Review of Midterm

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1. First Price Auction

   Recall the first price auction model. 2 players value the good $v_i \geq 0$ for $i = 1, 2$. The bidders simultaneously bid a price. The one with the higher price wins the good and pays his own bid, and the loser gets nothing and does not pay. If there is a tie in bids, then the good is randomly allocated to the two bidders with equal probability. We restrict attention to pure strategies.

   (a) Model this as a normal-form game.

   (b) Assume that now the loser gets nothing but need to pays his own bid. Write down their payoff function in this case.

   (c) Return to the baseline model in the following questions. Find all pure NE if $v_1 = v_2 = v$.

   (d) Prove that there is no pure NE if $v_1 \neq v_2$.

2. NE

   (a) Find all NE (pure and mixed).

	l	r
T	-1, 2	3, 2
B	2, 3	0, 1

   (b) Find all NE (pure and mixed).

	l	m	r
T	8, 1	0, 2	4, 3
M	3, 1	4, 4	0, 0
B	5, 0	3, 3	1, 4

   (c) Find all NE (pure and mixed).

	w	~~x~~	y	z
~~A~~	5, 6	3, 7	0, 4	1, 0
B	8, 3	2, 1	5, 2	3, 3
C	7, 5	4, 4	5, 6	0, 0
~~D~~	3, 4	7, 1	3, 3	1, 5

3. Air Conditioner

There are n roommates. Each of them decides how much money to contribute to buy an air conditioner. Agent i's valuation of the air conditioner is $v_i > 0$ and the cost of the air conditioner is $c > 0$. Let $x_i \in R_+$ be agent i's contribution. If the total contribution exceeds the cost, i.e. $\sum x_i \geq c$, the air conditioner will be installed and agent i's payoff is $v_i - x_i$. If the total contribution does not cover the cost, the air conditioner will not be installed and agent i's payoff is $-x_i$. Let $n = 2$ and assume $\max\{v_1, v_2\} < c < v_1 + v_2$. We restrict attention to pure strategies.

In the following three questions, assume they simultaneously decide how much money to contribute.

(a) Argue that there is no NE in which $x_1^* + x_2^* > c$.

(b) Argue that there is no NE in which $0 < x_1^* + x_2^* < c$.

(c) Find all NE.

In the following three questions, assume they sequentially decide how much to contribute in the order of roommate 1 and 2.

(a) Write their strategy spaces.

(b) Find a SPE. Is it unique?

(c) Find a NE which is not the SPE.