Exercises

E10.1 An adaptive filter ADALINE is shown in Figure E10.1. Suppose that the weights of the network are given by

$$w_{1,1} = 1, \; w_{1,2} = -4, \; w_{1,3} = 2,$$

and the input to the filter is

$$\{y(k)\} = \{\ldots, 0, 0, 0, 1, 1, 2, 0, 0, \ldots\}.$$

Find the response $\{a(k)\}$ of the filter.

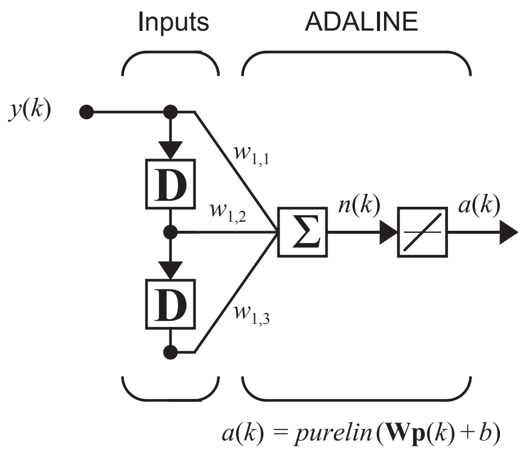


Figure E10.1 Adaptive Filter ADALINE for Exercise E10.1

E10.2 In Figure E10.2 two classes of patterns are given.

i. Use the LMS algorithm to train an ADALINE network to distinguish between class I and class II patterns (we want the network to identify horizontal and vertical lines).

ii. Can you explain why the ADALINE network might have difficulty with this problem?

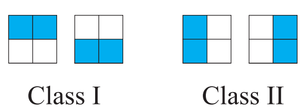


Figure E10.2 Pattern Classification Problem for Exercise E10.2

E10.3 Suppose that we have the following two reference patterns and their targets:

$$\left\{\mathbf{p}_1 = \begin{bmatrix}1\\1\end{bmatrix}, t_1 = 1\right\}, \left\{\mathbf{p}_2 = \begin{bmatrix}1\\-1\end{bmatrix}, t_2 = -1\right\}.$$

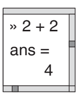
In Problem P10.3 these input vectors to an ADALINE were assumed to occur with equal probability. Now suppose that the probability of vector $\mathbf{p}_1$ is 0.75 and that the probability of vector $\mathbf{p}_2$ is 0.25. Does this change of probabilities change the mean square error surface? If yes, what does the surface look like now? What is the maximum stable learning rate?

E10.4 In this exercise we will modify the reference pattern $\mathbf{p}_2$ from Problem P10.3:

$$\left\{\mathbf{p}_1 = \begin{bmatrix}1\\1\end{bmatrix}, t_1 = 1\right\}, \left\{\mathbf{p}_2 = \begin{bmatrix}-1\\-1\end{bmatrix}, t_2 = -1\right\}.$$

i. Assume that the patterns occur with equal probability. Find the mean square error and sketch the contour plot.

ii. Find the maximum stable learning rate.



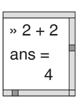
iii. Write a MATLAB M-file to implement the LMS algorithm for this problem. Take 40 steps of the algorithm for a stable learning rate. Use the zero vector as the initial guess. Sketch the trajectory on the contour plot.

iv. Take 40 steps of the algorithm after setting the initial values of both parameters to 1. Sketch the final decision boundary.

v. Compare the final parameters from parts (iii) and (iv). Explain your results.

E10.5 We again use the reference patterns and targets from Problem P10.3, and assume that they occur with equal probability. This time we want to train an ADALINE network with a bias. We now have three parameters to find: $w_{1,1}$, $w_{1,2}$ and b.

i. Find the mean square error and the maximum stable learning rate.



ii. Write a MATLAB M-file to implement the LMS algorithm for this problem. Take 40 steps of the algorithm for a stable learning rate. Use the zero vector as the initial guess. Sketch the final decision boundary.

iii. Take 40 steps of the algorithm after setting the initial values of all parameters to 1. Sketch the final decision boundary.

iv. Compare the final parameters and the decision boundaries from parts (iii) and (iv). Explain your results.

E10.6 We have two categories of vectors. Category I consists of

$$\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \end{bmatrix} \right\}.$$

Category II consists of

$$\left\{ \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \end{bmatrix} \right\}.$$

We want to train a single-neuron ADALINE network without a bias to recognize these categories ($t = 1$ for Category I and $t = -1$ for Category II). Assume that each pattern occurs with equal probability.

i. Draw the network diagram.

ii. Take four steps of the LMS algorithm, using the zero vector as the initial guess. (one pass through the four vectors above - present each vector once). Use a learning rate of 0.1.

iii. What are the optimal weights?

iv. Sketch the optimal decision boundary.

v. How do you think the boundary would change if the network were allowed to have a bias? If the boundary would change, indicate the approximate new position on your sketch of part iv. You do not need to perform any calculations here - just explain your reasoning.

E10.7 Suppose that we have the following three reference patterns and their targets:

$$\left\{ \mathbf{p}_1 = \begin{bmatrix} 3 \\ 6 \end{bmatrix}, t_1 = \begin{bmatrix} 75 \end{bmatrix} \right\}, \left\{ \mathbf{p}_2 = \begin{bmatrix} 6 \\ 3 \end{bmatrix}, t_2 = \begin{bmatrix} 75 \end{bmatrix} \right\}, \left\{ \mathbf{p}_3 = \begin{bmatrix} -6 \\ 3 \end{bmatrix}, t_3 = \begin{bmatrix} -75 \end{bmatrix} \right\}.$$

Each pattern is equally likely.

i. Draw the network diagram for an ADALINE network with no bias that could be trained on these patterns.

ii. We want to train the ADALINE network with no bias using these patterns. Sketch the contour plot of the mean square error performance index.

iii. Find the maximum stable learning rate for the LMS algorithm.

iv. Sketch the trajectory of the LMS algorithm on your contour plot. Assume a very small learning rate, and start with all weights equal to zero. This does not require any calculations.

E10.8 Suppose that we have the following two reference patterns and their targets:

$$\left\{\mathbf{p}_1 = \begin{bmatrix}1\\2\end{bmatrix}, t_1 = \begin{bmatrix}-1\end{bmatrix}\right\}, \left\{\mathbf{p}_2 = \begin{bmatrix}-2\\1\end{bmatrix}, t_2 = \begin{bmatrix}1\end{bmatrix}\right\}.$$

The probability of vector $\mathbf{p}_1$ is 0.5 and the probability of vector $\mathbf{p}_2$ is 0.5.We want to train an ADALINE network without a bias on this data set.

i. Sketch the contour plot of the mean square error performance index.

ii. Sketch the optimal decision boundary.

iii. Find the maximum stable learning rate.

iv. Sketch the trajectory of the LMS algorithm on your contour plot. Assume a very small learning rate, and start with initial weights $\mathbf{W}(0) = \begin{bmatrix}0 & 1\end{bmatrix}$.

E10.9 We have the following input/target pairs:

$$\left\{\mathbf{p}_1 = \begin{bmatrix}4\\2\end{bmatrix}, t_1 = 5\right\}, \left\{\mathbf{p}_2 = \begin{bmatrix}2\\-4\end{bmatrix}, t_2 = -2\right\}, \left\{\mathbf{p}_3 = \begin{bmatrix}-4\\4\end{bmatrix}, t_3 = 9\right\}.$$

The first two pair each occurs with probability of 0.25, and the third pair occurs with probability 0.5. We want to train a single-neuron ADALINE network without a bias to perform the desired mapping.

i. Draw the network diagram.

ii. What is the maximum stable learning rate?

iii. Perform one iteration of the LMS algorithm. Apply the input $\mathbf{p}_1$ and use a learning rate of $\alpha = 0.1$. Start from the initial weights $\mathbf{x}_0 = \begin{bmatrix}0 & 0\end{bmatrix}^T$.

E10.10 Repeat E10.9 for the following input/target pairs:

$$\left\{\mathbf{p}_1 = \begin{bmatrix} 2 \\ -4 \end{bmatrix}, t_1 = 1\right\}, \left\{\mathbf{p}_2 = \begin{bmatrix} -4 \\ 4 \end{bmatrix}, t_2 = -1\right\}, \left\{\mathbf{p}_3 = \begin{bmatrix} 4 \\ 2 \end{bmatrix}, t_3 = 1\right\}.$$

The first two pair each occurs with probability of 0.25, and the third pair occurs with probability 0.5. We want to train a single-neuron ADALINE network without a bias to perform the desired mapping.

E10.11 We want to train a single-neuron ADALINE network without a bias, using the following training set, which categorizes vectors into two classes. Each pattern occurs with equal probability.

$$\left\{\mathbf{p}_1 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}, t_1 = -1\right\} \left\{\mathbf{p}_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, t_2 = -1\right\} \left\{\mathbf{p}_3 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, t_3 = 1\right\} \left\{\mathbf{p}_4 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, t_4 = 1\right\}$$

i. Draw the network diagram.

ii. Take one step of the LMS algorithm (present $\mathbf{p}_1$ only) starting from the initial weight $\mathbf{W}(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}$. Use a learning rate of 0.1.

iii. What are the optimal weights? Show all calculations.

iv. Sketch the optimal decision boundary.

v. How do you think the boundary would change if the network were allowed to have a bias? Indicate the approximate new position on your sketch of part iv.

vi. What is the maximum stable learning rate for the LMS algorithm?

vii. Sketch the contour plot of the mean square error performance surface.

viii. On your contour plot of part vii, sketch the path of the LMS algorithm for a very small learning rate (e.g., 0.001) starting from the initial condition $\mathbf{W}(0) = \begin{bmatrix} 2 & 0 \end{bmatrix}$. This does not require any calculations, but explain how you obtained your answer.

E10.12 Suppose that we have the following three reference patterns and their targets:

$$\left\{\mathbf{p}_1 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}, t_1 = \begin{bmatrix} 26 \end{bmatrix}\right\}, \left\{\mathbf{p}_2 = \begin{bmatrix} 4 \\ 2 \end{bmatrix}, t_2 = \begin{bmatrix} 26 \end{bmatrix}\right\}, \left\{\mathbf{p}_3 = \begin{bmatrix} -2 \\ -2 \end{bmatrix}, t_3 = \begin{bmatrix} -26 \end{bmatrix}\right\}.$$

The probability of vector $\mathbf{p}_1$ is 0.25, the probability of vector $\mathbf{p}_2$ is 0.25 and the probability of vector $\mathbf{p}_3$ is 0.5.

i. Draw the network diagram for an ADALINE network with no bias that could be trained on these patterns.

ii. Sketch the contour plot of the mean square error performance index.

iii. Show the optimal decision boundary (for the weights that minimize mean square error) and verify that it separates the patterns into the appropriate categories.

iv. Find the maximum stable learning rate for the LMS algorithm. If the target values are changed from 26 and -26 to 2 and -2, how would this change the maximum stable learning rate?

v. Perform one iteration of the LMS algorithm, starting with all weights equal to zero, and presenting input vector $\mathbf{p}_1$. Use a learning rate of $\alpha = 0.5$.

vi. Sketch the trajectory of the LMS algorithm on your contour plot. Assume a very small learning rate, and start with all weights equal to zero.

E10.13 Consider the adaptive predictor in Figure E10.3.

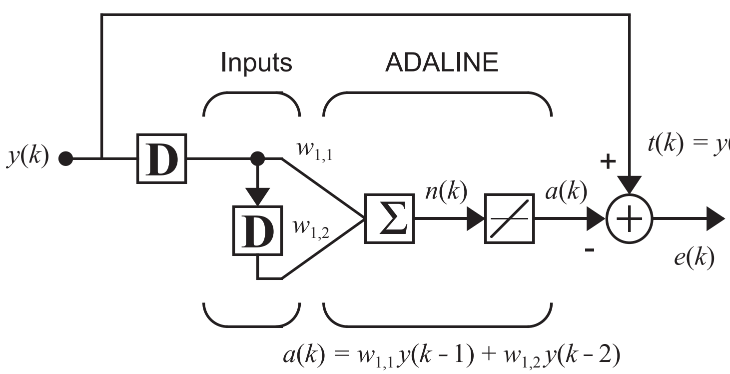


Figure E10.3 Adaptive Predictor for Exercise E10.13

Assume that $y(k)$ is a stationary process with autocorrelation function

$$C_y(n) = E[y(k)(y(k+n))].$$

i. Write an expression for the mean square error in terms of $C_y(n)$.

ii. Give a specific expression for the mean square error when

$$y(k) = \sin\left(\frac{k\pi}{5}\right).$$

iii. Find the eigenvalues and eigenvectors of the Hessian matrix for the mean square error. Locate the minimum point and sketch a rough contour plot.

iv. Find the maximum stable learning rate for the LMS algorithm.

v. Take three steps of the LMS algorithm by hand, using a stable learning rate. Use the zero vector as the initial guess.

vi. Write a MATLAB M-file to implement the LMS algorithm for this problem. Take 40 steps of the algorithm for a stable learning rate and sketch the trajectory on the contour plot. Use the zero vector as the initial guess. Verify that the algorithm is converging to the optimal point.

vii. Verify experimentally that the algorithm is unstable for learning rates greater than that found in part (iv).

E10.14 Repeat Problem P10.9, but use the numerals “1”, “2” and “4”, instead of the letters “T”, “G” and “F”. Test the trained network on each reference pattern and on noisy patterns. Discuss the sensitivity of the network. (*Use the Neural Network Design Demonstration* Linear Pattern Classification (`nnd10lc`).)