



2025 年数理经济学笔记

授课: 杨佳楠老师

作者: 徐靖

组织: PKU

时间: February 27, 2025

声明: 请勿用于个人学习外其他用途!

目录

第 1 章 Linear Algebra¹

内容提要

- Leading Principal Minor : 顺序主子式
- Orthogonal matrix : 正交矩阵
- Symmetric matrix : 对称矩阵
- Positive definite matrix : 正定矩阵
- Positive semi-definite matrix : 半正定矩阵
- Determinant : 行列式

定义 1.1

For $N \times N$ matrix $A = (a_{ij})$, using any row or column:

$$\det A = \sum_{i=1}^N (-1)^{i+j} a_{ij} \det A_{ij}$$

where A_{ij} is the $(N-1) \times (N-1)$ matrix obtained by deleting the i -th row and j -th column of A .



定理 1.1

$$A^{-1} = \frac{1}{\det A} \tilde{A}$$

where $a_{mn}^{\tilde{}} = (-1)^{m+n} \det A_{nm}$.



定义 1.2

Orthogonal matrix : $P^T P = I$.

Symmetric matrix : $A^T = A$.

Positive definite matrix : $x^T A x > 0$ for all $x \neq 0$.

Positive semi-definite matrix : $x^T A x \geq 0$ for all x .



定义 1.3

Leading Principal Minor : determinant of the first $k \times k$ submatrix of A . For real symmetric matrix A , A is positive definite if and only if all its leading principal minors are positive.



定义 1.4

$$Av = \lambda v$$

where v is **eigenvector**, λ is **eigenvalue**. λ is a root of the **characteristic polynomial** $\det(A - \lambda I) = 0$.



定义 1.5

Complex inner product :

$$\langle x, y \rangle = x^* y = \sum_{i=1}^n \bar{x}_i y_i$$

where \bar{x} is the **complex conjugate** and x^* is the **conjugate transpose** (adjoint).



Hermitian matrix : $A^* = A$.

For real matrices, Hermitian matrix is symmetric.

For Hermitian matrix, all eigenvalues are real.

¹ 只记一些矩阵分解吧, 以防忘了

定理 1.2 (Diagonalization of Symmetric Matrices)

$$P^T A P = \text{diag}\{\lambda_1, \dots, \lambda_n\}$$

where P is orthogonal matrix, λ_i are eigenvalues of A .

