c) $$\chi^2(\vec{\theta}) = \sum_{i=1}^{N} \left(\frac{y_i - M(x_i,\vec{\theta})}{\sigma_i} \right)^2$$ donde: $$M = \frac{\theta_0}{\theta_1 + e^{-\theta_2 \cdot x}}$$

$$\chi^2(\vec{\theta}) = \sum_{i=1}^{N} \left(\frac{y_i - \left(\frac{\theta_0}{\theta_1 + e^{-\theta_2 x_i}} \right)}{\sigma_i} \right)^2 \Bigg]$$

$$\rightarrow \frac{\partial \chi^2(\vec{\theta})}{\partial \theta_i} = \sum_{j=1}^{N} \left(\frac{y_j - M(x,\vec{\theta})}{\sigma_j} \right)^2 \frac{\partial}{\partial \theta_i}$$ $\rightarrow$ usando regla de la Cadena.

Como:
$y_j = cte$
$x_j = cte$

$$\longrightarrow \frac{\partial \chi^2(\vec{\theta})}{\partial \theta_i} = \sum_{j=1}^{N} \underbrace{2 * \left(\frac{y_i - M(x_i,\vec{\theta})}{\sigma_i} \right)^1}_{\text{derivada externa}} \cdot \underbrace{- \frac{\partial M}{\partial \theta_i}}_{\text{derivada interna}}$$

Tomando propiedades de la suma.

$$\frac{\partial \chi^2(\vec{\theta})}{\partial \theta_i} = -2 \cdot \sum_{j=1}^{N} \left(\frac{y_i - M(x_i,\vec{\theta})}{(\sigma_i = 1)} \right)^1 \cdot \frac{\partial M(x_i,\vec{\theta})}{\partial \theta_i}$$

d)

Se sabe que $$\nabla_\theta M(x_i,\vec{\theta}) = \left[\frac{\partial M(x_i,\vec{\theta})}{\partial \theta_0}, \frac{\partial M(x_i,\vec{\theta})}{\partial \theta_1}, \frac{\partial M(x_i,\vec{\theta})}{\partial \theta_2} \right]$$

Si el descenso se define como $\vec{x}^1 = \vec{x}^0 - \gamma \nabla \vec{F}(\vec{x}_0)$

Pero: $\vec{x}^1 = \vec{\theta}^1$, $\vec{x}^0 = \vec{\theta}^0$, $\nabla \vec{F}(\vec{x}_0) = \nabla_\theta M(x_i,\theta)$

$$\therefore \quad \vec{\theta}^1 = \vec{\theta}^0 - \gamma \left[-2 \cdot \sum_{j=1}^{N} y_i - \left(M(x_i,\vec{\theta}) \cdot \frac{\partial M(x_i,\vec{\theta})}{\partial \theta_i} \right) \right]$$

Pero $\tilde{\vec{\theta}}^{1} = \tilde{\vec{\theta}}_{j+1}$ y $\tilde{\vec{\theta}}^{0} = \tilde{\vec{\theta}}_{j}$

$$\longrightarrow \tilde{\vec{\theta}}_{j+1} = \tilde{\vec{\theta}}_{j} - \gamma \left[-2 \cdot \sum_{j=1}^{N} y_i - \left(M(x_i, \vec{\theta}) \cdot \frac{\partial M(x_i, \vec{\theta})}{\partial \theta_i} \right) \right]$$

Como $\frac{\partial M(x_i, \vec{\theta})}{\partial \theta_i} \rightarrow$ es $\frac{\partial M(x_i, \vec{\theta})}{\partial \theta_0}, \frac{\partial M(x_i, \vec{\theta})}{\partial \theta_1}, \frac{\partial M(x_i, \vec{\theta})}{\partial \theta_2} = \nabla M$

$$\therefore \tilde{\vec{\theta}}_{j+1} = \tilde{\vec{\theta}}_{j} - \gamma \left[-2 \cdot \sum_{j=1}^{N} y_i - \left(M(x_i, \vec{\theta}) \cdot \nabla_{\theta} M(x_i, \vec{\theta}_i) \right) \right]$$