<u>Thm</u> Let $\{a_n\}$ be monot in $\mathbb{R}_\infty$ in $\mathbb{R}$. Then

$$\lim_{n\to\infty} a_n = \begin{cases} \sup\{a_k\} & \text{if } a_1 \le a_2 \le a_3 \le \cdots \\ \inf\{a_k\} & \text{if } a_1 \ge a_2 \ge \cdots \end{cases}$$

The limit is in $\mathbb{R}$ iff $\{a_n\}$ is bounded.

<u>Pf.</u> W.l.o.g. spse $a_1 \le a_2 \le \cdots$

Take any $b < \sup\{a_k\}$. Then $\exists\, k_* < \infty$ s.t. $b < a_{k_*}$. For all $n \ge k_*$, $a_n \ge a_{k_*}$ so $a_n > b$. But also $a_n \le \sup\{a_k\}$