Game Theory Lecture Notes 1
A Brief Introduction

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Individual Decision Problems v.s. Strategic Interactions

- Recall from your Intermediate Microeconomics.
- Consumer's utility maximization problem:

$$\max_{x_1, x_2 \geq 0} \; u(x_1, x_2)$$
$$\text{s.t. } p_1 x_1 + p_2 x_2 \leq w.$$

- Competitive firm's profit maximization problem:

$$\max_{y \geq 0} \; py - c(y).$$

- These two problems are examples of **individual decision problems**: decision maker's utility/payoff is completely determined by his own choice.

Individual Decision Problems v.s. Strategic Interactions

- In many situations, decision makers interact with each other:
  - chess;
  - oligopoly competition;
  - auction;
  - political campaign;
  - ...
- In these situations, one participant's utility/payoff not only depend on his own choice but also on others'.
- We call these kinds of situations as **strategic interactions**.

Individual Decision Problems v.s. Strategic Interactions

- Individual decision problems are easy to solve, e.g., first order condition, Lagrange multiplier.
- Strategic interactions are much more difficult to analyze.
- Rock-paper-scissors:
  - If I predict that you will choose “Rock”, I will choose “Paper”.
  - But if you predict my prediction, you will choose “Scissors”.
  - If I predict your prediction, I will choose “Rock”.
  - If you predict my prediction, ...
- Not a simple optimization problem.

Individual Decision Problems v.s. Strategic Interactions

- From *Theory of Games and Economic Behavior* by von Neumann and Morgenstern (1944) on strategic interactions:

  "[I]t is unlikely that a mere repetition of the tricks which served us so well in physics will do for the social phenomena too."

  "[The] process of mathematization is not at all obvious. ... We shall find it necessary to draw upon techniques of mathematics which have not been used heretofore in mathematical economics, and ... further study may result in the future in the creation of new mathematical disciplines."

- Game theory provides a solution.

Game Theory

- Game theory provides a language to describe a strategic interaction.
  - We will learn how to write down a formal model.
- Game theory also offers a prediction about what rational players will do in a given strategic interaction.
  - We will also learn how to solve the model.

Review of Expected Utilities

- We always assume that agents are **rational**.
- Rational players try to maximize their utilities/payoffs.

Review of Expected Utilities

Definition 1.1

A simple lottery over outcomes $X = \{x_1, x_2, \ldots, x_n\}$ is defined as a probability distribution $p = (p(x_1), \ldots, p(x_n))$, where $p(x_k) \geq 0$ is the probability that x_k occurs and $\sum_{k=1}^{n} p(x_k) = 1$.

- Consider literally a lottery that pays \$0 with probability 99% and \$100 with probability 1%:
  - $X = \{0, 100\}$;
  - probability distribution p with $p(0) = 0.99$ and $p(100) = 0.01$.
- Tomorrow's weather can be either sunny, cloudy or raining with probabilities 80%, 15% and 5% respectively:
  - $X = \{s, c, r\}$;
  - probability distribution p with $p(s) = 0.8$, $p(c) = 0.15$ and $p(r) = 0.05$.

Review of Expected Utilities

Definition 1.2

Let $u(x)$ be the player's payoff function over outcomes in $X = \{x_1, x_2, \ldots, x_n\}$, and let $p = (p_1, \ldots, p_n)$ be a lottery over X. Then we define the player 's **expected payoff from the lottery** p as

$$\mathbb{E}[u(x)] \equiv p_1 u(x_1) + \ldots + p_n u(x_n) = \sum_{k=1}^{n} p_k u(x_k).$$

- Expected payoff is the *expectation* of the payoffs from the outcomes with respect to the lottery p.
- In other words, it is the *weighted average* of the payoffs from the outcomes.

Review of Expected Utilities

- Consider the previous weather example.
- Suppose an agent has two choices: carry an umbrella (u) and not carry an umbrella (n).
- Assume the agent's payoff is -1 if it does not rain and he carries an umbrella; 10 if it rains and he carries an umbrella; 0 if it does not rain and he does not carry an umbrella; -10 if it rains but he does not carry an umbrella.
- Then his expected payoff from carrying an umbrella is

$$0.8 \times (-1) + 0.15 \times (-1) + 0.05 \times 10 = -0.45,$$

  and his expected payoff from not carrying an umbrella is

$$0.8 \times 0 + 0.15 \times 0 + 0.05 \times (-10) = -0.5.$$

- Carrying an umbrella is optimal.