Game Theory HW04

Harassment

1.
   - Strategy space: $S_i = \{R, K\}, \forall i = 1, 2, \ldots, n$
   - Players set: $\{1, 2, \ldots, n\}$
   - Payoff:
     - $v_i(R, s_{-i}) = 1 - c$
     - $v_i(K, s_{-i}) = \begin{cases} 0 & \text{if } s_{-i} = (K, K, \ldots, K) \\ 1 & else \end{cases}$

2.
   - Firstly, $(K, K, \ldots, K)$ is not a NE for everyone gets a payoff at 0 now, but they can get better off by choosing R because :$v_i(R, s_{-i}) = 1 - c > 0$
   - Secondly, an S with more than 1 R cannot be a NE for that keeping others still, the one who choose R can get better off by choosing K, from which he/she can get a payoff 1 rather than 1-c.
   - Finally, we only get S with only 1 R and this is the exact NE because the one choosing R will not choose K. If he/she turns to K, then he/she will get 0 instead of 1-c.
   - So, $(R, K, \ldots, K), (K, R, K, \ldots, K), \cdots, (K, K, \ldots, K, R)$ are all NE. There are n NE in total.

3. We suppose that everyone mixes with a same p to report, then we can derive with the indifferent condition that

   $(1-p)^{n-1} \times 0 + 1 - (1-p)^{n-1} = 1 - c$

$\Rightarrow p^* = 1 - c^{\frac{1}{n-1}}$

We should verify that this is an NE. By symmetry, we only should confirm that $v_1(\sigma^*, \sigma^*, \ldots, \sigma^*) \geq v_1(s, \sigma^*, \ldots, \sigma^*), \forall s \in S_1$

$v_1(\sigma^*, \sigma^*, \ldots, \sigma^*) = p^*(1-c) + (1-p^*)(1-(1-p^*)^{n-1}) = 1 - cp^* - (1-p^*)^n$

$v_1(R, \sigma^*, \ldots, \sigma^*) = 1 - c$ and $v_1(K, \sigma^*, \ldots, \sigma^*) = 1 - (1-p^*)^{n-1} = 1 - c$

$v_1(\sigma^*, \sigma^*, \ldots, \sigma^*) \geq v_1(R, \sigma^*, \ldots, \sigma^*) \Leftrightarrow c \geq (1-p^*)^{n-1} = c$

$v_1(\sigma^*, \sigma^*, \ldots, \sigma^*) \geq v_1(K, \sigma^*, \ldots, \sigma^*) \Leftrightarrow c \geq (1-p^*)^{n-1} = c$

离散式全支付拍卖

1.

	0	1	2
0	1.5，2.5	0，4	0，3
1	2，0	0.5，1.5	-1，3
2	1，0	1，-1	-0.5，0.5

- For player 2, choosing 2 dominates choosing 0;
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	1	2
0	0，4	0，3
1	0.5，1.5	-1，3
2	1，-1	-0.5，0.5

- For player 1, choosing 2 dominates choosing 1;
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	1	2
0	0，4	0，3
2	1，-1	-0.5，0.5

2. We can tell that in the reduced game, there is not a pure strategy NE.

- Consider that player 1 choose 0 at probability p and player 2 choose 1 at q.
- Indifferent condition:
  - $q \times (-1) + (1-q) \times 0.5 = 0 \Rightarrow q = \frac{1}{3}$
  - $4p - (1-p) = 3p + 0.5(1-p) \Rightarrow p = \frac{3}{5}$
- （既然nash均衡存在，那么根据必要条件推出的唯一条件就决定了这个nash均衡吗？）
- So NE is $(\frac{3}{5} \circ 0 + \frac{2}{5} \circ 2, \frac{1}{3} \circ 1 + \frac{2}{3} \circ 2)$