

Homework 7

Due: May 9th, 2025 (in class)

Problem 1

Consider the problem

$$\begin{aligned} \text{maximize} \quad & x_1 + \log x_2 - \frac{1}{2x_3^2} \\ \text{subject to} \quad & x_1 + p_2x_2 + p_3x_3 \leq w, \end{aligned}$$

where $x_2, x_3 > 0$ but x_1 is unconstrained and $p_2, p_3, w > 0$ are constants.

1. Show that the objective function is concave.
2. Write down the Lagrangian.
3. Are the Karush-Kuhn-Tucker conditions sufficient for a solution? Write down the KKT conditions and find the solution.

Problem 2

Solve

$$\begin{aligned} \text{maximize} \quad & \langle b, x \rangle \\ \text{subject to} \quad & \langle x, Ax \rangle \leq r^2, \end{aligned}$$

where $0 \neq b \in \mathbb{R}^N$, A is an $N \times N$ symmetric positive definite matrix, and $r > 0$.