\infty} |a_{N+k}|$$

$$= \sum_{k=1}^{\infty} |a_N| \prod_{j=1}^k \left| \frac{a_{N+j}}{a_{N+j-1}} \right|$$

$$\leq |a_N| \sum_{k=1}^{\infty} \lambda^k < \infty$$

a series  
converges