1 Set Theory

To demystify mathematics consider

(i) What is a theorem?

(ii) What is a proof?

What if we don't know the answer?
To begin we need

(a) an example(s)

(b) a nearly related concept

3

To demystify mathematics, consider

(i) What is a Theorem?

(ii) What is a Proof?

What if we don't know the answer?

To begin we need

(a) an example(s)

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Related Concept: Greek Syllogism
<u>example:</u>

1. All men are mortal.
2. Socrates is a man.
3. Therefore, Socrates must die.

To analyze, recast in set theoretic terms via Venn Diagram.

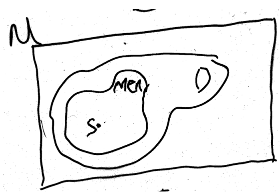
4

Related Concept:
Greek Syllogism

<u>Example:</u>

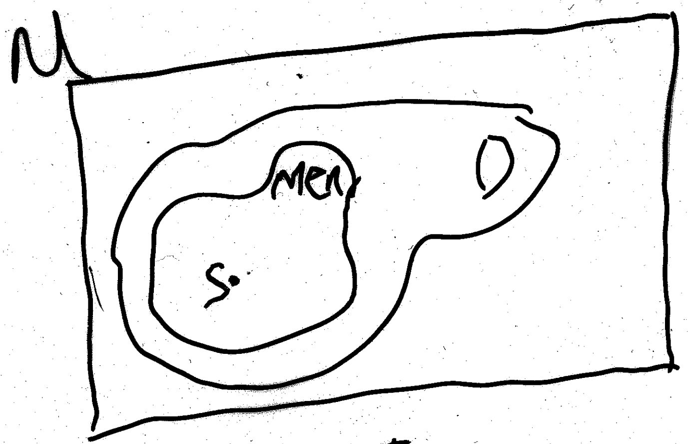
(1) All men are mortal
(2) Socrates is a man
$\therefore$ (3) Socrates must die

To analyze, recast
in set theoretic terms
via Venn diagram



S: Socrates
M: Set of Men
D: Things that will die
$\mathcal{U}$: Things on Earth

5



S : socrates
M : set of men
D : things that die
U : things on earth

2 Generate $\mathbb{N}$

3 From $\mathbb{Z}$ to $\mathbb{R}$ via ordering

4 Sequence and Limits

Theorem Suppose $a_n \to a$ and $b_n \to b$. Then $a_n + b_n \to a + b$ (and $\lambda a_n \to \lambda a$)and $a_n b_n \to a * b$

Theorem if $a_n \to a \neq 0$, then $\frac{1}{a_n} \to \frac{1}{a}$

corollary if $a_n \to a$ and $b_n \to b \neq 0$ then $\frac{a_n}{b_n} \to \frac{a}{b}$

(57)

Thm Spse $a_n \to a$ and $b_n \to b$. Then $a_n + b_n \to a + b$ (and $\lambda a_n \to \lambda a$) and $a_n b_n \to a \cdot b$

Thm If $a_n \to a \neq 0$, then $\frac{1}{a_n} \to \frac{1}{a}$

Cor ~~then~~ If $a_n \to a$ and $b_n \to b \neq 0$ then $\frac{a_n}{b_n} \to \frac{a}{b}$

A method of proving corner:

monotone sequences

Lat $\{a_n\}$ be a sequence of reals. We say $\{a_n\}$ is monotonic if either $a_1 \leq a_2 ... \leq a_a \leq ...$ or else $a_1 \geq a_2 ... \geq a_a \geq ...$ In the first case $\{a_n\}$ is said to be (monotone) non-decreasing and in the second case it is (monotone) non-increasing. Theorem suppose a_n is monotonic and bowled. Then it conveys in R.

A Method (S3) of Proving Conv:

Monotone Sequences

Let $\{a_n\}$ be a seq of reals. We say $\{a_n\}$ is monotonic iff either

$$a_1 \leq a_2 \leq \cdots \leq a_n \leq \cdots$$

or else

$$a_1 \geq a_2 \geq \cdots \geq a_n \geq \cdots$$

In the first case $\{a_n\}$ is said to be (monotone) non-decreasing and in the second case it is (monotone) non-increasing.

Thm Spse $\{a_n\}$ is monotonic and bounded. Then it converges in $\mathbb{R}$.

Theorem Let $\{a_n\}$ be monotonic in R. Then in R_∞

$$\lim_{n\to\infty} a_n = \begin{cases} sup\ \{a_k\} & if a_1 \leq a_2 \leq a_3 \leq ... \\ inf\ \{a_k\} & if a_1 \geq a_2 \geq ... \end{cases}$$

Pf: W l.o.g suppose $a_1 \leq a_2 \leq$ take away $b < suppose a_k$ Then $\exists k < \infty s.t. b <$ $\{a_k\}$. For all $n \geq k, a_n \geq a_n so\ a_n > b$. But also $a_n/leqsuppose\ \{a_k\}$

(52)

Thm Let $\{a_n\}$ be monot in R. Then in R_∞

$$\lim_{n\to\infty} a_n = \begin{cases} \sup\{a_k\} \text{ if } a_1 \leq a_2 \leq a_3 \leq \cdots \\ \inf\{a_k\} \text{ if } a_1 \geq a_2 \geq \cdots \end{cases}$$

Pf. W.l.o.g supse $a_1 \leq a_2 \leq \cdots$ Take any $b < \sup\{a_k\}$ Then $\exists\ k_* < \infty$ s.t. $b < a_{k_*}$. For all $n \geq k_*$, $a_n \geq a_{k_*}$ so $a_n > b$. But also $a_n \leq \sup\{a_k\}$

Theorem Let $\{a_n\}$ be any sequence in $\mathbb{R}$ then $\exists 1 \leq n_1 < n_2 < ...s.t.a_{n_1}, a_{n_2}, a_{n_3}, ...$ is monotonic (i.e. a_n has a monotonic sub sequence)
Pf: Either $a_1 \leq a_n$ for infinity many $n > 1$. Applying to each aj Let J $= \{j \geq 1 : a_n \geq a_j$ for infinity many $n > j\}$ Cosel $|J| = \infty$ Let $n_1 =$ smallest $j \in J$ $\exists n_2 > n_1$ with $n_2/inJ, ...$ ete get n_k

(52A)

Thm Let $\{a_n\}$ be any seq in $\mathbb{R}$
Then $\exists\ 1 \leq n_1 < n_2 < \cdots$ s.t.
$a_{n_1}, a_{n_2}, a_{n_3}, \ldots$ is monot.
(i.e. $\{a_n\}$ has a monotonic subsequence)

Pf: Either $a_1 \leq a_n$ for infinitely many $n > 1$ or $a_1 \geq a_n$ for infinitely many $n > 1$.
Applying this question to each a_j:
Let $J = \{ j \geq 1 : a_n \geq a_j$ for infinitely many $n > j \}$

Case1 $|J| = \infty$
Let $n_1 =$ smallest $j \in J$
$\exists\ n_2 > n_1$ with $n_2 \in J, \ldots$ etc get $\{n_k\}$

and $a_{n_1} \leq a_{n_2} \leq ...$

<u>Case 2</u> $|J| < \infty$ take away $n_1 >$ one and max. $j \leftarrow J$

By construction, if n_2 is large enough $n_2 > n_1$ and $a_{n_2} < a_{n_1}$ By induction, having constructed $n_1 < n_2 < ... < n_k$ p.t. $a_{n_1} > a_{n_2} > ... > a_{n_k}$ $\exists n_{k+1} > n_k$ s.t. $a_{n_{k+1}} < a_{n_k}$

and so the theorem holds.

52B

and $a_{n_1} \leq a_{n_2} \leq \dots$

<u>Case 2</u> $|J| < \infty$

Take any $n_1 >$ one and max. $j \in J$

By construction, if n_2 is large enough $n_2 > n_1$ and $a_{n_2} < a_{n_1}$

By induction, having constructed $n_1 < n_2 < \dots < n_k$ p.t. $a_{n_1} > a_{n_2} > \dots > a_{n_k}$

$\exists\ n_{k+1} > n_k$ s.t. $a_{n_{k+1}} < a_{n_k}$.

and so the Thm holds.

5 Limit and Convergence

6 Infinite Series

7 Metric Spaces Part 1

8 Metric Spaces Part 2