



# Differences Between Two Groups

# Statistical Golems

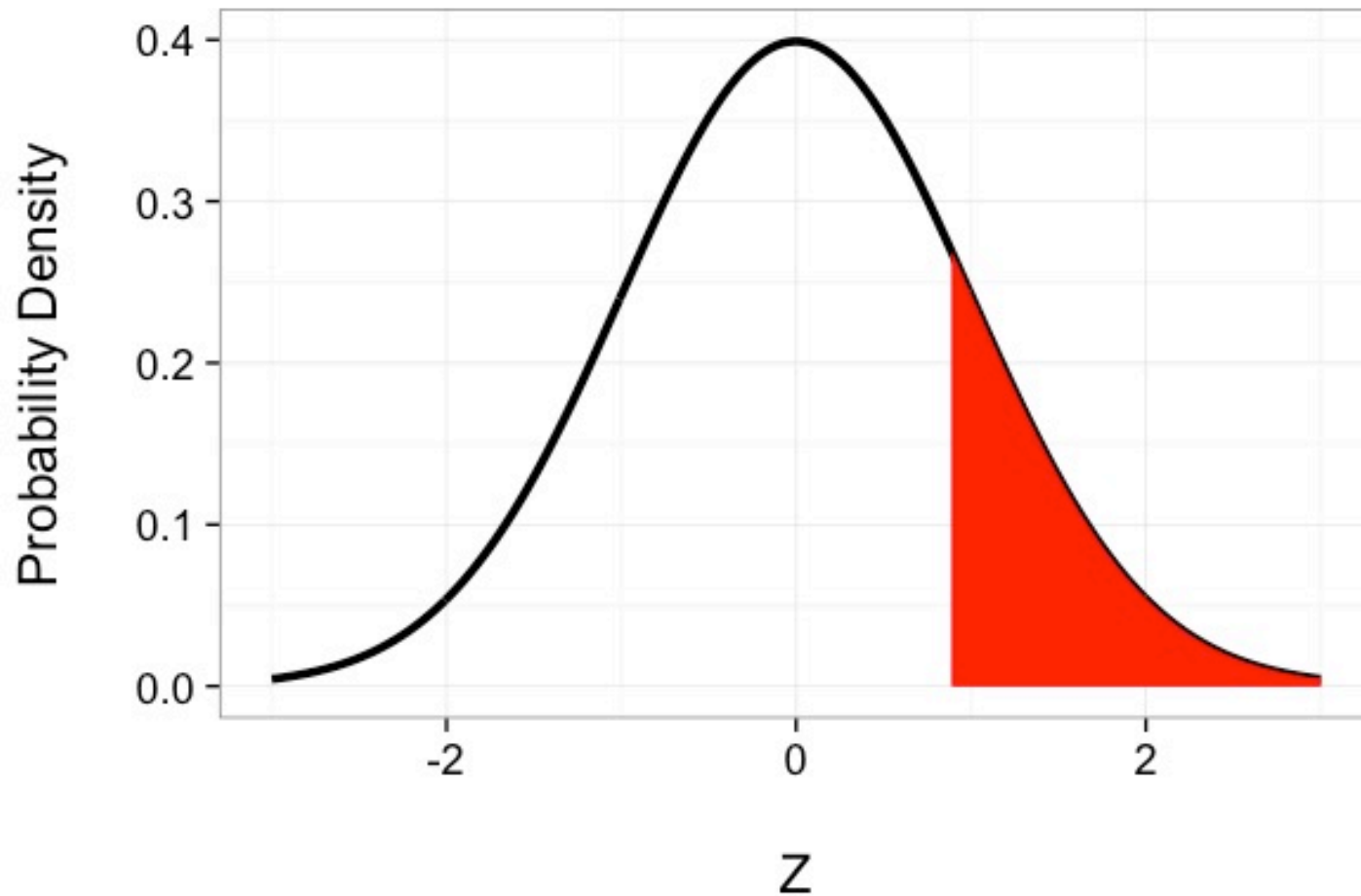
(sensu Richard McElreath)



ravenscar45

# This is a Golem

- What is my data generating process?
- What is my error generating process?



# What Drives My Golem?



Drawn from Data

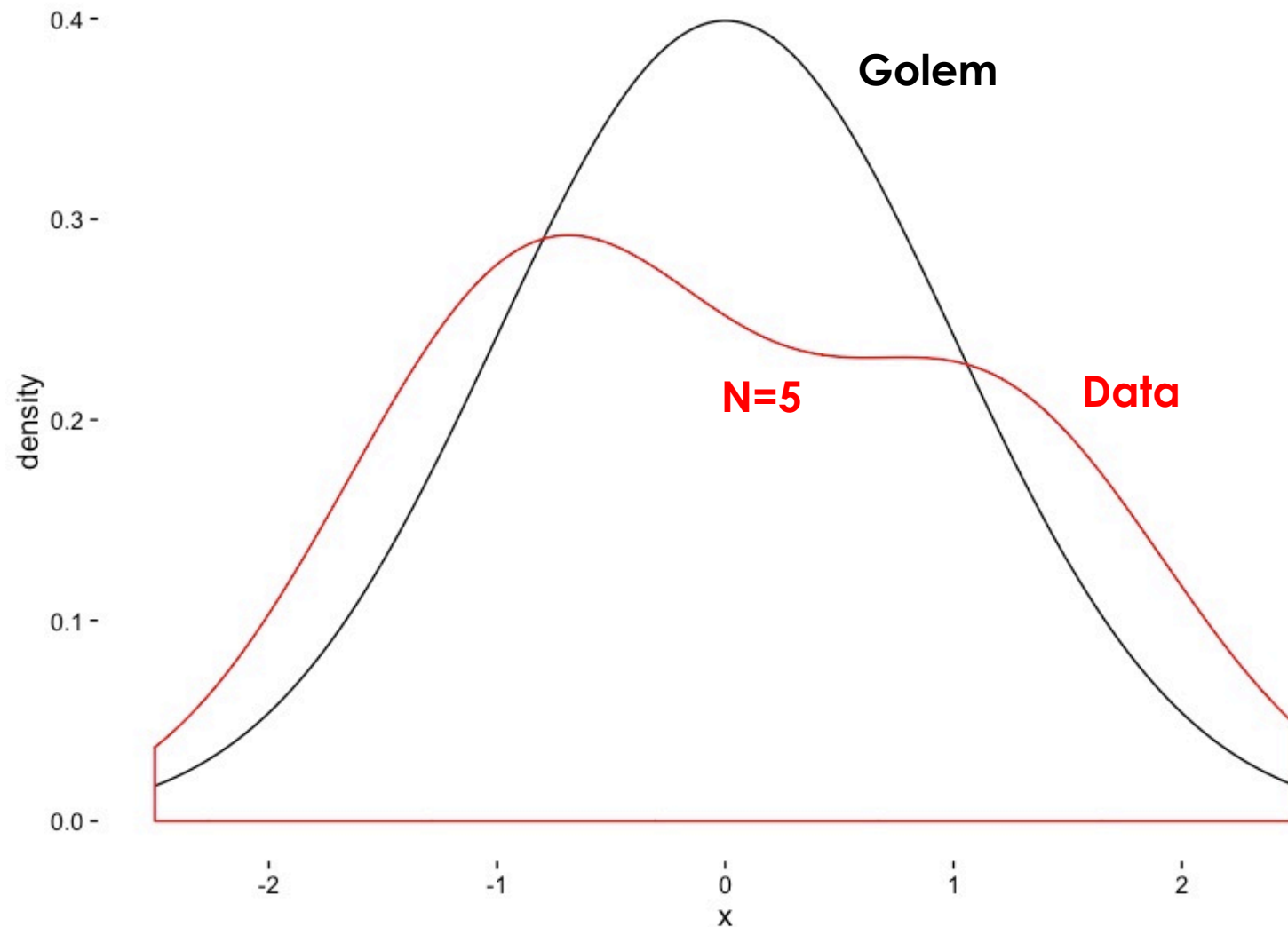
Assumption about Population

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

Assumption about Population

Drawn from Data

# Is this a Good Golem for Realistic Sample Sizes?







*Chemist & Statistician*

**WILLIAM SEALY  
GOSSET**

1876-1937

Chief Brewer

*Student 't' test*



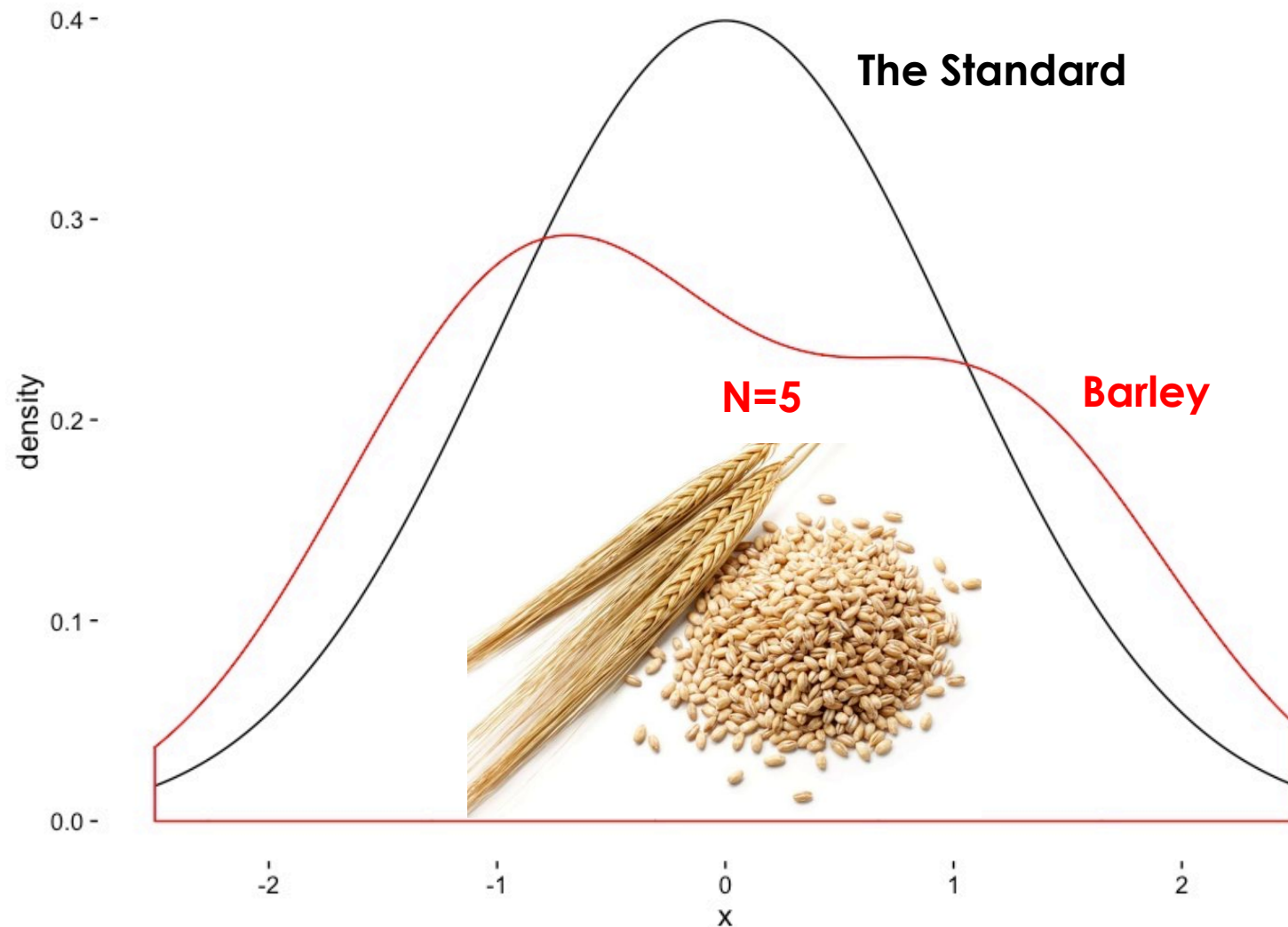
# INGREDIENTS

**To understand what makes GUINNESS special,  
you have to start with the raw ingredients.**

Water, barley, hops and yeast: four natural ingredients, carefully selected to ensure that they are of the highest quality. Each ingredient is special in its own right but when mixed together according to our secret recipe, the result is simply extraordinary.



# Does This Farm Produce Barley at the Right Moisture?



# What Does My Golem Know?

The diagram shows the t-test formula with four red arrows pointing to its components, each labeled with its source:

- An arrow from the top-left points to  $\bar{x}$  with the label "Drawn from Data".
- An arrow from the top-right points to  $\mu$  with the label "Assumption about Population".
- An arrow from the bottom-left points to  $s$  with the label "Drawn from Data".
- An arrow from the bottom-right points to  $n$  with the label "Drawn from Data".

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

Evaluate against T Distribution with  $n-1$  Degrees of Freedom

# T Versus N

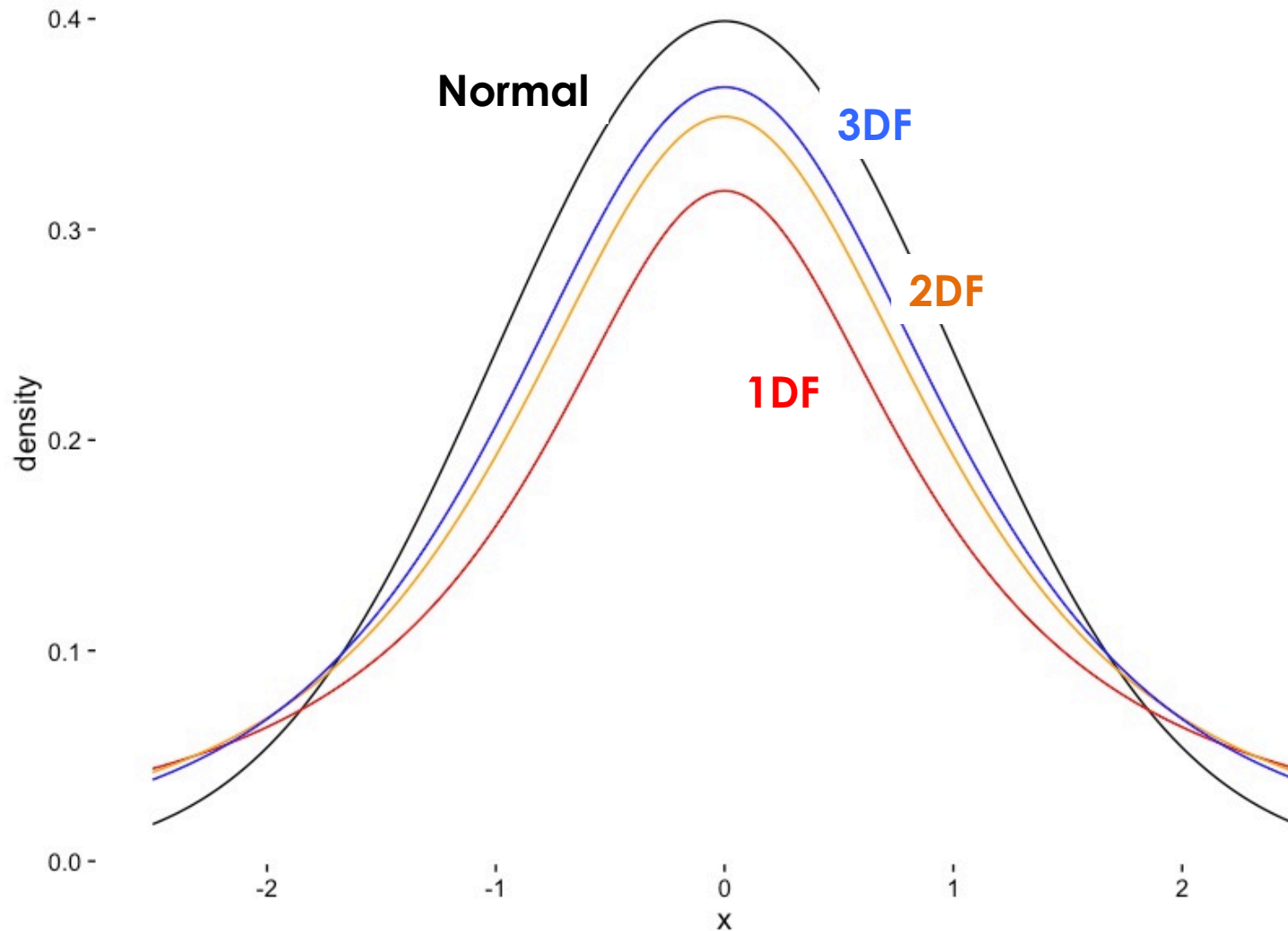
- A Normal Distribution is defined by a mean and a SD
- A T-Distribution assumes a mean of 0, a SD of 1, but changes shape based on its Degrees of Freedom



# Degrees of What?

- Let's say you **estimate** a mean
- $\text{Mean} = (x_1 + x_2 + x_3)/3$
- If you know the mean,  $x_1$ , and  $x_2$ , you can calculate  $x_3$
- How much unique information is there in calculating a parameter?

# T Distribution Versus Normal



# Comparing Paired Groups

$H_0$ : Difference = 0

Drawn from Data

$$t = \frac{\bar{x}_d}{s / \sqrt{n}}$$

Drawn from Data

Drawn from Data

The diagram illustrates the components of the t-statistic formula  $t = \frac{\bar{x}_d}{s / \sqrt{n}}$ . Three red arrows point from the text 'Drawn from Data' to the components of the formula: the sample mean  $\bar{x}_d$ , the sample standard deviation  $s$ , and the sample size  $n$ .

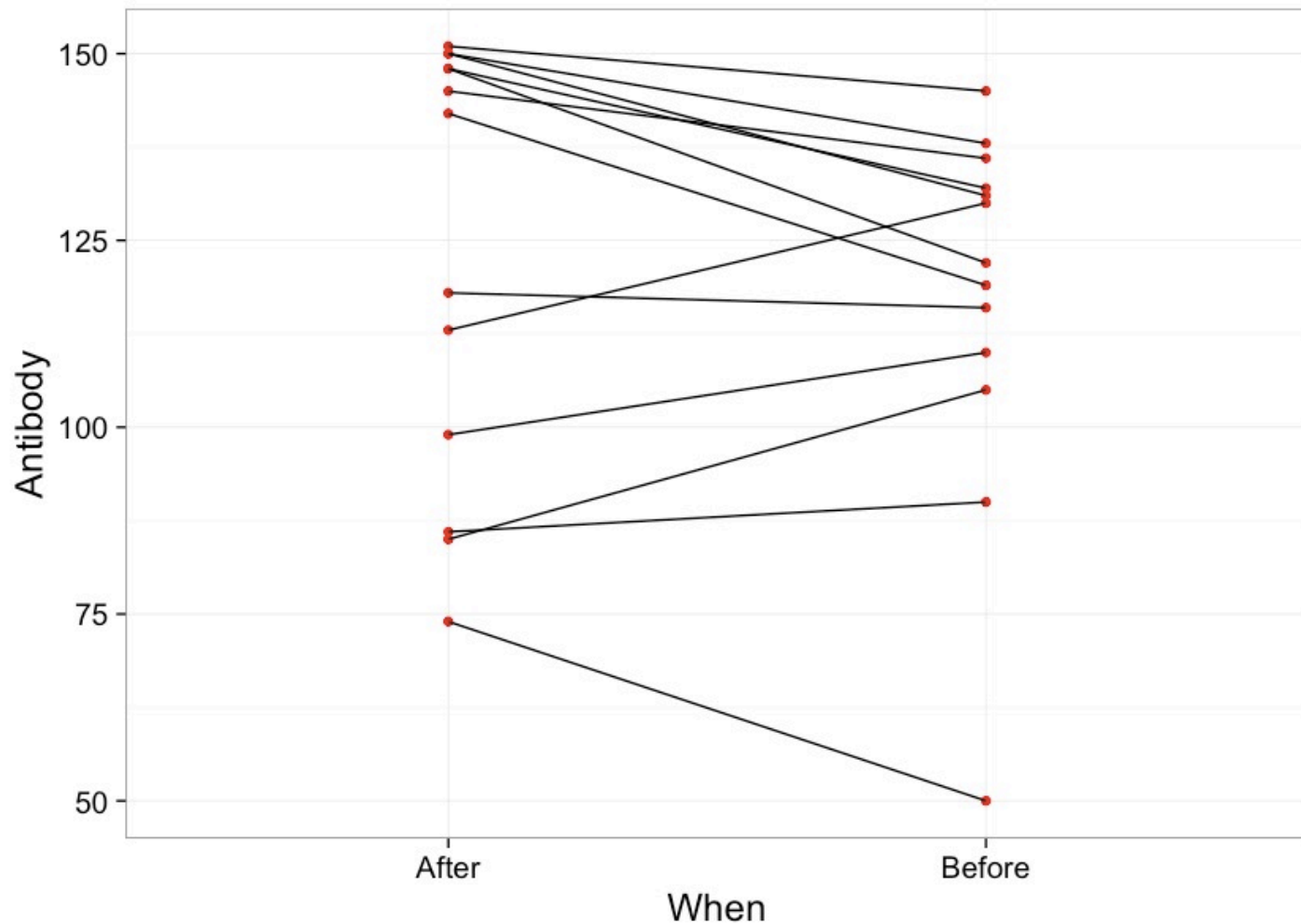
Evaluate against T Distribution with  $n-1$  Degrees of Freedom  
 $N$  is the sample size per group



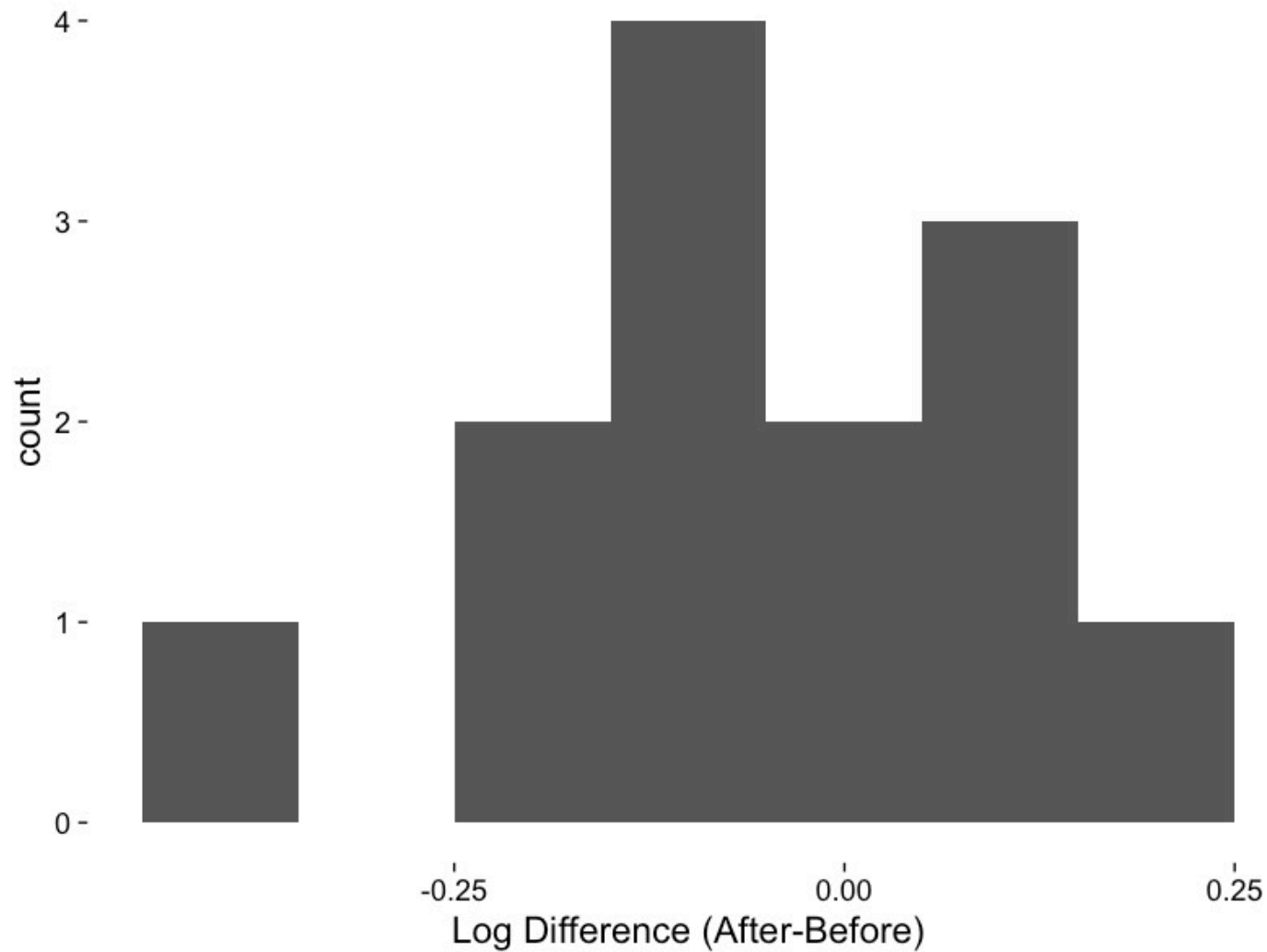
# Testosterone and Birds



# Differences in Antibody Performance

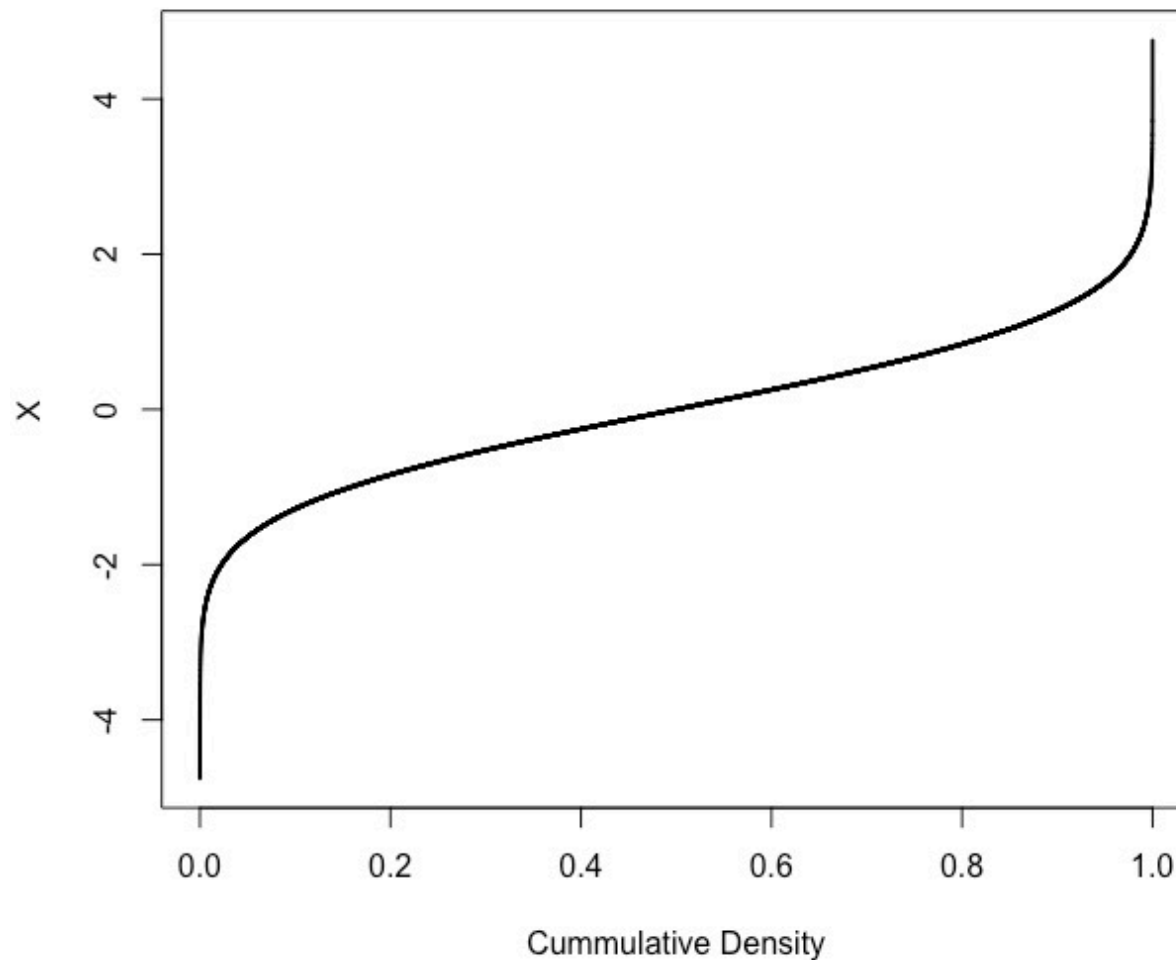


# Is the Log Difference Different from 0?





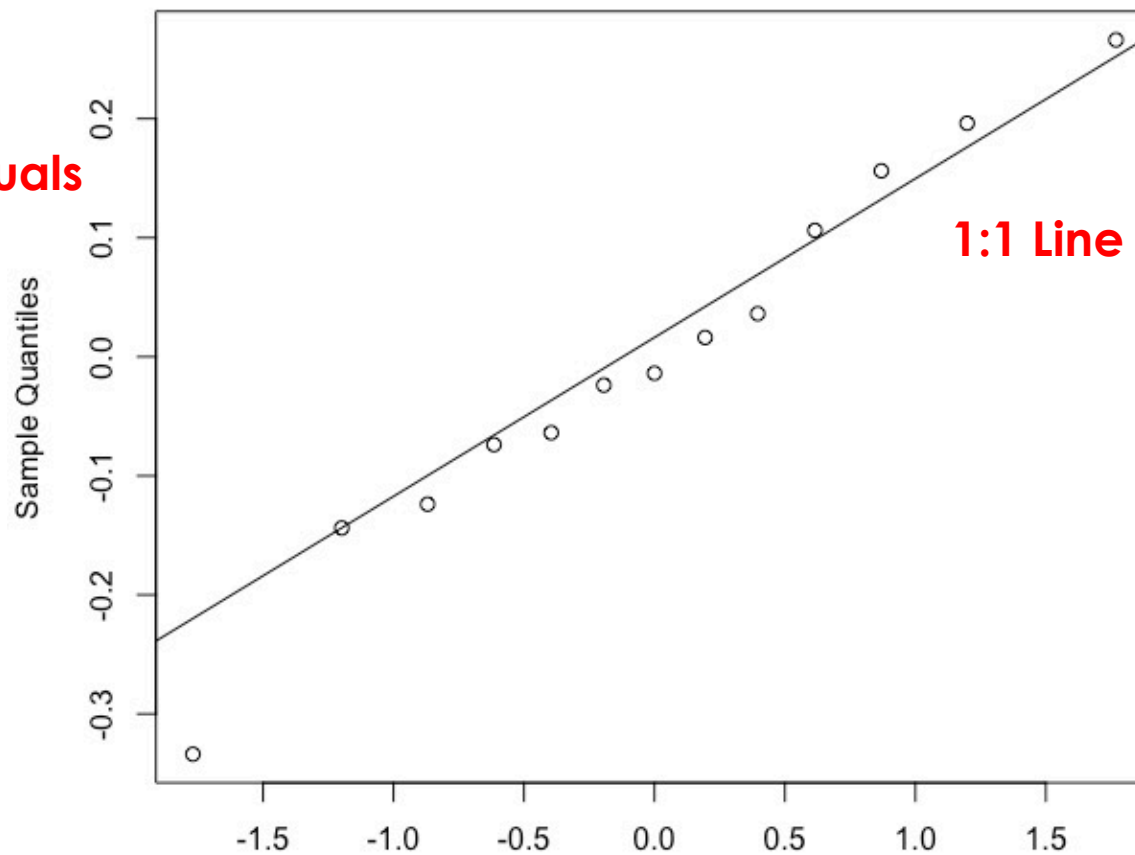
# One Way to Evaluate If Your Golem is Good: Quantiles



# QQ Plot to Evaluate If Residuals are Normal

Normal Q-Q Plot

From Residuals



1:1 Line

Theoretical Quantiles From a Normal Distribution

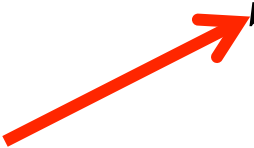
# General Testing Workflow

1. Build a Test
2. Evaluate Assumptions of Test
3. Evaluate Results
4. Visualize Results



# Comparing Groups

$H_0$ : Difference = 0

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_{12} / \sqrt{1/n}}$$


**Pooled Sample SD** =  $\sqrt{(s_1 + s_2)}$

Evaluate against T Distribution with  $n-1$  Degrees of Freedom

$N$  is the sample size per group

Assumes equal sample size and equal variance of populations

# Troubleshooting Your Golem

1. Unequal Sample Sizes
  - Alternate Formula for Denominator
2. Unequal Population Variances
  - Welch's T-Test (different denominator and DF)
3. Residuals Not Normal
  - Transform
  - Non-Parametric Test
  - Golem with a different error structure