Game Theory, Fall 2022
Problem Set 4

Zhuoyuan He and Sihao Tang*

1. There are $n \geq 2$ victims of sexual harassment by a perpetrator. Each of the victim simultaneously and independently decides whether to report the harassment or keep quiet. As long as at least one victim reports, the perpetrator is punished and each of the victim gets a payoff of 1. If no victim reports, the perpetrator is not punished and the victims get 0. Reporting has a cost $c \in (0, 1)$, while keeping quiet costs 0. Model this situation as a normal form game of the n victims.

   (a) Model this situation as a normal form game of the n victims.

   Soln: There are n players (victims), and the set of players is $N = \{1, 2, ..., n\}$. Each has two strategies, reporting and keeping quiet, defined as 1 and 0. The strategy space is $S_i = \{1, 0\}$ for $i \in N$. The payoff function can be derived as:

$$v_i(s_1, ..., s_n) = \begin{cases} 1 - s_i \cdot c, & \text{if } \sum_{i=1}^{n} s_i > 0, \\ 0, & \text{if } \sum_{i=1}^{n} s_i = 0. \end{cases}$$

   (b) Find all pure strategy Nash equilibria.

   Soln: All victims keeping quiet is not a Nash equilibrium, since every victim has an incentive to deviate and report. Moreover, there is no Nash equilibrium in which more than one victims report, since every reporting victim has an incentive to deviate and keep quiet. Doing so can save the reporting cost without affecting punishing the perpetrator. Every strategy profile in which one and only one victim reports is a Nash equilibrium. The quiet victims do not have an incentive to deviate because the perpetrator is already punished. The reporting victim has no incentive to deviate either, because reporting yields a payoff $1 - c$ while keeping quiet yields 0. So there are in total n pure strategy Nash equilibria.

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(c) Find a Nash equilibrium in mixed strategies.

Soln: There might be many such equilibria. For example, we consider a symmetric equilibrium. Suppose every victim mixes between keeping quiet with probability $p \in (0,1)$ and reporting with probability $1-p \in (0,1)$. Consider one victim. If he/she keeps quiet, the probability that all the opponents keep quiet is p^{n-1} and the probability that at least one opponent reports is $1-p^{n-1}$. Thus, his/her payoff from keeping quiet is

$$p^{n-1} \times 0 + (1-p^{n-1}) \times 1$$

If instead, he/she reports, the perpetrator is punished for sure. So his/her payoff is $1-c$ from reporting. For this victim to mix, he/she must be indifferent. That is

$$p^{n-1} \times 0 + (1-p^{n-1}) \times 1 = 1-c$$

Solving this equation, we get $p = c^{\frac{1}{n-1}}$. Therefore, every victim mixes between keeping quiet with probability $c^{\frac{1}{n-1}}$ and reporting with probability $1-c^{\frac{1}{n-1}}$ is a Nash equilibrium.

2. ST Exercise 6.9.

(a) **Soln:** Its matrix representation is in Figure 1.

		Player 2		
		0	1	2
	0	$\frac{3}{2}, \frac{5}{2}$	$0, 4$	$0, 3$
Player 1	1	$2, 0$	$\frac{1}{2}, \frac{3}{2}$	$-1, 3$
	2	$1, 0$	$1, -1$	$-\frac{1}{2}, \frac{1}{2}$

Figure 1: Normal form game for Question 2a

In the first round, 0 for player 2 is strictly dominated by 2. In the second round, 1 for player 1 is strictly dominated by 2. No strategy is strictly dominated in the remaining game. Hence $\{0, 2\} \times \{1, 2\}$ survives IESDS.

(b) **Soln:** We only need to focus on the reduced game. Clearly, there is no pure strategy Nash equilibrium. Moreover, it is easy to see that there is no Nash equilibrium in which only one bidder mixes. So, we consider equilibrium in which both bidders mix.

For bidder 1 to mix between 0 and 2, we must have

$$\sigma_2(1) - \frac{1}{2}(1 - \sigma_2(1)) = 0,$$

implying

$$\sigma_2(1) = \frac{1}{3} \text{ and } \sigma_2(2) = \frac{2}{3}.$$

For bidder 2 to mix between 1 and 2, we must have

$$4\sigma_1(0) - (1 - \sigma_1(0)) = 3\sigma_1(0) + \frac{1}{2}(1 - \sigma_1(0)),$$

implying

$$\sigma_1(0) = \frac{3}{5} \text{ and } \sigma_1(2) = \frac{2}{5}.$$

Therefore, there is a unique Nash equilibrium $(\frac{3}{5} \circ 0 + \frac{2}{5} \circ 2, \frac{1}{3} \circ 1 + \frac{2}{3} \circ 2)$.