Homework 3

Due: March 21th, 2025 (in class)

Problem 1

Use implicit differentiation to find the partial derivatives $\partial y/\partial x_1$ and $\partial y/\partial x_2$ implied by the relationship:

$$F(x_1, x_2, y) = 3x_1 x_2 + x_2 y^2 + x_1^2 x_2 y - 10 = 0$$

Problem 2

Consider a simple Cournot duopoly model, in which the inverse demand for a good is

$$P(q) = q^{-1/\eta}$$

and the two firms producing the good face cost functions

$$C_i(q_i) = \frac{1}{2} c_i q_i^2, \quad \text{for } i = 1, 2$$

1. Write the profit for firm i

2. Illustrate the first order conditions of the equilibrium output level

Problem 3

For each of the following functions, show whether it is convex, concave, or neither.

1. $f(x_1, x_2) = x_1^2 + 3x_1 x_2 + 2x_2^2$.

2. $f(x_1, x_2) = e^{x_1} + e^{x_2}$.

3. $f(x_1, x_2) = \log(e^{x_1} + e^{x_2})$,

Problem 4

Let $f(x_1, x_2) = x_1^2 - x_1 x_2 + 2x_2^2 - x_1 - 3x_2$.

1. Compute the gradient and the Hessian of f.

2. Determine whether f is convex, concave, or neither.

3. Find the stationary point(s) of f.

4. Determine whether each stationary point is a maximum, minimum, or neither.