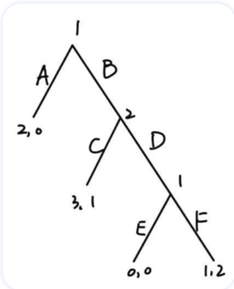
Game Theory HW05

2100010850 娄峻赫

7.2 策略和均衡

1. The extensive form can be shown by:



   This is a game of perfect information, because every information set is a singleton.
2. There are 4 terminal nodes and 3 information sets.
3. For player 1: $|\{A, B\} \times \{E, F\}| = 4$, there are 4 pure strategies.

   For player 2: there are only 2 pure strategies: $\{C\}, \{D\}$
4.

	C	D
AE	2,0	2,0
AF	2,0	2,0
BE	3,1	0,0
BF	3,1	1,2

- From the normal form we can easily find NEs:$(BE, C), (AE, D), (AF, D)$
- Mixed Strategies:
  - Given 2 chooses C for sure, 1 will only mix between BE and BF. For 2, he/she has no intencive to choose D only when: $\sigma_1(BE) + 1 - \sigma_1(BE) > 2(1 - \sigma_1(BE)) \Rightarrow \sigma_1(BE) > 0.5$
  - So $(\sigma_1(BE) \circ BE + (1 - \sigma_1(BE)) \circ BF, C)$, $\sigma_1(BE) > 0.5$ is an NE .
  - Given 2 chooses D for sure, 1 will only mix between AE and AF. For 2, he/she has no intencive to choose C. So $(\sigma_1(AE) \circ AE + (1 - \sigma_1(AE)) \circ AF, D)$ is an NE
  - Given 2 mixes between C and D, for 1, BF strictly dominates BE so that BE will not be mixed. If 1 mix BF, then choosing D is apparently different with choosing C for 2 so 2 will not mix. So 1 will only mix between AE and AF. To assure that 1 will not deviate,
  - $2 \geq 3\sigma_2(C)$,
  - $2 \geq 3\sigma_2(C) + (1 - \sigma_2(C))$
  - $\Rightarrow \sigma_2(C) \leq 0.5$
  - So $(\sigma_1(AE) \circ AE + (1 - \sigma_1(AE)) \circ AF, \sigma_2(C) \circ C + (1 - \sigma_2(C)) \circ D)$, $\sigma_2(C) \leq 0.5$ is an NE.
  - (AF, D) is the most appealing NE because it survives the backward induction, thus it is the most reasonable NE.

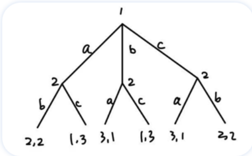
7.3 一字棋

1. This is a game of perfect information for that everyone know what has happened before exactly
2. There are 18 information sets because player 1 can have 9 different choices after choosing O or X.
3. There are $18 \times 8 = 144$ information sets because player 2 has 8 different choices after 1 chooses.

4. For player1, there are $1 + 18 \times 8 + 18 \times 8 \times 7 \times 6 + 18 \times 8 \times 7 \times 6 \times 5 \times 4 + 18 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 = 852913$ information sets. For player 2, there are $18 + 18 \times 8 \times 7 + 18 \times 8 \times 7 \times 6 \times 5 + 18 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 = 394146$ information sets.
5. There are $18 \times 8! = 725760$ terminal nodes.

7.5 否决权

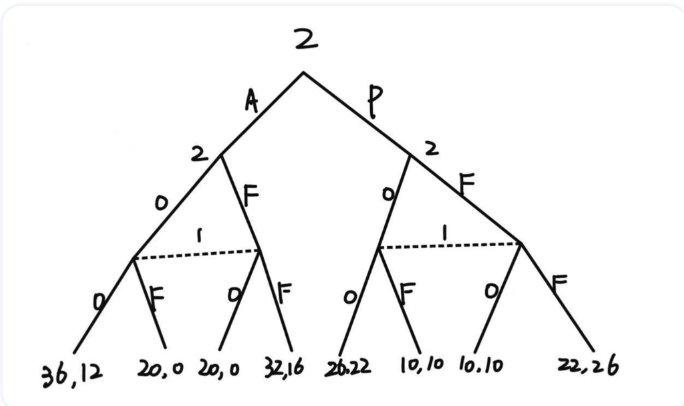
1. Let us denote $v_1(a) = v_2(c) = 3, v_1(b) = v_2(b) = 2, v_1(c) = v_2(a) = 1$



1. 1 has 3 pure strategies : $\{a, b, c\}$ and 2 has $|\{b, c\} \times \{a, c\} \times \{b, a\}| = 8$ pure strategies.
2. Considering that we can not find mixed NEs for that we do not have the precise utility function, we only discuss pure strategy NEs here:
   - $BR_1(s_2) = \begin{cases} b & \text{if } s_2 \in \{bab, cab\} \\ c & \text{if } s_2 \in \{bca, cca, ccb\} \\ \{a, c\} & \text{if } s_2 = bcb \\ \{b, c\} & \text{if } s_2 \in \{baa, caa\} \end{cases}$
   - $BR_2(s_1) = \begin{cases} \{cca, ccb, caa, cab\} & \text{if } s_1 = a \\ \{bca, bcb, cca, ccb\} & \text{if } s_1 = b \\ \{bcb, bab, ccb, cab\} & \text{if } s_1 = c \end{cases}$
   - So NEs are $(c, bcb), (c, ccb)$

7.8 兄弟

1. We denote giving 20 as A(All) and giving 10 as P(Partly)



2. For 1: $\{OO, OF, FO, FF\}$

   For 2: $\{AOO, AOF, AFO, AFF, POO, POF, PFO, PFF\}$
3.

	AOO	AOF	AFO	AFF	POO	POF	PFO	PFF
OO	36,12	36,12	20,0	20,0	26,22	10,10	26,22	10,10
OF	36,12	36,12	20,0	20,0	10,10	22,26	10,10	22,26
FO	20,0	20,0	32,16	32,16	26,22	10,10	26,22	10,10
FF	20,0	20,0	32,16	32,16	10,10	22,26	10,10	22,26

4. We can consider pure strategy as a special type of mixed strategy with a p at 1. So we can easily tell that every pure strategy in the form of “Axx” is strictly dominated by $(0.501 \circ POO + 0.499 \circ POF)$, so we can solve this problem without thinking 2 choosing A.

Now the problem is:

	PO	PF
O	26,22	10,10
F	10,10	22,26

Pure strategies NEs are: $(O, PO), (F, PF)$

Consider 2 mix PO with p and 1 mix O with q, so we have:

$26p + 10(1-p) = 10p + 22(1-p) \Rightarrow p = \frac{3}{7}$

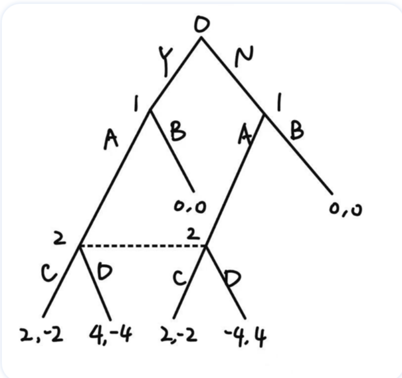
$22q + 10(1-q) = 10q + 26(1-q) \Rightarrow q = \frac{4}{7}$

So back to the original game, we have NEs:

$(p \circ OO + (1-p) \circ FO, q \circ PFO + (1-q) \circ POO) \cup (p \circ OF + (1-p) \circ FF, q \circ POF + (1-q) \circ PFF) \cup$
$(c \circ OO + (\frac{4}{7} - c) \circ FO + d \circ OF + (\frac{3}{7} - d) \circ FF, a \circ POO + (\frac{3}{7} - a) \circ PFO + b \circ POF + (\frac{4}{7} - b) \circ PFF), \forall a, d$

7.9 教务长的困境

1. The game tree is:



2. Normal form game is:

	C	D
AA	2,-2	0,0
AB	1,-1	2,-2
BA	1,-1	-2,2
BB	0,0	0,0

3. We first examine the pure strategies: None

Then we assume 2 mixs C with p and we discuss strategies of 1.

- BB is strictly dominated by AB and BA is strictly dominated by $(0.5 \circ AA + 0.5 \circ AB)$, so we can discuss choices of 1 without thinking BA and BB.
- Now we assume 1 mix AA with q, so we have:
- $2p + 0 = p + 2(1-p) \Rightarrow p = \frac{2}{3}$
- $-2q - (1-q) = -2(1-q) \Rightarrow q = \frac{1}{3}$

- As the theorum tells us, there exists an NE, so the only NE is $(\frac{1}{3} \circ AA + \frac{2}{3} \circ AB, \frac{2}{3} \circ C + \frac{1}{3} \circ D)$

Find all NE

Consider the following normal form representation of the centipede game we covered in class. Find all Nash equilibria.

	nn	nc	cn	cc
NN	$1,1$	$1,1$	$1,1$	$1,1$
NC	$1,1$	$1,1$	$1,1$	$1,1$
CN	$0,3$	$0,3$	$2,2$	$2,2$
CC	$0,3$	$0,3$	$1,4$	$3,3$

1. 4 pure strategy NE are :
   $(NN, nn), (NN, nc), (NC, nn), (NC, nc)$
2. Consider the mixed NE:
   - We can tell that: If $(p \circ NN + (1-p) \circ NC, \sigma_1 \circ nn + \sigma_2 \circ nc + \sigma_3 \circ cn + \sigma_4 \circ cc)$ is an NE, then 1 has no intention to deviate only when $1 \geq 2(\sigma_3 + \sigma_4)$ and $1 \geq \sigma_3 + 3\sigma_4$
     - So $(p \circ NN + (1-p) \circ NC, \sigma_1 \circ nn + \sigma_2 \circ nc + \sigma_3 \circ cn + \sigma_4 \circ cc)$
       $\forall p \in (0,1), (\sigma_1, \sigma_2, \sigma_3, \sigma_4) \in \{[0,1]^4 | \sum \sigma_i = 1, \sigma_3 + \sigma_4 \leq 0.5, \sigma_3 + 3\sigma_4 \leq 1\}$ are mixed NEs.
   - for that each pure stategy of this mixed strategy is the best response for the action of the other player(as shown in pure strategies)
   - Then we try to show the rest mixed strategies do not satisfy the indifferent condition.
   1. If 1 mixes 4 strategies, which means each strategy has got a positive possibility, then 2 will not choose cn or cc beacuse there exists no possibility of 1 to make the choices between cn and cc are indifferent. So:
      1. 2 will not use pure strategy because 1 will use his best responce of that pure strategy, which violates the assumption. （The same is true in other cases.）
      2. If 2 only mixes nn and nc, there exists no possibility of 1 to make the choices between NN or NC and CN or CC are indifferent.
      3. If 2 only mixes cn and cc, CC will not be chosen by 1 because there exists no possibility of 1 to make the choices between NN/NC and CC are indifferent.
      4. If 2 mixes between $\{nn, nc\}$ and $\{cn, cc\}$, CN and CC must be different for their payoff vary between $(0, 2)$ against $(0, 1)$, or $(0, 2)$ against $(0, 3)$
   2. If 1 mixes 3 strategies
      1. If 1 mixes NN, NC, CN: $\{nn, nc\}$ and $\{cn, cc\}$ are different to 2, so 2 only can mixes between one set of the two. Whether 2 choose which of the two, CN and NN/NC are apparently different.(2.1)
      2. If 1 mixes NN, NC, CC: for 2, only cn is the best response given that $\sigma_1(CC) > 0$.(2.2)
      3. If 1 mixes NN/NC, CN, CC: for 2, cc is different for there exists no possibility of 1 to make cc to have the same payoff with other strategies. Now, however 2 mix between nn, nc, cc, CC will be different from NN/NC
   3. If 1 mixes 2 strategies:
      1. If 1 mixes NN/NC with CN: the same with (2.1)
      2. If 1 mixes NN/NC with CC: the same with (2.2)
      3. If 1 mixes CN with CC: for 2, cc is different.Now, however 2 mix between nn, nc, cc, CN will be different from CC.