Game Theory, Fall 2022
Problem Set 1

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1. ST Exercise 3.2.[1]

   Soln: There are two players, the kicker (player 1) and the goalie (player 2). Each has two strategies, L or R, where obviously L denotes "left" and R denotes "right". Thus, the strategy space is $S_i = \{L, R\}$ for $i = 1, 2$. Suppose the winner obtains payoff 1 and the loser obtains payoff -1. Then, we have $v_1(L, R) = v_1(R, L) = v_2(L, L) = v_2(R, R) = 1$ and $v_1(L, L) = v_1(R, R) = v_2(L, R) = v_2(R, L) = -1$. The matrix is in Figure 1.

		Player 2	
		L	R
Player 1	L	$-1, 1$	$1, -1$
	R	$1, -1$	$-1, 1$

Figure 1: The normal form game for Question 1

2. ST Exercise 3.3.

   Soln: There are two players, 1 and 2. The strategy space for player i is $S_i = \{S, C\}$, where S represents Sutro Tower and C represents Coit Tower. Suppose meeting up yields payoff 1 to each of the players and not meeting up yields -1. Therefore, $v_i(S, S) = v_i(C, C) = 1$ and $v_i(S, C) = v_i(C, S) = -1$ for $i = 1, 2$. The matrix is in Figure 2.

3. ST Exercise 3.4.

   Soln: Its game matrix is in Figure 3, where S stands for stag and H stands for hare.

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[1] "ST" refers to our textbook: *Game Theory: An Introduction* by Steven Tadelis.

		Player 2	
		S	C
Player 1	S	$1, 1$	$-1, -1$
	C	$-1, -1$	$1, 1$

Figure 2: The normal form game for Question 2

		Player 2	
		S	H
Player 1	S	$3, 3$	$0, 1$
	H	$1, 0$	$1, 1$

Figure 3: The normal form game for Question 3

4. Consider the first/second price auction enviornment we covered in class. Instead of 2 bidders, assume there are n bidders with value $v_1, \ldots, v_n$.

   (a) Extend the first price auction to this case. As usual, if more than one bidder bid the same highest price, a winning bidder is randomly drawn from them with equal probabilities. (Use this tie breaking rule also for the next question).

   Soln: There are n bidders, $N = \{1, 2, ..., n\}$. Every bidder can bid a nonnegative price $b_i \geq 0$. Thus, bidder i's strategy space is $S_i = [0, +\infty)$. The payoff function for bidder i is

$$v_i(b_i, b_{-i}) = \begin{cases} \frac{v_i - b_i}{|\{j|b_j = b_i\}|}, & \text{if } b_i \geq b_j \text{ for all } j, \\ 0, & \text{otherwise.} \end{cases}$$

   (b) Extend the second price auction to this case.

   Soln: The set of players and strategy space are the same as first price auction, $N = \{1, 2, ..., n\}$, and $S_i = [0, +\infty)$, for $i \in N$. The payoff function for player i is

$$v_i(b_i, b_{-i}) = \begin{cases} \frac{v_i - \max_{j \neq i} b_j}{|\{j|b_j = b_i\}|}, & \text{if } b_i \geq b_j \text{ for all } j, \\ 0, & \text{otherwise.} \end{cases}$$

5. ST Exercise 4.3.

   (a) **Soln:** There are two players, 1 and 2. The strategy space for player i is $S_i = \{0, 1, 2\}$. The game matrix is in Figure 4.

   (b) It is easy to check that none of player 1's strategy is strictly dominated. For player 2, $b_2 = 0$ is strictly dominated by $b_2' = 2$.

Player 2

		0	1	2
	0	$1.5, 2.5$	$0, 4$	$0, 3$
Player 1	1	$2, 0$	$1, 2$	$0, 3$
	2	$1, 0$	$1, 0$	$0.5, 1.5$

Figure 4: The normal form game for Question 5

(c) In the first round, 0 for player 2 is deleted. In the second round, $b_1 = 0$ for player 1 is strictly dominated by $b_1' = 2$, and thus is deleted. In the third round, $b_2 = 1$ for player 2 is strictly dominated by $b_2' = 2$. Hence it is deleted in this round. In the last round, $b_1 = 1$ for player 1 is strictly dominated by $b_1' = 2$. Therefore, only $\{2\} \times \{2\}$ survives IESDS.

6. ST Exercise 4.5.

Soln: There are two players in the game. We delete all strictly dominated strategies in each round. In the first round, U for player 1 is strictly dominated by M. In the second round, C for player 2 is strictly dominated by R. Hence $\{M, D\} \times \{L, R\}$ survives iterated deletion of strictly dominated strategies.

Player 2

		L	C	R
	U	$6, 8$	$2, 6$	$8, 2$
Player 1	M	$8, 2$	$4, 4$	$9, 5$
	D	$8, 10$	$4, 6$	$6, 7$

Figure 5: The normal form game for Question 6