Homework 6

April 12, 2025

Problem 1

Given the consumer utility functions as:

(a) $u = x_1 x_2$

(b) $u = \sqrt{x_1} + x_2$

With budget constraint $p_1 x_1 + p_2 x_2 = m$, derive the corresponding Slutsky decomposition for $\frac{\partial x_1^*}{\partial p_1}$ and $\frac{\partial x_1^*}{\partial p_2}$. Discuss the sign of income and substitution effect.

Problem 2

A competitive firm seeks to maximize its profit given by:

$$\pi = py - wL - rK$$

where y is output, L is labor input, K is capital input, and p, w, and r are output price, wage rate, and capital rental rate, respectively. The production function is given by $y = f(L, K)$.

(a) Set up the Lagrangian for this constrained optimization problem.

(b) Derive the first-order conditions.

(c) Use the envelope theorem to find expressions for $\frac{\partial V}{\partial p}$, $\frac{\partial V}{\partial w}$, and $\frac{\partial V}{\partial r}$, where $V(p, w, r)$ is the value function (maximum profit).

(d) Provide economic interpretations for these derivatives.