Introduction to Digital Circuits

Which Book will be Used?

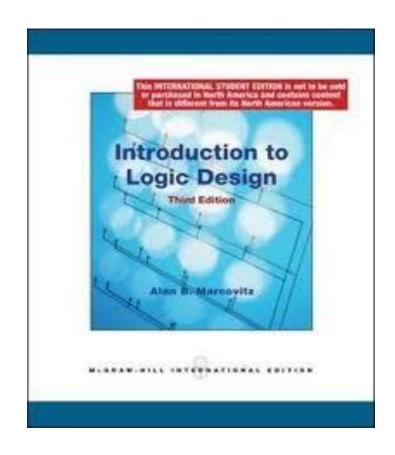
Introduction to Logic Design

❖ Alan B. Marcovitz

♦ Third Edition

♦ McGraw Hill

♦ 2010



What will I Learn in this Course?

- Towards the end of this course, you should be able to:
 - ♦ Represent numbers and perform arithmetic in various number systems.
 - List the basic identities of Boolean algebra and perform algebraic manipulations of Boolean expressions.
 - ♦ Simplify functions using the K-map method.
 - ♦ Design efficient combinational circuits utilizing basic functional blocks such as multiplexors, encoders, decoders, adders, and comparators.
 - Analyze and design efficient Mealy and Moore sequential circuits.
 - Model simple combinational and sequential circuits using Verilog HDL and use tools to simulate and verify correctness of design.
 - Design registers and counters and understand their applications.

Is it Worth the Effort?

- Absolutely!
- Digital circuits are employed in the design of:
 - ♦ Digital computers
 - ♦ Data communication
 - ♦ Digital phones
 - ♦ Digital cameras
 - ♦ Digital TVs, etc.
- This course provides the fundamental concepts and the basic tools for the design of digital circuits and systems

Presentation Outline

- Analog versus Digital Circuits
- Digitization of Analog Signals
- Binary Numbers and Number Systems
- Number System Conversions
- Representing Fractions
- Binary Codes

Analog versus Digital

- Analog means continuous
- Analog parameters have continuous range of values
 - → Example: temperature is an analog parameter
 - → Temperature increases/decreases continuously
 - ♦ Like a continuous mathematical function, No discontinuity points
 - ♦ Other examples?
- Digital means using numerical digits
- Digital parameters have fixed set of discrete values

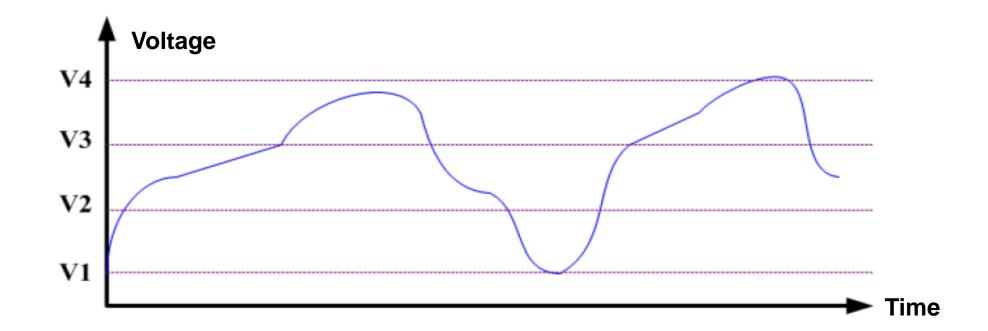
 - → Thus, the month number is a digital parameter (cannot be 1.5!)
 - ♦ Other examples?

Analog versus Digital System

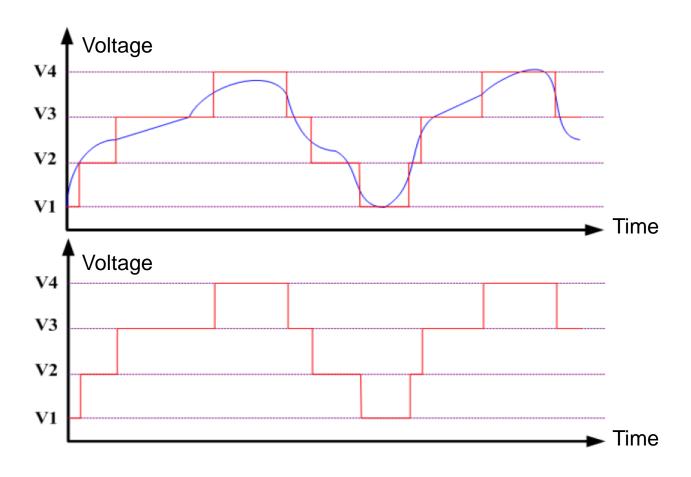
- Are computers analog or digital systems?
 - Computer are digital systems
- Which is easier to design an analog or a digital system?
 - Digital systems are easier to design, because they deal with a limited set of values rather than an infinitely large range of continuous values
- The world around us is analog
- It is common to convert analog parameters into digital form
- This process is called digitization

Digitization of Analog Signals

- Digitization is converting an analog signal into digital form
- Example: consider digitizing an analog voltage signal
- Digitized output is limited to four values = {V1,V2,V3,V4}



Digitization of Analog Signals - cont'd

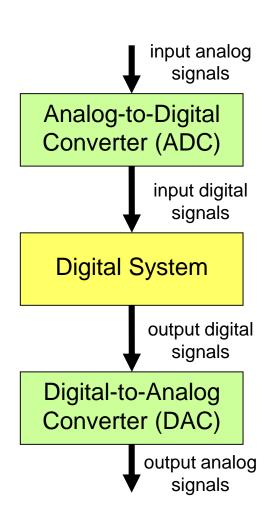


- Some loss of accuracy, why?
- How to improve accuracy?

Add more voltage values

ADC and DAC Converters

- Analog-to-Digital Converter (ADC)
 - ♦ Produces digitized version of analog signals
- Digital-to-Analog Converter (DAC)
 - ♦ Regenerate analog signal from digital form
 - ♦ Digital input => Analog output
- Our focus is on digital systems only
 - ♦ Both input and output to a digital system are digital signals



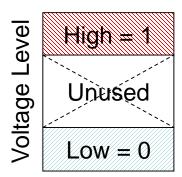
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How do Computers Represent Digits?

- Binary digits (0 and 1) are the simplest to represent
- Using electric voltage
 - ♦ Used in processors and digital circuits
 - → High voltage = 1, Low voltage = 0
- Using electric charge

 - ♦ Charged memory cell = 1, discharged memory cell = 0
- Using magnetic field
 - ♦ Used in magnetic disks, magnetic polarity indicates 1 or 0
- Using light
 - ♦ Used in optical disks, optical lens can sense the light or not

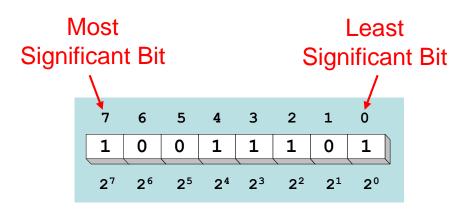


Binary Numbers

- Each binary digit (called a bit) is either 1 or 0
- ❖ Bits have no inherent meaning, they can represent ...
 - ♦ Unsigned and signed integers
 - ♦ Fractions
 - ♦ Characters
 - ♦ Images, sound, etc.

Bit Numbering

- ♦ Most significant bit (MSB) is leftmost (bit 7 in an 8-bit number)



Decimal Value of Binary Numbers

- Each bit represents a power of 2
- Every binary number is a sum of powers of 2
- **!** Decimal Value = $(d_{n-1} \times 2^{n-1}) + ... + (d_1 \times 2^1) + (d_0 \times 2^0)$
- **Solution** Binary $(10011101)_2 = 2^7 + 2^4 + 2^3 + 2^2 + 1 = 157$

7	6	5	4	3	2	1	0
1	0	0	1	1	1	0	1
		2 ⁵	`	`	`		

2ⁿ 2ⁿ **Decimal Value Decimal Value** 2^{8} 2^{0} 256 2^{1} 512 2^{2} 2^{10} 1024 2^3 2^{11} 2048 2^{4} 2^{12} 4096 16 2^{13} 32 8192 26 2^{14} 16384 64 2^{7} 2^{15} 128 32768

Some common powers of 2



Positional Number Systems

Different Representations of Natural Numbers

XXVII Roman numerals (not positional)

27 Radix-10 or decimal number (positional)

11011₂ Radix-2 or binary number (also positional)

Fixed-radix positional representation with *n* digits

Number N in radix
$$r = (d_{n-1}d_{n-2} \dots d_1d_0)_r$$

 N_r Value = $d_{n-1} \times r^{n-1} + d_{n-2} \times r^{n-2} + \dots + d_1 \times r + d_0$
Examples: $(11011)_2 = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2 + 1 = 27$
 $(2107)_8 = 2 \times 8^3 + 1 \times 8^2 + 0 \times 8 + 7 = 1095$

Convert Decimal to Binary

- Repeatedly divide the decimal integer by 2
- Each remainder is a binary digit in the translated value
- ❖ Example: Convert 37₁₀ to Binary

	Remainder	Quotient	Division
least significant bit	1 ←	18	37 / 2
	0	9	18 / 2
$37 = (100101)_2$	1	4	9/2
[0	2	4/2
	0	1	2/2
most significant bit	1 ←	0	1/2
stop when quotient is zero			

Decimal to Binary Conversion

$$N = (d_{n-1} \times 2^{n-1}) + ... + (d_1 \times 2^1) + (d_0 \times 2^0)$$

❖ Dividing *N* by 2 we first obtain

$$\Rightarrow$$
 Quotient₁ = $(d_{n-1} \times 2^{n-2}) + ... + (d_2 \times 2) + d_1$

- \Rightarrow Remainder₁ = d_0
- ♦ Therefore, first remainder is least significant bit of binary number

Dividing first quotient by 2 we first obtain

$$\Rightarrow$$
 Quotient₂ = $(d_{n-1} \times 2^{n-3}) + ... + (d_3 \times 2) + d_2$

- \Rightarrow Remainder₂ = d_1
- Repeat dividing quotient by 2
 - ♦ Stop when new quotient is equal to zero
 - ♦ Remainders are the bits from least to most significant bit

Popular Number Systems

- ❖ Binary Number System: Radix = 2
 - ♦ Only two digit values: 0 and 1
 - ♦ Numbers are represented as 0s and 1s
- ❖ Octal Number System: Radix = 8
- ❖ Decimal Number System: Radix = 10
 - → Ten digit values: 0, 1, 2, ..., 9
- ❖ Hexadecimal Number Systems: Radix = 16

 - \Rightarrow A = 10, B = 11, ..., F = 15
- Octal and Hexadecimal numbers can be converted easily to Binary and vice versa

Octal and Hexadecimal Numbers

- ❖ Octal = Radix 8
- Only eight digits: 0 to 7
- Digits 8 and 9 not used
- ❖ Hexadecimal = Radix 16
- ❖ 16 digits: 0 to 9, A to F
- **❖** A=10, B=11, ..., F=15
- First 16 decimal values (0 to 15) and their values in binary, octal and hex. Memorize table

Decimal Radix 10	Binary Radix 2	Octal Radix 8	Hex Radix 16
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	Α
11	1011	13	В
12	1100	14	С
13	1101	15	D
14	1110	16	Е
15	1111	17	F

Binary, Octal, and Hexadecimal

Binary, Octal, and Hexadecimal are related:

Radix
$$16 = 2^4$$
 and Radix $8 = 2^3$

- ❖ Hexadecimal digit = 4 bits and Octal digit = 3 bits
- Starting from least-significant bit, group each 4 bits into a hex digit or each 3 bits into an octal digit
- * Example: Convert 32-bit number into octal and hex

3	3		5			3			()			5			5			2			3			6			2			4		Octal
1	1	1	0	1	0	1	1	0))	1	0	1	1	0	1	0	1	0	0	1	1	1	1	0	0	1	0	1	0	0	32-bit binary
	E	1]	В				1					6			i	A			•	7			(9			4	4		Hexadecimal

Converting Octal & Hex to Decimal

- ***** Octal to Decimal: $N_8 = (d_{n-1} \times 8^{n-1}) + ... + (d_1 \times 8) + d_0$
- **!** Hex to Decimal: $N_{16} = (d_{n-1} \times 16^{n-1}) + ... + (d_1 \times 16) + d_0$
- Examples:

$$(7204)_8 = (7 \times 8^3) + (2 \times 8^2) + (0 \times 8) + 4 = 3716$$

$$(3BA4)_{16} = (3 \times 16^3) + (11 \times 16^2) + (10 \times 16) + 4 = 15268$$

Converting Decimal to Hexadecimal

- Repeatedly divide the decimal integer by 16
- * Each remainder is a hex digit in the translated value
- Example: convert 422 to hexadecimal

Division	Quotient	Remainder	
422 / 16	26	6	least significant digit
26 / 16	1	A	
1 / 16	0	1 ←	most significant digit
422 = (1A6	5) ₁₆	stop whe	

❖ To convert decimal to octal divide by 8 instead of 16

Important Properties

- ❖ How many possible digits can we have in Radix r?
 r digits: 0 to r − 1
- ❖ What is the result of adding 1 to the largest digit in Radix r?
 Since digit r is not represented, result is (10)_r in Radix r

Examples:
$$1_2 + 1 = (10)_2$$
 $7_8 + 1 = (10)_8$ $9_{10} + 1 = (10)_{10}$ $F_{16} + 1 = (10)_{16}$

❖ What is the largest value using 3 digits in Radix r?

In binary:
$$(111)_2 = 2^3 - 1$$

In octal:
$$(777)_8 = 8^3 - 1$$

In decimal:
$$(999)_{10} = 10^3 - 1$$

In Radix r.

largest value =
$$r^3 - 1$$

Important Properties - cont'd

How many possible values can be represented ...

Using *n* binary digits? 2^n values: 0 to $2^n - 1$

Using *n* octal digits 8^n values: 0 to $8^n - 1$

Using *n* decimal digits? 10^n values: 0 to $10^n - 1$

Using *n* hexadecimal digits 16^n values: 0 to $16^n - 1$

Using *n* digits in Radix r? r^n values: 0 to $r^n - 1$

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Representing Fractions

 \clubsuit A number N_r in *radix r* can also have a fraction part:

$$N_r = d_{n-1}d_{n-2} \dots d_1d_0 \cdot d_{-1}d_{-2} \dots d_{-m+1}d_{-m}$$
 $0 \le d_i < r$
Integer Part Fraction Part

Radix Point

 \clubsuit The number N_r represents the value:

$$N_{r} = d_{n-1} \times r^{n-1} + \dots + d_{1} \times r + d_{0} +$$
 (Integer Part)
$$d_{-1} \times r^{-1} + d_{-2} \times r^{-2} \dots + d_{-m} \times r^{-m}$$
 (Fraction Part)
$$N_{r} = \sum_{i=0}^{j=n-1} d_{i} \times r^{i} + \sum_{j=-m}^{j=-1} d_{j} \times r^{j}$$

Examples of Numbers with Fractions

$$(2409.87)_{10}$$
 = $2 \times 10^3 + 4 \times 10^2 + 9 + 8 \times 10^{-1} + 7 \times 10^{-2}$

$$(1101.1001)_2$$
 = $2^3 + 2^2 + 2^0 + 2^{-1} + 2^{-4} = 13.5625$

$$(703.64)_8 = 7 \times 8^2 + 3 + 6 \times 8^{-1} + 4 \times 8^{-2} = 451.8125$$

$$(A1F.8)_{16}$$
 = $10 \times 16^2 + 16 + 15 + 8 \times 16^{-1} = 2591.5$

$$4\times (423.1)_5$$
 = $4\times 5^2 + 2\times 5 + 3 + 5^{-1} = 113.2$

♦ (263.5)₆ Digit 6 is NOT allowed in radix 6

Converting Decimal Fraction to Binary

- \bullet Convert N = 0.6875 to Radix 2
- Solution: Multiply N by 2 repeatedly & collect integer bits

Multiplication	New Fraction	Bit	
$0.6875 \times 2 = 1.375$	0.375	1 -	→ First fraction bit
$0.375 \times 2 = 0.75$	0.75	0	
$0.75 \times 2 = 1.5$	0.5	1	
$0.5 \times 2 = 1.0$	0.0	1 -	→ Last fraction bit

- Stop when new fraction = 0.0, or when enough fraction bits are obtained
- **Therefore**, $N = 0.6875 = (0.1011)_2$
- **!** Check $(0.1011)_2 = 2^{-1} + 2^{-3} + 2^{-4} = 0.6875$

Converting Fraction to any Radix r

❖ To convert fraction N to any radix r

$$N_r = (0.d_{-1} d_{-2} \dots d_{-m})_r = d_{-1} \times r^{-1} + d_{-2} \times r^{-2} \dots + d_{-m} \times r^{-m}$$

 \clubsuit Multiply *N* by *r* to obtain d_{-1}

$$N_r \times r = d_{-1} + d_{-2} \times r^{-1} \dots + d_{-m} \times r^{-m+1}$$

- \clubsuit The integer part is the digit d_{-1} in radix r
- The new fraction is $d_{-2} \times r^{-1} \dots + d_{-m} \times r^{-m+1}$
- ❖ Repeat multiplying the new fractions by r to obtain d₂ d₃ ...
- Stop when new fraction becomes 0.0 or enough fraction digits are obtained

More Conversion Examples

- The integer and fraction parts are converted separately

Division	Quotient	Remainder
139 / 8	17	3
17 / 8	2	1
2/8	0	2

Multiplication	New Fraction	Digit
$0.6875 \times 8 = 5.5$	0.5	5
$0.5 \times 8 = 4.0$	0.0	4

- **Therefore**, $139 = (213)_8$ and $0.6875 = (0.54)_8$
- Now, join the integer and fraction parts with radix point

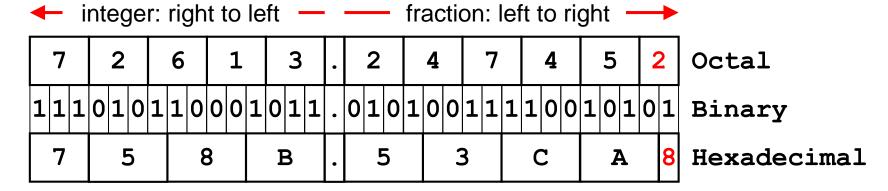
$$N = 139.6875 = (213.54)_8$$

Conversion Procedure to Radix r

- ❖ To convert decimal number *N* (with fraction) to radix *r*
- Convert the Integer Part
 - Repeatedly divide the integer part of number N by the radix r and save the remainders. The integer digits in radix r are the remainders in reverse order of their computation. If radix r > 10, then convert all remainders > 10 to digits A, B, ... etc.
- Convert the Fractional Part
 - → Repeatedly multiply the fraction of N by the radix r and save the integer digits that result. The fraction digits in radix r are the integer digits in order of their computation. If the radix r > 10, then convert all digits > 10 to A, B, ... etc.
- ❖ Join the result together with the radix point

Simplified Conversions

- Converting fractions between Binary, Octal, and Hexadecimal can be simplified
- Starting at the radix pointing, the integer part is converted from right to left and the fractional part is converted from left to right
- Group 4 bits into a hex digit or 3 bits into an octal digit



Use binary to convert between octal and hexadecimal

Important Properties of Fractions

- ❖ How many fractional values exist with *m* fraction bits?
 2^m fractions, because each fraction bit can be 0 or 1
- ❖ What is the largest fraction value if m bits are used? Largest fraction value = $2^{-1} + 2^{-2} + ... + 2^{-m} = 1 - 2^{-m}$ Because if you add 2^{-m} to largest fraction you obtain 1
- ❖ In general, what is the largest fraction value if m fraction digits are used in radix r?

Largest fraction value = $r^{-1} + r^{-2} + ... + r^{-m} = 1 - r^{-m}$

For decimal, largest fraction value = $1 - 10^{-m}$

For hexadecimal, largest fraction value = $1 - 16^{-m}$

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Binary Codes

- How to represent characters, colors, etc?
- Define the set of all represented elements
- Assign a unique binary code to each element of the set
- ❖ Given n bits, a binary code is a mapping from the set of elements to a subset of the 2ⁿ binary numbers
- Coding Numeric Data (example: coding decimal digits)
 - ♦ Coding must simplify common arithmetic operations
 - → Tight relation to binary numbers
- Coding Non-Numeric Data (example: coding colors)
 - ♦ More flexible codes since arithmetic operations are not applied

Example of Coding Non-Numeric Data

- Suppose we want to code 7 colors of the rainbow
- ❖ As a minimum, we need 3 bits to define 7 unique values
- ❖ 3 bits define 8 possible combinations
- Only 7 combinations are needed
- Code 111 is not used
- Other assignments are also possible

•	
Color	3-bit code
Red	000
Orange	001
Yellow	010
Green	011
Blue	100
Indigo	101
Violet	110

Minimum Number of Bits Required

❖ Given a set of *M* elements to be represented by a binary code, the minimum number of bits, *n*, should satisfy:

$$2^{(n-1)} < M \le 2^n$$

- $n = \lceil \log_2 M \rceil$ where $\lceil x \rceil$, called the ceiling function, is the integer greater than or equal to x
- How many bits are required to represent 10 decimal digits with a binary code?
- ❖ Answer: log₂ 10 = 4 bits can represent 10 decimal digits

Decimal Codes

- Binary number system is most natural for computers
- But people are used to the decimal number system
- Must convert decimal numbers to binary, do arithmetic on binary numbers, then convert back to decimal
- To simplify conversions, decimal codes can be used
- Define a binary code for each decimal digit
- Since 10 decimal digits exit, a 4-bit code is used
- ❖ But a 4-bit code gives 16 unique combinations
- 10 combinations are used and 6 will be unused

Binary Coded Decimal (BCD)

- Simplest binary code for decimal digits
- Only encodes ten digits from 0 to 9
- BCD is a weighted code
- ❖ The weights are 8,4,2,1
- Same weights as a binary number
- There are six invalid code words
 1010, 1011, 1100, 1101, 1110, 1111
- Example on BCD coding:

13 ⇔	(0001)	0011)	BCD
------	--------	-------	-----

Decimal	BCD
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
	1010
Unused	• • •
	1111

Warning: Conversion or Coding?

Do NOT mix up conversion of a decimal number to a binary number with coding a decimal number with a binary code

$$4 \cdot 13_{10} = (1101)_2$$

This is conversion

❖ 13 ⇔ (0001 0011)_{BCD}

This is coding

- ❖ In general, coding requires more bits than conversion
- ❖ A number with *n* decimal digits is coded with 4*n* bits in BCD

Other Decimal Codes

- ❖ BCD, 5421, 2421, and 8 4 -2 -1 are weighted codes
- Excess-3 is not a weighted code
- ❖ 2421, 8 4 -2 -1, and Excess-3 are self complementary codes

Decimal	BCD 8421	5421 code	2421 code	8 4 -2 -1 code	Excess-3 code
0	0000	0000	0000	0000	0011
1	0001	0001	0001	0111	0100
2	0010	0010	0010	0110	0101
3	0011	0011	0011	0101	0110
4	0100	0100	0100	0100	0111
5	0101	1000	1011	1011	1000
6	0110	1001	1100	1010	1001
7	0111	1010	1101	1001	1010
8	1000	1011	1110	1000	1011
9	1001	1100	1111	1111	1100
Unused	•••	•••	•••	•••	•••

Character Codes

Character sets

- ♦ Standard ASCII: 7-bit character codes (0 127)
- ♦ Unicode: 16-bit character codes (0 65,535)
- ♦ Unicode standard represents a universal character set
 - Defines codes for characters used in all major languages
 - Each character is encoded as 16 bits
- → UTF-8: variable-length encoding used in HTML
 - Encodes all Unicode characters
 - Uses 1 byte for ASCII, but multiple bytes for other characters

Null-terminated String

♦ Array of characters followed by a NULL character

Printable ASCII Codes

	0	1	2	3	4	5	6	7	8	9	Α	В	С	D	Е	F
2	space	į	**	#	\$	୧	&	1	()	*	+	,	_	•	/
3	0	1	2	3	4	5	6	7	8	9	•	;	<	=	>	?
4	9	A	В	С	D	E	F	G	Н	I	J	K	L	М	N	0
5	P	Q	R	S	Т	U	v	W	X	Y	Z	[\]	^	_
6	•	a	b	С	d	е	f	g	h	i	j	k	1	m	n	0
7	р	q	r	s	t	u	v	W	x	У	Z	{		}	~	DEL

Examples:

- ♦ ASCII code for space character = 20 (hex) = 32 (decimal)
- \Rightarrow ASCII code for 'L' = 4C (hex) = 76 (decimal)
- \Rightarrow ASCII code for 'a' = 61 (hex) = 97 (decimal)

Control Characters

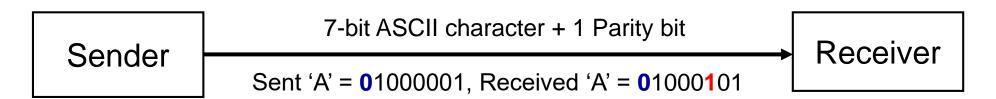
- The first 32 characters of ASCII table are used for control
- Control character codes = 00 to 1F (hexadecimal)
 - ♦ Not shown in previous slide
- Examples of Control Characters
 - ♦ Character 0 is the NULL character ⇒ used to terminate a string
 - ♦ Character 9 is the Horizontal Tab (HT) character
 - ♦ Character 0A (hex) = 10 (decimal) is the Line Feed (LF)
 - ♦ Character 0D (hex) = 13 (decimal) is the Carriage Return (CR)
 - ♦ The LF and CR characters are used together
 - They advance the cursor to the beginning of next line
- One control character appears at end of ASCII table
 - ♦ Character 7F (hex) is the Delete (DEL) character

Parity Bit & Error Detection Codes

- Binary data are typically transmitted between computers
- Because of noise, a corrupted bit will change value
- ❖ To detect errors, extra bits are added to each data value
- Parity bit: is used to make the number of 1's odd or even
- Even parity: number of 1's in the transmitted data is even
- ❖ Odd parity: number of 1's in the transmitted data is odd

7-bit ASCII Character	With Even Parity	With Odd Parity			
'A' = 1000001	0 1000001	1 1000001			
'T' = 1010100	1 1010100	0 1010100			

Detecting Errors



- Suppose we are transmitting 7-bit ASCII characters
- ❖ A parity bit is added to each character to make it 8 bits
- Parity can detect all single-bit errors
 - ♦ If even parity is used and a single bit changes, it will change the parity to odd, which will be detected at the receiver end
 - → The receiver end can detect the error, but cannot correct it because it
 does not know which bit is erroneous
- Can also detect some multiple-bit errors
 - ♦ Error in an odd number of bits