Homework 4

Due: March 28th, 2025 (in class)

Problem 1

Let $f(x_1, x_2) = x_1^2 + 2x_2^2 - 2x_1x_2 - 2x_2$ and consider the optimization problem

$$minimize \quad f(x_1, x_2)$$

Try start the gradient algorithm from $x_0 = (0, 0)$, and perform four iterations to find x_3

Problem 2

Consider a simple Cournot duopoly model in the previous homework, in which the inverse demand for a good is

$$P(q) = q^{-1/\eta}$$

and the two firms producing the good face cost functions

$$C_i(q_i) = \frac{1}{2}c_i q_i^2, \quad \text{for } i = 1, 2$$

Base on your answer to Problem 2 in Homework 3, assuming $\eta = 1.6$, $c_1 = 0.6$, and $c_2 = 0.8$, with an initial guess of $q_1 = q_2 = 0.2$, write down the procedure of finding the numerical answer with Newton Method with tolerance level 1.510^8.

Problem 3

Solve the following constrained maximization and minimization problems:

1. $\max y = x_1^{0.25}x_2^{0.75}$ subject to $2x_1^2 + 5x_2^2 = 10$
2. $\min y = 2x_1 + 4x_2$ subject to $x_1^{0.25}x_2^{0.75} = 10$
3. $\max y = x_1 + x_2$ subject to $x_1^2 + 2x_2^2 + x_3^2 = 1, \quad x_1 + x_2 + x_3 = 1$