Univariate and Mulitvariate Tests

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5.11 Test H_0: $\mu' = (6,11)$

Formula for multivariate test statistic T^2

$$T^2 = n(\bar{y} - \mu_0)' S^{-1} (\bar{y} - \mu_0)$$

```
# Make Y matrix from textbook
Y <- matrix(c(3,10,6,12,5,14,10,9), nrow = 4, ncol = 2, byrow = TRUE); Y
```

```
##      [,1] [,2]
## [1,]    3   10
## [2,]    6   12
## [3,]    5   14
## [4,]   10    9
```

```
# Number of rows and columns in Y
n <- nrow(Y)
p <- ncol(Y)
```

```
# Vector of the means
# The number 2 tells R to look at columns (1 for rows)
ybar <- apply(Y,2,mean); ybar
```

```
## [1]  6.00 11.25
```

```
# Mean to be tested
mu <- c(6,11); mu
```

```
## [1]  6 11
```

```
# Unbiased variance/covariance of Y matrix
S <- var(Y); S
```

```
##           [,1]      [,2]
## [1,]  8.666667 -2.666667
## [2,] -2.666667  4.916667
```

```
# Inverse of S
S_inv <- solve(S); S_inv
```

```
##             [,1]       [,2]
## [1,] 0.13849765 0.07511737
## [2,] 0.07511737 0.24413146
```

```
# Deviation vector
ydev <- (ybar - mu) ; ydev
```

```
## [1] 0.00 0.25
```

```
# Transpose of deviation vector
ydev_t <- t(ydev); ydev_t
```

```
##      [,1] [,2]
## [1,]    0 0.25
```

```
# Test statistic
Tsqr <- n*(ydev_t %*% S_inv %*% ydev); Tsqr
```

```
##             [,1]
## [1,] 0.06103286
```

```
# Critical value for $T^2$ -distribution
alpha <- 0.05
n <- 4
p <- 2
( Fcrit05 <- (((n-1)*p)/(n-p))*qf(1-alpha,p,n-p) )
```

```
## [1] 57
```

Our test statistic (T^2) is 0.061. This value is not greater than our critical value for alpha 0.05, therefore we do not reject the null hypothesis.

5.14 Use the ramus bone data in Table 3.7:

```
getwd()
```

```
## [1] "/Users/isabellachittumuri/Documents/Hunter College/Fall 2020/Stat 717/HW"
```

```
bonelength <- read.csv("Table 3.7.csv")
```

(a) Test H_0: μ = (48,49, 50,51)’

```
# Mean to be tested
mu_1 <- matrix(c(48,49,50,51), nrow = 4, ncol = 1, byrow = F); mu_1
```

```
##      [,1]
## [1,]   48
## [2,]   49
## [3,]   50
## [4,]   51
```

```r
# Individual variable means
ybar1 <- mean(bonelength$y1)
ybar2 <- mean(bonelength$y2)
ybar3 <- mean(bonelength$y3)
ybar4 <- mean(bonelength$y4)
```

```r
# Put all ybars into a 4x1 matrix
ybar_1<- matrix(c(ybar1, ybar2, ybar3, ybar4), nrow = 4, ncol = 1, byrow = F); ybar_1
```

```
##          [,1]
## [1,] 48.77895
## [2,] 49.73158
## [3,] 50.53158
## [4,] 51.43158
```

```r
# Covariance matrix of dataframe
S_bone <- cov(bonelength); S_bone
```

```
##          y1       y2       y3       y4
## y1 6.357310 6.254035 6.198480 5.904591
## y2 6.254035 6.567836 6.581725 6.293947
## y3 6.198480 6.581725 7.271170 7.317281
## y4 5.904591 6.293947 7.317281 7.872281
```


```r
# Covariance matrix of dataframe
S_bone <- cov(bonelength); S_bone
```

```
##          y1       y2       y3       y4
## y1 6.357310 6.254035 6.198480 5.904591
## y2 6.254035 6.567836 6.581725 6.293947
## y3 6.198480 6.581725 7.271170 7.317281
## y4 5.904591 6.293947 7.317281 7.872281
```


```r
# Inverse of S_bone
S_bone_inv <- solve(S_bone); S_bone_inv
```

```
##            y1        y2         y3         y4
## y1  2.5350448 -2.731365  0.4208607 -0.1088488
## y2 -2.7313653  5.463742 -4.3788838  1.7505238
## y3  0.4208607 -4.378884  8.3112293 -4.5399986
## y4 -0.1088488  1.750524 -4.5399986  3.0290392
```

```r
# Deviation vector
ydev_1 <- (ybar_1 - mu_1); ydev_1
```

```
##           [,1]
## [1,] 0.7789474
## [2,] 0.7315789
## [3,] 0.5315789
## [4,] 0.4315789
```

```
# Transpose of deviation vector
ydev_t_1 <- t(ybar_1 - mu_1); ydev_t_1
```

```
##           [,1]      [,2]      [,3]      [,4]
## [1,] 0.7789474 0.7315789 0.5315789 0.4315789
```

```
# Multivariate test statistic
Tsq <- 19*(ydev_t_1 %*% S_bone_inv %*% ydev_1); Tsq
```

```
##          [,1]
## [1,] 2.925034
```

```
# Critical value for $T^2$ -distribution
alpha <- 0.05
n <- 19
p <- 4
( Fcrit05 <- (((n-1)*p)/(n-p))*qf(1-alpha,p,n-p) )
```

```
## [1] 14.66673
```

Our multivariate test statistic is 2.925. This value is not greater than our critical value which is 14.667. Therefore, we do not reject the null hypothesis.

(b) If Ho is rejected, test each variable separately.

Formula for univariate test statistic t^2

$$t = \frac{\bar{y} - \mu_0}{s/\sqrt{n}} = \frac{\sqrt{n}(\bar{y} - \mu_0)}{s}$$

```
# Standard deviation (sd) of each variable
bone_sd <- apply(bonelength, 2, sd); bone_sd
```

```
##       y1       y2       y3       y4
## 2.521371 2.562779 2.696511 2.805758
```

```
# Call each sd separately
y1_sd <- bone_sd[1]; y1_sd
```

```
##       y1
## 2.521371
```

```r
y2_sd <- bone_sd[2]; y2_sd
```

```
##       y2
## 2.562779
```

```r
y3_sd <- bone_sd[3]; y3_sd
```

```
##       y3
## 2.696511
```

```r
y4_sd <- bone_sd[4]; y4_sd
```

```
##       y4
## 2.805758
```

```r
# Recall mu and ybars from part (a)
mu_1
```

```
##      [,1]
## [1,]   48
## [2,]   49
## [3,]   50
## [4,]   51
```

```r
ybar_1
```

```
##          [,1]
## [1,] 48.77895
## [2,] 49.73158
## [3,] 50.53158
## [4,] 51.43158
```

```r
# Univariate test statistic using values for corresponding variables
(48.77895 - 48)/(y1_sd/sqrt(19))
```

```
##       y1
## 1.346634
```

```r
(49.73158 - 49)/(y2_sd/sqrt(19))
```

```
##       y2
## 1.244307
```

```r
(50.53158- 50)/(y3_sd/sqrt(19))
```

```
##       y3
## 0.859297
```

```r
(51.43158- 51)/(y4_sd/sqrt(19))
```

```
##        y4
## 0.6704831
```

```r
# Critical value for $t^2$ -distribution (alpha=.05, n=19)
qt(.05, (19-1), lower.tail = F)
```

```
## [1] 1.734064
```

None of our univariate test statistic is greater than our critical value which is 1.734. Therefore this confirms our result in part (a), that we do not reject the null hypothesis.