MATLAB exercise
Estimation techniques

Overview: In this exercise, you will construct several estimators and compare the results. You will implement a Bayesian MMSE estimator, and an MLE estimator for a few different scenarios. Put your code in a MATLAB publisher file, you will be graded as follows:

80%: Technical Correctness of the code
10%: Quality of figures, meaning they should be annotated fully (legends, axes labels). You should also make judicious use of overlaid plots, or subplots, as appropriate. In general you should be striving to use only a few, well-annotated plots.
10%: Quality of comment in code. Your code should be well commented, with a few paragraphs describing your solution, referring to figures, as appropriate. The overall goal here is a single document that contains both your code and all necessary information so that someone with a general EE background could understand the document.

Scenario 1: Consider this system:

$$\boldsymbol{x} = h\boldsymbol{\theta} + \boldsymbol{v}$$

Where **v** is Gaussian with zero-mean, and variance σ^2, and h is a known parameter. Let h = .5 for this exercise.

a) If θ is assumed to be a Gaussian random variable with mean μ and variance σ_o^2, find the Bayes MMSE estimate of θ.
b) If θ is assumed to be a deterministic but unknown parameter, find the Maximum Likelihood Estimate.
c) Implement both estimators in MATLAB. Plot the convergence of the mean-squared error as a function of the number of measurements taken. In order to do this, you'll need to run many iterations of the estimator. Think carefully about what the difference between iterations and measurements is.
d) Additionally, run the Bayes MMSE estimator, but with some incorrect prior information on θ. To do this, keep the model the same, but draw θ from a different distribution. For one incorrect prior, use a uniform random variable. The choice of the second incorrect prior is up to you; feel free to experiment with several. What effect does choosing an incorrect prior have on the estimate?

Scenario 2:

For this scenario, you will re-derive chapter 1 of my Ph.D. thesis. Consider a BPSK signal that is corrupted by interference that is also BPSK modulated. Assume that all timing, phase, frequency synchronization has been performed, and that the interfering signal is in phase, and has equal power as the desired signal. However, the interfering signal is shorter than the desired signal. It begins at time t_1 and ends at time t_2, which are unknown parameters. Since nothing is known about the distribution of t_1 and t_2, a Maximum Likelihood estimation approach is appropriate. In this exercise, you will derive and implement a ML estimator for the parameters t_1 and t_2.

a) Make a mathematical model of this system. Assume that the interference is one contiguous block, and that $t_2 > t_1$. Also assume there is additive white Gaussian noise, and that the variance of the noise is known. Make a model and from that you can get the likelihood function. Hints: What does the signal look like when interference is present, and when it is not?
b) Implement a maximum likelihood estimator. As far as I know, an analytical solution to the ML estimator is not possible, so you need to do this computationally by exhaustively computing all possible outcomes and picking the most likely one. This can get rather intensive computationally wise, so use small values for your signal length. Say 100 symbols for the desired signal, and let the interference range from 10-80 symbols.
c) Summarize your results; include informative, enlightening plots that prove that your implementation works.

Scenario 3:

The data in the .mat file, data.mat, has been drawn from either an exponential distribution, or a Rayleigh distribution. Both distributions have a single parameter, λ.

a) Derive and implement max-likelihood estimates for both distributions of the parameter, λ.
b) Compute the max-likelihood estimates of the parameter. Which distribution do you think the data was drawn from? Justify your answer.