(5.3c)

($\Leftarrow$) Spse $\{a_n\}$ is Cauchy.
Then $\exists\ B < \infty$ s.t. $|a_n| \le B$
for all n. Moreover
$\exists\ 1 \le n_1 < n_2 < \cdots$ s.t.
$\{a_{n_k} : k \ge 1\}$ is monotonic
w.l.o.g. spse $a_{n_1} \le a_{n_2} \le \cdots$
Let $L = \sup\{a_{n_k} : k \ge 1\}$
Then $\lim_{k\to\infty} a_{n_k} = L \ (\le B)$

Conjecture: $a_j \to L$

Pf: Take any $\varepsilon > 0$. $\exists K_\varepsilon < \infty$
s.t. for $k \ge K_\varepsilon$ $|a_{n_k} - L| < \varepsilon/2$
$\exists N_\varepsilon \ge K_\varepsilon$ s.t. $|a_i - a_j| < \varepsilon/2$
for $i, j \ge N_\varepsilon$. So for $j \ge N_\varepsilon$ and
$k_\varepsilon \ge N_\varepsilon$, $|a_j - L| \le \underbrace{|a_j - a_{n_{k_\varepsilon}}|}_{<\varepsilon} + |a_{n_{k_\varepsilon}} - L|$