Problem Set 2

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1 ST Exercise 5.4

Answer:

(a)

Assume the players can ask for amounts that are not restricted to integers, then the best response is

$$BR_i(s_j) = \begin{cases} 8 - s_j, & s_j \in [0, 8), \\ [0, 8], & s_j = 8. \end{cases}$$

In fact, the integer case is the subset of above fomula.

(b)

From the best response correspondence, we find every $(i, 8-i), i \in [0, 8]$ will be a Nash equilibrium.
And there is another Nash equilibrium $(8, 8)$. That is because there are no difference among all of one's requests including 8, when the other player is asking for 8 slices.

2 ST Exercise 5.5

Answer:

(a)

Let s_1, s_2, s_3 represent whether to contribute, 1 if contribute, 0 if not, then the best responses is

$$BR_i(s_{-i}) = \begin{cases} 0, & -s_i + \sum_{j \in [3]} s_j = 2, \\ 1, & -s_i + \sum_{j \in [3]} s_j = 1, \\ 0, & -s_i + \sum_{j \in [3]} s_j = 0, \end{cases}$$

(b)

Traversing all $2^3 = 8$ strategy combinations, it was found that only $(0,0,0),(0,1,1),(1,0,1),(1,1,0)$ are pure strategy Nash equilibria.

3 ST Exercise 5.9

Answer:

(a)

Assume $n \geq 2$, one will of course maximize his clean time when $c > 1$ regardless of the other's choice. Thus the unique Nash equilibrium is $s_i = 5, \forall i \in [n]$.

(b)

One will of course minimize his clean time when $c > 1$ regardless of the other's choice. Thus the unique Nash equilibrium is $s_i = 0, \forall i \in [n]$.

(c)

The Nash equilibrium is obviously not Pareto efficient. Let $s_i = 5$ and we have $v_i = -2s_i + \sum_{i=1}^{5} s_i = 15 > 0$. In this case everyone is better.

4 ST Exercise 5.10

Answer:

(a)

First order condition:

$$a + \mathrm{e}_j - 2\mathrm{e}_i = 0$$

Best response function:

$$BR_i(\mathrm{e}_j) = \frac{a + \mathrm{e}_j}{2}$$

(b)

The best response of player i is increasing in the choice of player j whereas in the Cournot model it is decreasing in the choice of player j. That's because in this the choices of the two players are strategic complements while ins the Cournot game they are strategic substitutes.

(c)

Solve the first order condition

$$a + \mathrm{e}_1 - 2\mathrm{e}_2 = 0, +\mathrm{e}_2 - 2\mathrm{e}_1 = 0,$$

we get

$$e_1 = e_2 = a$$

The solution is unique because it is the only point at which these two best response functions cross.

5 ST Exercise 5.12

Answers:

(a)

Another Nash equilibrium is $(1.5, 1.51)$. In this equilibrium firm 1 fulfills market demand at the price of 1.5 and has no incentive the change the price in every direction. Firm 2 is indifferent between the current price and any higher price, and no incentive to decrease price because it will lose money if it sells the product at a price less than 2.

(b)

Consider the best response of firms,

$$BR_1(p_2) = \begin{cases} (p_2, +\infty], & p_2 < 1, \\ [1, +\infty], & p_2 = 1, \\ p_2 - 0.01, & p_2 > 1 \end{cases}$$

$$BR_2(p_1) = \begin{cases} (p_1, +\infty], & p_1 < 2, \\ [2, +\infty], & p_1 = 2, \\ p_1 - 0.01, & p_2 > 2 \end{cases}$$

And we find there are 100 Nash equilibria from the above formula. They start with $(1.00, 1.01)$ and go all the way up with one-cnet increases to $(1.99, 2.00)$. The same logic to (a) explain why they are.

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Answer:
Nash equilibrium exists only when p_1 is between firm 1 and firm 2 because:

- When $p_1 < 0$, both firms lose money at this price, and they will definitely raise prices in turn
- When $p_1 > \text{firm 2's marginal cost} = 2$, both firms make money at this price, and they will definitely lower prices in turn to gain market share

Consider the case of $1 \geq p_1 \geq 2$, at which firm 2 makes no profit and firm 1 makes no loss. For firm 2, p_2 has no difference within $[p_1, +\infty)$, and is better than a price lower than p_1. For firm 1, the best response is p_2, so $p_1 = p_2 \in [1, 2]$ are Nash equilibrium

7 ST Exercise 5.16

Answer:

(a)

For x^*, we have

$$v - p_1 - x^* = v - p_2 - (1 - x^*) \Rightarrow x^* = \frac{1 + p_2 - p_1}{2}$$

Buyers in $[0, x^*)$ are served by 1 and the other by 2, thus we have

$$v_i(p_1, p_2) = (\frac{1 + p_j - p_i}{2})p_i$$

Solve the first order condition,we get

$$p_1 = \frac{1+p_2}{2}, p_2 = \frac{1+p_1}{2}$$

(b)

The case of everyone will be served by one of the firms is discussed above and we could get a unique Nash equilirium $p_1 = p_2 = 1$. But when $v = 1$ the utility of $x* = \frac{1}{2}$ is $v - p_1 - \frac{1}{2} = -\frac{1}{2}$ thus he will not buy. Assume x^* will not choose either one.

Then 1 firm maximizes

$$\max_{p_1} = (1 - p_1)p_1$$

which yields the solution $p_1 = \frac{1}{2}$, which means everyone in the interval $x \in [0, \frac{1}{2}]$ wished to buy from firm 1. This implies if both firms set their prices $\frac{1}{2}$ then each would maximize profits ignoring the other firm, which is the unique Nash equilibrium.

(c)

We have $x^* = \frac{1}{2} + p_2 - p_1$ Thus profits of two firms are:

$$v_i(p_1, p_2) = (\frac{1}{2} + p_j - p_i)p_i$$

And the response functions are:

$$p_1 = \frac{1+2p_2}{4}, p_2 = \frac{1+2p_1}{4}$$

which yields $p_1 = p_2 = \frac{1}{2}$ and x^* 's utility is 0. That means it is the Nash equilibrium.

(d)

About x^*:

$$v - p_1 - \alpha x^* = v - p_2 - \alpha(1 - x^*) \Rightarrow x^* = \frac{1}{2} + \frac{1}{2\alpha}(p_2 - p_1)$$

Then the profits :

$$v_i = \left(\frac{1}{2} + \frac{1}{2\alpha}(p_2 - p_1)\right) p_1$$

Then the best response:

$$p_i = \frac{\alpha}{2} + \frac{p_j}{2}$$

which yields $p_i = \alpha$, From the analysis in (c) above we know that for any $\alpha \in \left[0, \frac{1}{2}\right)$, x^* will strictly prefer to buy over not buying and so will every other customer.