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Thm 2 (The Integral Test)

Ifsa $a_1 \geq a_2 \geq \cdots \geq 0$

Extend a_n to $a(x)$ where

$a(x)$ is continuous,

$a(n) = a_n$ and $a(x) \searrow$

Then $\sum_n a_n < \infty$ iff $\int_1^\infty a(x)\,dx < \infty$

Pf:

$$\int_1^\infty a(x)\,dx = \sum_{n=1}^\infty \int_n^{n+1} a(x)\,dx$$

$$\leq \sum_{n=1}^\infty \int_n^{n+1} a_n\,dx$$

$$= \sum_{n=1}^\infty a_n$$