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Since $\bigcup_{n=1}^{\infty} F_n = \mathbb{N}$,

$\exists N' \geq N$ s.t.

$$\{1, 2, \dots, N\} \subseteq F_{N'}$$

Hence for all $n \geq N'$

$$\sum_{j \in F_n} a_j \geq \sum_{j=1}^N a_j > \Delta$$

Moreover, since F_n

is finite, $\exists n^* \geq \max\{j \in F_n\}$

Hence $\sum_{j \in F_n} a_j \leq \sum_{j=1}^{n^*} a_j \leq \Delta_{\infty}$