Demostrar :

$$\frac{d^2 f(x_i)}{dx^2} = \frac{f(x_{i+2}) - 2f(x_i) + f(x_{i-2})}{4h^2}$$

Se toman las series de taylor evaluadas en $x+h$ y $x-h$:

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2} f''(x) + \frac{h^3}{3!} f'''(x) + \cdots$$

$$f(x-h) = f(x) - hf'(x) + \frac{h^2}{2} f''(x) - \frac{h^3}{3!} f'''(x) \cdots$$

Si se suman ambas expresiones $f(x+h) + f(x-h)$:

$$f(x+h) = f(x) + \cancel{hf'(x)} + \frac{h^2}{2} f''(x) + \cancel{\frac{h^3}{3!} f'''(x)} + \cancel{\cdots}$$

$$f(x-h) = f(x) - \cancel{hf'(x)} + \frac{h^2}{2} f''(x) - \cancel{\frac{h^3}{3!} f'''(x)} \cancel{\cdots}$$

notamos que muchos factores se cancelan :

$$f(x+h) + f(x-h) = 2f(x) + \cancel{2}\frac{h^2}{\cancel{2}} f''(x)$$

$$f(x+h) + f(x-h) - 2f(x) = \cancel{2}\frac{h^2}{\cancel{2}} f''(x)$$

despejando la segunda derivada f'' :

$$\frac{f(x+h) + f(x-h) - 2f(x)}{h^2} = f''$$