1

(a)

Expected return:

$$E(r_B) = 70\% r_A + 30\% r_f = 15\%$$

Standard deviation:

$$\sigma_B = 70\% \sigma_A = 19.6\%$$

(b)

Investment proportions:

- 30% in T-bills
- $70\% \times 25\% = 17.5\%$ in Stock A
- $70\% \times 32\% = 22.4\%$ in Stock A
- $70\% \times 43\% = 30.1\%$ in Stock A

(c)

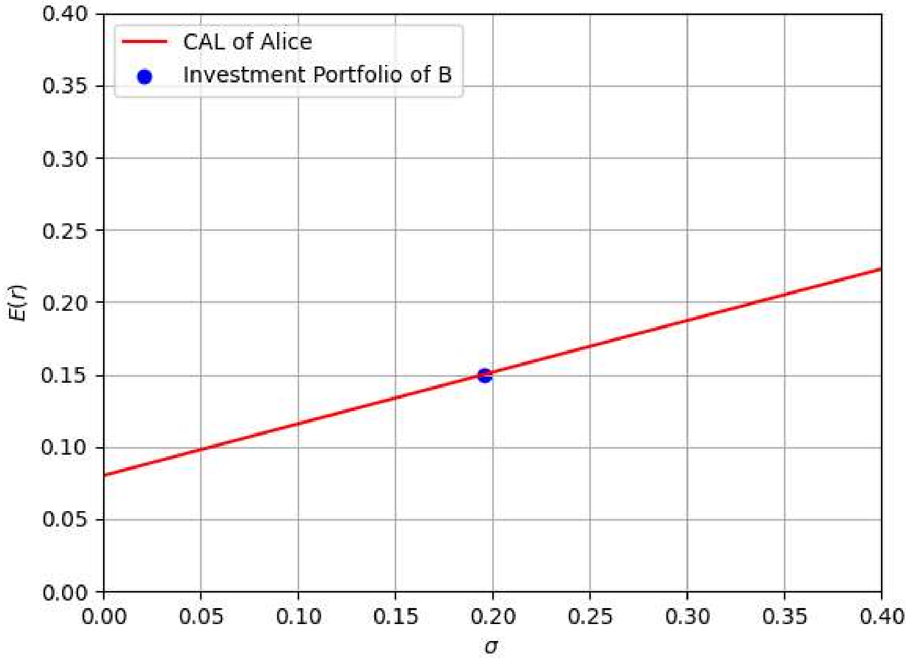
Sharpe ratio of A:

$$\frac{E(r_A) - r_f}{\sigma_A} = \frac{0.18 - 0.08}{0.28} = 0.357$$

Sharpe ratio of B:

$$\frac{E(r_B) - r_f}{\sigma_B} = \frac{0.15 - 0.08}{0.196} = 0.357$$

(d)



- 斜率为0.357,截距为0.08

(e)

i.

$$E(r_B)' = r_f + yS\sigma_A = 0.16 \Rightarrow y = 0.8$$

ii.

$$\sigma_B' = 80\%\sigma_A = 22.4\%$$

(e)

i.

$$\sigma_B'' = y\sigma_A \leq 18\% \Rightarrow y \leq 9/14$$

由于$E(r_B)''$严格随y增大而增大,因此最大化收益率时,y取其上限$9/14 = 0.64$

ii.

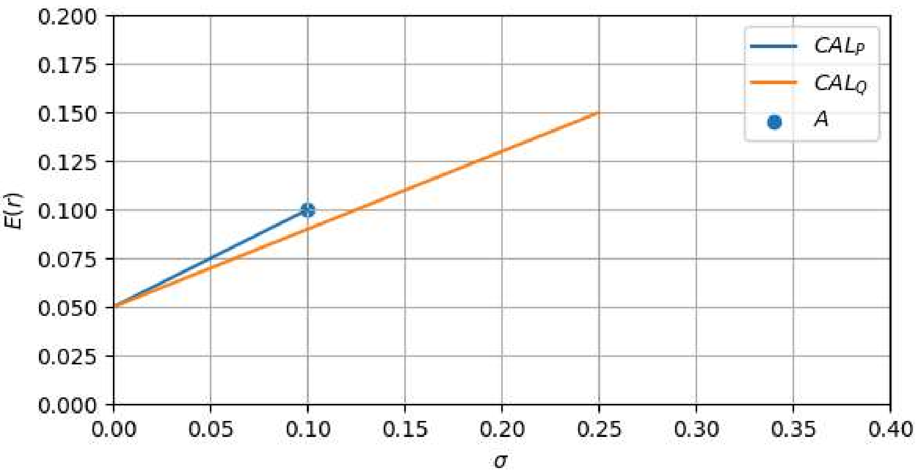
$$E(r_B)'' = r_f + yS\sigma_A = 14.4\%$$

2

(a)

$$\frac{E(r_p) - r_f}{\sigma_p} = 0.5 > 0.4 = \frac{E(r_q) - r_f}{\sigma_q}$$

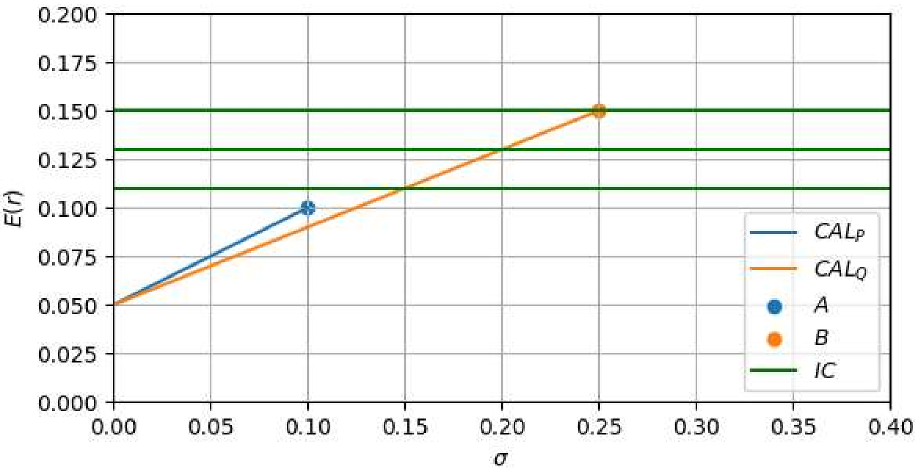
P 具有较高的Sharpe ratio. 每当P可用，投资者会严格倾向于P而不是Q.



由于投资者不允许借贷,因此 CAL_P 不会超过A点,投资者无法借入保证金以实现P比1更高的权重.因此,如果他想实现回报率高于10%,他需要投资Q.这对应于Q权$> 50\%$的情况

(b)

风险中性投资者有水平的无差异线(图中以绿色表示).由于无借用假设,最高无差异线上的点是 B.表示投资者更愿意只投资于资产Q,且不持有无风险资产.



(c)

定义:

- $f(A) = U_p^*(A) - U_q^*(A)$
- $U_x^*(A) = \max_{y_x} U_x(y_x)$
- $y_x \in [0,1]$为选择资产x时,资产x的占比

注意到,

$$
\begin{aligned}
U_x &= E(r) - A\sigma^2 \\
&= r_f + y_x(E(r_x) - r_f) - Ay_x^2\sigma_x^2 \\
\frac{dU_x}{dy_x} &= E(r_x) - r_f - 2A\sigma_x^2 y_x
\end{aligned}
$$

因此,

$$
\frac{dU_x}{dy_x} > 0 \Leftrightarrow 0 < y_x < \min\{\frac{E(r_x) - r_f}{2A\sigma_x^2}, 1\}
$$

也即

$$
y_x^* = \underset{y_x}{\operatorname{argmax}} U_x = \min\{\frac{E(r_x) - r_f}{2A\sigma_x^2}, 1\}
$$

代入得,

$$
y_p = \min\{\frac{5}{2A}, 1\}, y_q = \min\{\frac{0.8}{A}, 1\}
$$

$$
f(A) = \begin{cases}
\frac{21A}{400} - \frac{1}{20} & 0 < A \le 0.8 \\
\frac{1}{20} - \frac{A}{100} - \frac{1}{25A} & 0.8 < A \le 2.5 \\
\frac{9}{200} & 2.5 < A
\end{cases}
$$

解 $f(A) < 0$ 得 $A < 1$, 也即当 $A < 1$ 时,投资者会选择 Q 而不是 P 来和短期国库券构建投资组合

3

(a)

Size of new portfoilo is given by,

$$
90 + 10 = 100
$$

That's one million dollars.

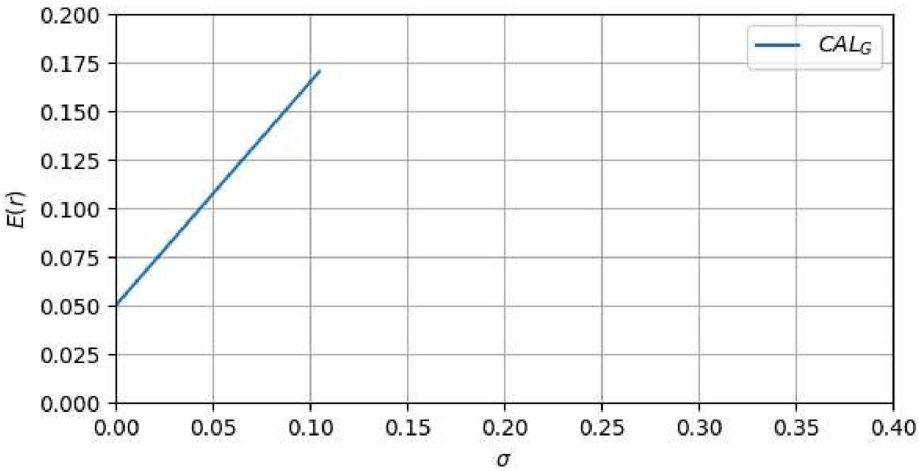
Expected rate of return and standand error of new portfolio are given by:

$$
\begin{aligned}
E(r_G) &= w_A r_A + w_g r_g = 0.1 \times 15\% + 0.9 \times 10\% = 10.5\% \\
\sigma_G^2 &= w_A^2\sigma_A^2 + w_g^2\sigma_g^2 + 2w_A w_g \sigma_{Ag} \\
&= w_A^2\sigma_A^2 + w_g^2\sigma_g^2 + 2w_A w_g \rho_{Ag}\sigma_A\sigma_g \\
&= 0.002449 \\
\sigma_G &= 0.0495
\end{aligned}
$$

(b)

Sharpe ratio is given by:

$$S_G = \frac{E(r_G) - r_f}{\sigma_G} = 1.313$$



(c)

这种说法是错误的,因为B公司与原投资组合收益率的相关系数未知,无法断定置换为B公司股票后新投资组合收益率的标准差不变

(d)

Given that,

$$\sigma_f = 0, \sigma_{fg} = 0$$

Expected rate of return and standand error of new portfolio are given by:

$$\begin{aligned} E(r'_G) &= w_f r_f + w_g r_g = 0.1 \times 4\% + 0.9 \times 10\% = 9.4\% \\ \sigma'^2_G &= w_g^2 \sigma_g^2 \\ &= 0.002025 \\ \sigma'_G &= 0.045 \end{aligned}$$

Sharpe ratio is given by:

$$S'_G = \frac{E(r'_G) - r_f}{\sigma'_G} = 1.2$$

4

(a)

Variance of returns is given by,

$$\begin{aligned} var(R_i) &= var(\alpha_i + \beta_i R_m + e_i) \\ &= \beta_i^2 \sigma_m^2 + \sigma^2(e_i) \end{aligned}$$

Percentage of idiosyncratic risk is given by,

$$\frac{var(e_i)}{var(R_i)}$$

Substitute the data to get,

i	**1**	**2**	**3**	**4**	**5**
收益率方差	0.02128	0.02128	0.245	0.245	0.245
特有风险占比	94.0%	94.0%	81.6%	81.6%	81.6%

(b)

The portfolio return is

$$R_p = \frac{1}{5}\sum_{i=1}^{5} \alpha_i + \beta_i R_m + e_i$$

Variance of this return is given by

$$\begin{aligned} var(R_p) &= var(\frac{1}{5}\sum_{i=1}^{5} \alpha_i + \beta_i R_m + e_i) \\ &= var(\frac{1}{5}\sum_{i=1}^{5} \beta_i R_m) + var(\frac{1}{5}\sum_{i=1}^{5} e_i) \\ &= (\frac{1}{5}\sum_{i=1}^{5} \beta_i)^2 \sigma_m^2 + \sum_{i=1}^{5} (\frac{1}{5})^2 \sigma^2(e_i) \\ &= 0.00698 \end{aligned}$$

Percentage of idiosyncratic risk is given by,

$$\frac{var(e_i)}{var(R_i)} = 57.33\%$$

(c)

$$\begin{cases} w_A + w_C = 1 \\ w_A \beta_A + w_C \beta_C = 0 \end{cases} \Rightarrow \begin{cases} w_A = 15/7 \\ w_C = -8/7 \end{cases}$$

Variance of this return is given by

$$var(R_p) = var(w_A\alpha_A + w_C\alpha_C + w_Ae_A + w_Ce_C) = 0.118$$

Percentage of systemic risk is zero

5

(a)

To construct an arbitrage portfolio without borrowing, consider a portfolio S which consists of weight 0.5 in Q and 0.5 in bills. The beta of portfolio S is

$$\beta_S = 0.5\beta_Q = 1$$

which is the same as β_P . The expected return of S is

$$E(r_S) = 0.5E(r_Q) + 0.5r_f = 13\%$$

which is greater than the expected return of P . Hence

$$r_S = E(r_S) + \beta_S F = 13\% + F$$

Selling P and using the proceeds to buy S gives a return $rS - rP = 2\%$ regardless of F.

(b)

The condition for no arbitrage is

$$r_P = r_S \Rightarrow E(r_Q) = (E(r_P) - r_f)\beta_Q/\beta_P + r_f = 20\%$$

(c)

The sharpe ratio is given by:

$$S = \frac{E(P) - r_f}{\sigma_P} = \frac{E(P) - r_f}{\beta_P\sigma_F} = 8\%$$

(d)

$$\frac{E(P) - r_f}{\beta_P} = \frac{E(r_S) - r_f}{\beta_S}$$
$$\Rightarrow E(r_S) = 8\%\beta_S + 4\%$$

6

(a)

Expected rate of return and standand error of new portfolio are given by:

$$
\begin{aligned}
E(r_P) &= w_X r_X + w_M r_M = 0.25 \times 16\% + 0.75 \times 15\% = 15.25\% \\
\sigma_P^2 &= w_X^2 \sigma_X^2 + w_M^2 \sigma_M^2 + 2 w_X w_M \sigma_{XM} \\
&= w_X^2 \sigma_X^2 + w_M^2 \sigma_M^2 + 2 w_X w_M \rho_{XM} \sigma_X \sigma_M \\
&= 0.0265 \\
\sigma_P &= \sqrt{265}/100
\end{aligned}
$$

(b)

$$
S_P = \frac{E(r_P) - r_f}{\sigma_P} = 41\sqrt{265}/1060 = 0.630
$$

(c)

i	w
X	20%*25%=5%
M	20%*75%=15%
T-bills	80%

Expected rate of return and standand error of new portfolio are given by:

$$
\begin{aligned}
E(r_C) &= w_P r_P + w_f r_f = 0.2 \times 15.25\% + 0.80 \times 5\% = 7.05\% \\
\sigma_c^2 &= w_P^2 \sigma_P^2 \\
\sigma_c &= \sqrt{265}/500 = 0.0326
\end{aligned}
$$

In [ ]: