(53A)

<u>Prop</u> Let $\{a_n\}$ be a Cauchy seq in $\mathbb{R}$. Then $\{a_n\}$ is <u>bdd</u>.

<u>Pf</u>: $\exists N < \infty$ s.t. for $j, k \geq N$

$|a_j - a_k| < 1$. Let

(*) $B = \max\{|a_1|, |a_2|, \ldots, |a_N|\} + 1$

For $1 \leq j \leq N$, $|a_j| < B$.

For $j > N$

$$|a_j| = |a_j - a_N + a_N|$$
$$\leq |a_N| + |a_j - a_N|$$
$$< |a_N| + 1 \leq B$$

Hence $\{a_j\}$ is <u>bounded</u>.