

(52A)

What if $\{a_n\}$ is
not monotonic?

Then Let $\{a_n\}$ be a seq in \mathbb{R} .

Then $\exists 1 \leq n_1 < n_2 < \dots$ s.t.
 a_{n_1}, a_{n_2}, \dots is monotonic

Pf: We need to consider
the tail of a seq. Let

$$J = \{j \geq 1 : a_j > a_{j+k} \text{ for all } k \geq 1\}$$

If J is unbounded
we may write $J = \{n_j : j \geq 1\}$
where $1 \leq n_1 < n_2 < \dots$