Cornerstone hypothesis tests: T and χ^2 !

Review

1. Functions
2. Z-Tests
3. Integrating Power, Z-Tests, and Functions

Today

1. One Sample T-Tests
2. The χ^2 test
3. Discussion: Are you a Neo-Fischerian?

Enter the Z-Test

$$Z = \frac{\bar{Y} - \mu}{\sigma / \sqrt{n}}$$

The Z-score compares a sample mean to an assumed (known) population mean.

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The Z-score compares a sample mean to an assumed (known) population mean.

We call it a Z-Score because we correct by the standard error based on a known POPULATION standard deviation.


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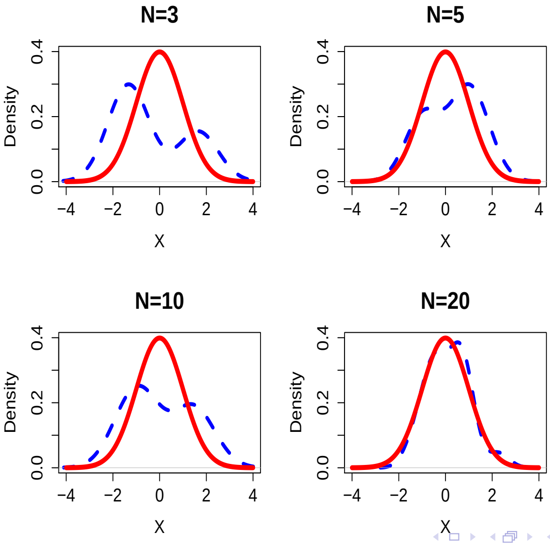
GUINNESS
DRAUGHT

William "Student" Gosset

- Quality control often works with small samples
- Small samples are seldom normal
- AND we rarely know the TRUE population SD

R. A. Fisher

Sample's Aren't Normal: Student's T



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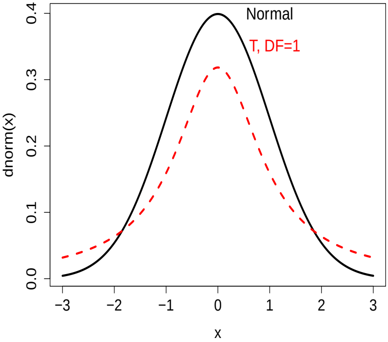

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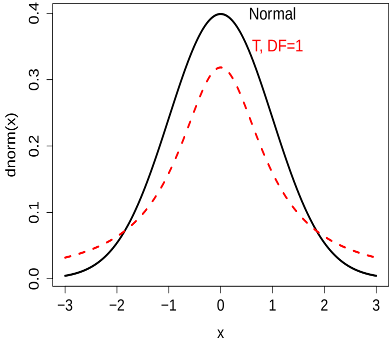

DF = Degrees of Freedom

= Sample Size - # of Estiamted Parameters

For t, this is n-1

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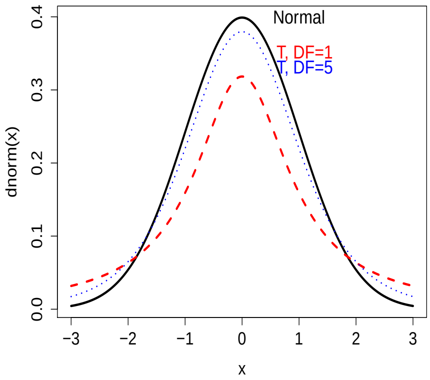


T Versus Z

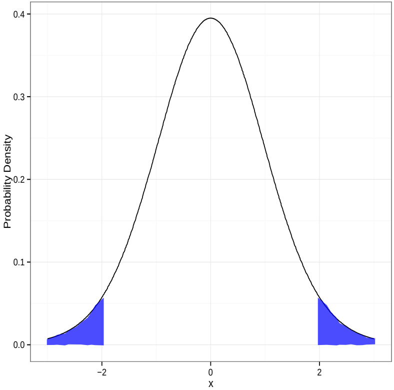
$$Z = \frac{\bar{Y} - \mu}{\sigma / \sqrt{n}}$$

$$T = \frac{\bar{Y} - \mu}{s / \sqrt{n}}$$

- For T, we calculate the sample standard error
- T is more useful if < 30 samples

Exercise: Gimme a T (function)

$$T = \frac{\bar{Y} - \mu}{SE_{\bar{Y}}}$$



Write a oneSampleT function using pt(). Write a SE function, too.
Challenge: make it return additional information of your choice.

Exercise: Gimme a T (function)

```
#SE first
se <- function(sample) sd(sample) / (sqrt(length(sample)))

#now T
oneSampleT <- function(sample, mu=0){
  t <- (mean(sample) - mu) / se(sample)

  2 * pt(abs(t), df=length(sample)-1, lower.tail=F)
}
```

Exercise: Gimme a T (function)

```
samp<-rnorm(50, 1)
t.test(samp)

#
#  One Sample t-test
#
# data:  samp
# t = 5.25, df = 49, p-value = 3.264e-06
# alternative hypothesis: true mean is not equal to 0
# 95 percent confidence interval:
#  0.4985 1.1167
# sample estimates:
# mean of x
#    0.8076

oneSampleT(samp)

# [1] 3.264e-06
```

Extensions of the One-Sample T-test

$$T = \frac{\bar{Y} - \mu}{s/\sqrt{n}}$$

- When values represent differences between paired treatments, this is a **paired T-test**
- Often used to assess whether a parameter is different from 0
- If $\bar{Y}$ is the difference between two means, s and n become pooled
- Pooling involves a variety of formulae accomodating differences in SE and N

Consider the χ^2

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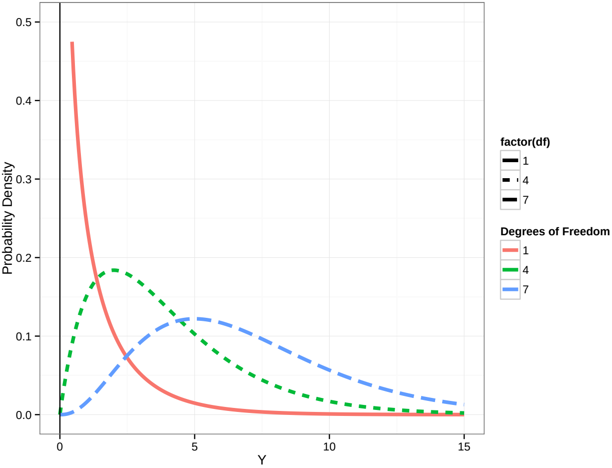
- Test **Goodness of fit** between observed and expected values
- If there is no difference, deviations should be normally distributed noise
- Summing up + and - differences is problematic - so we square!
- The square of a normal distribution is the χ^2 distribution!
- The χ^2 is defined by degrees of freedom = n-1!

Consider the χ^2

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$



When would you use a χ^2 test?

Assumptions of the χ^2

1. No expected values less that 1
2. 80% of the expected values must be $>$5

What happens if I violate the assumptions? Combine categories or use a different test.

Example of the χ^2

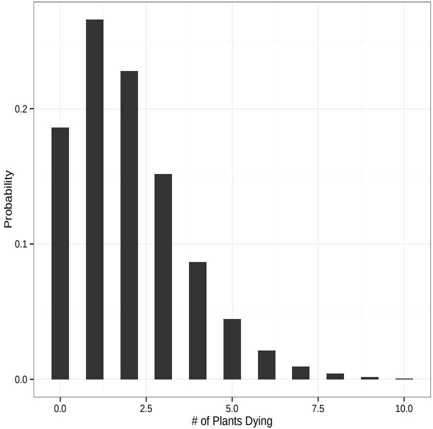
The number of tomato plants that failed due to disease in a farm should follow a negative binomial distribution in any given year with 2 plants dying and an overdispersion parameter of 5. Famer Dale seeded tomato plants in 100 different plots. Looking across his field, he finds the following pattern of mortality.

0 plants died: 9 plots
1 plants died: 23 plots
2 plants died: 22 plots
3 plants died: 18 plots
4 plants died: 19 plots
$>$4 plants died: 9 plots

Was this an anomolously bad year?

What are our Expectations?

Negative Binomial with a μ of 2 and dispersion parameter of 5.



Example of the χ^2

```
100*dnbinom(0:4, mu=2, size=5)

# [1] 18.593 26.562 22.767 15.178  8.673

100*(pnbinom(4, mu=2, size=5, lower.tail=FALSE))

# [1] 8.225
```

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100*dnbinom(0:4, mu=2, size=5)

# [1] 18.593 26.562 22.767 15.178  8.673

100*(pnbinom(4, mu=2, size=5, lower.tail=FALSE))

# [1] 8.225
```

0 plants died: 9 plots, 18.59 expected
1 plants died: 23 plots, 26,56 expected
2 plants died: 22 plots, 22.77 expected
3 plants died: 18 plots, 15.17 expected
4 plants died: 19 plots, 8.67 expected
$>$4 plants died: 9 plots, 8.23 expected

Example of the χ^2

```
observed <- c(9,23,22,18,19,9)

expected<- c(100*dnbinom(0:4, mu=2, size=5),
             100*(pnbinom(4, mu=2, size=5, lower.tail=FALSE)))

chisq<-sum((observed-expected)^2/expected)

chisq

# [1] 18.35

pchisq(chisq, df=6-1, lower.tail=FALSE)

# [1] 0.002543
```

Extensions of the χ^2 test

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

- Modification for small sample size
- Exact tests (e.g., Fisher's Exact) to look at exact predictions of probabilities
- Goodness of fit to evaluate whether we have a good model