

(52A)

Thm Let $\{a_n\}$ be any seq in \mathbb{R}

Then $\exists 1 \leq n_1 < n_2 < \dots$ s.t.

$a_{n_1}, a_{n_2}, a_{n_3}, \dots$ is monot.
(i.e. $\{a_n\}$ has a monotonic subsequence)

Pf: ~~Let~~ Either $a_1 \leq a_n$
for infinitely many
 $n > 1$ or $a_1 \geq a_n$ for
infinitely many $n > 1$.

Applying this question
to each a_j :

Let $J = \{j \geq 1 : a_n \geq a_j \text{ for}$
infinitely many
 $n > j\}$

Case $|J| = \infty$

Let $n_1 = \text{smallest } j \in J$

$\exists n_2' > n_1$ with $n_2 \in J, \dots$ etc
get $\{n_k\}$