Homework 5

Due: Apr 11th, 2025 (in class)

Problem 1

Maximize utility subject to a budget constraint

$$\max U(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha} \quad \text{s.t.} \quad p_1 x_1 + p_2 x_2 = m$$

where $0 < \alpha < 1$.

Use the bordered Hessian matrix, and discuss whether the tangency point between an indifference curve and budget line is a true local maximum.

Problem 2

Consider the following optimization problem

$$\max \text{ or } \min_{w.r.t.\, x,y,z} x^2 + y^2 + z^2$$
$$s.t.\, ax^2 + by^2 + cz^2 = 1$$

where a, b and c are constants satisfying $a > b > c > 0$. Write down the Lagrangian and the first-order conditions for this problem. Find all solutions to the first-order conditions. Check the second-order sufficient conditions, for both local maxima and minima, at each of the solutions of the first-order conditions.

Problem 3

Consider a firm with the following profit maximization problem:

$$\pi = pf(L, K) - wL - rK$$

where $f(L, K) = L^{\alpha} K^{\beta}$ is a Cobb-Douglas production function with $0 < \alpha < 1$, $0 < \beta < 1$, and $\alpha + \beta < 1$.

Determine how the optimal input demands L^* and K^* respond to changes in input prices w and r. Specifically: calculate $\frac{\partial L^*}{\partial w}$, $\frac{\partial L^*}{\partial r}$, $\frac{\partial K^*}{\partial w}$, and $\frac{\partial K^*}{\partial r}$. Interpret the economic meaning of these derivatives. Are the inputs gross substitutes, gross complements, or neither?