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Lower-bounding,

$$\int_1^\infty a(x)\,dx = \sum_{n=2}^{\infty} \int_{n-1}^{n} a(x)\,dx$$

$$\geq \sum_{n=2}^{\infty} \int_{n-1}^{n} a_n\,dx$$

$$= \sum_{n=2}^{\infty} a_n$$

Hence $\sum_{n=1}^{\infty} a_n$ and $\int_1^\infty a(x)\,dx$ converge or diverge together