



Convolutional Neural Networks

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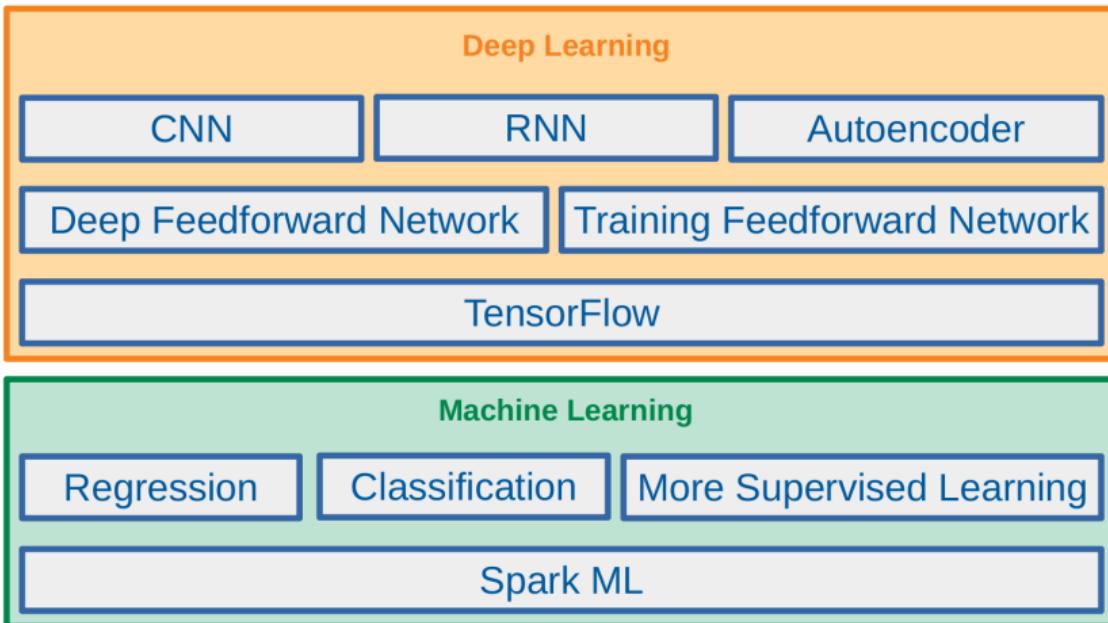


The Course Web Page

<https://id2223kth.github.io>

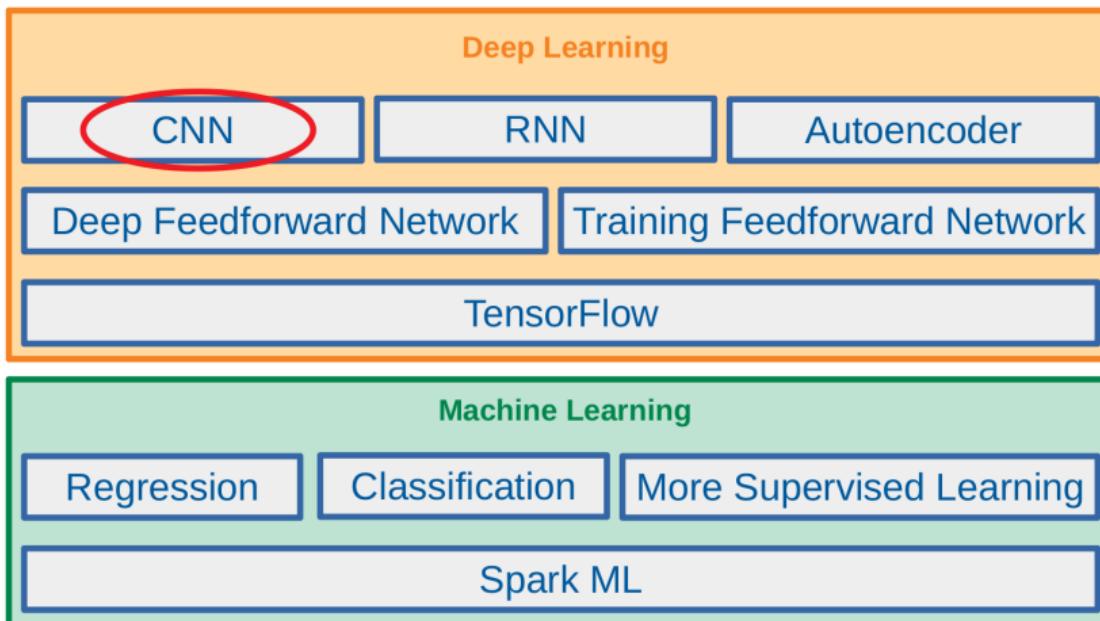


Where Are We?





Where Are We?





Let's Start With An Example



MNIST Dataset

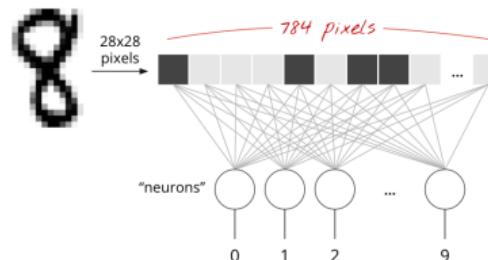
- ▶ Handwritten digits in the [MNIST](#) dataset are 28x28 pixel greyscale images.

A 10x10 grid of handwritten digits, likely from the MNIST dataset. The digits are arranged in rows, showing a variety of styles and orientations. The digits are black on a white background.

0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9	9

One-Layer Network For Classifying MNIST (1/4)

- ▶ Let's make a **one-layer** neural network for **classifying digits**.
- ▶ Each **neuron** in a neural network:
 - Does a **weighted sum** of all of its inputs
 - Adds a **bias**
 - Feeds the result through some **non-linear activation** function, e.g., **softmax**.



One-Layer Network For Classifying MNIST (2/4)



[<https://github.com/GoogleCloudPlatform/tensorflow-without-a-phd>]

One-Layer Network For Classifying MNIST (3/4)

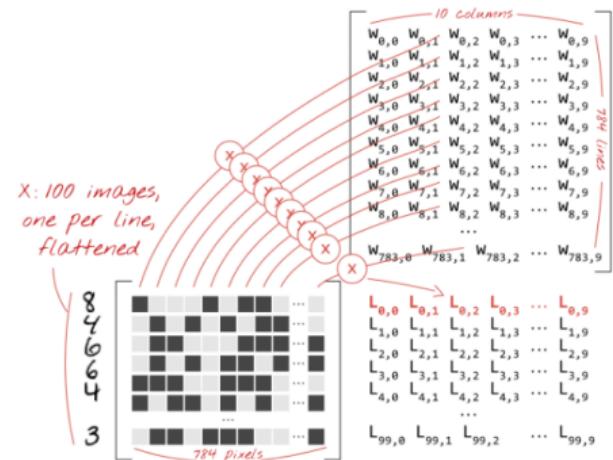
- ▶ Assume we have a **batch of 100 images** as the **input**.
- ▶ Using the **first column** of the **weights matrix \mathbf{W}** , we compute the **weighted sum** of all the **pixels** of the **first image**.
 - The **first neuron**:

$$L_{0,0} = w_{0,0}x_0^{(1)} + w_{1,0}x_1^{(1)} + \dots + w_{783,0}x_{783}^{(1)}$$
 - The **2nd neuron until the 10th**:

$$L_{0,1} = w_{0,1}x_0^{(1)} + w_{1,1}x_1^{(1)} + \dots + w_{783,1}x_{783}^{(1)}$$

$$\dots$$

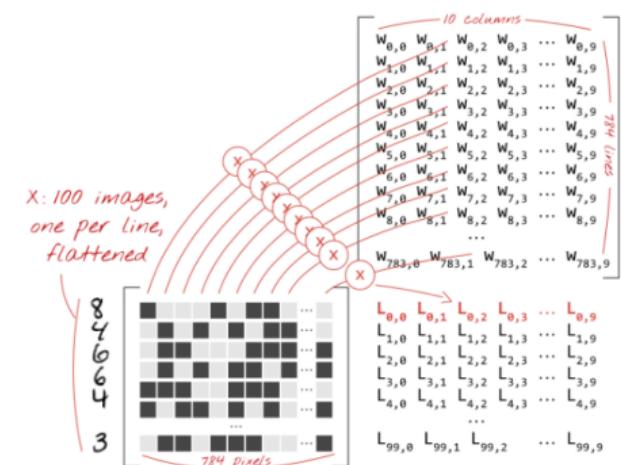
$$L_{0,9} = w_{0,9}x_0^{(1)} + w_{1,9}x_1^{(1)} + \dots + w_{783,9}x_{783}^{(1)}$$
 - Repeat the operation for the **other 99 images**, i.e., $\mathbf{x}^{(2)} \dots \mathbf{x}^{(100)}$



One-Layer Network For Classifying MNIST (4/4)

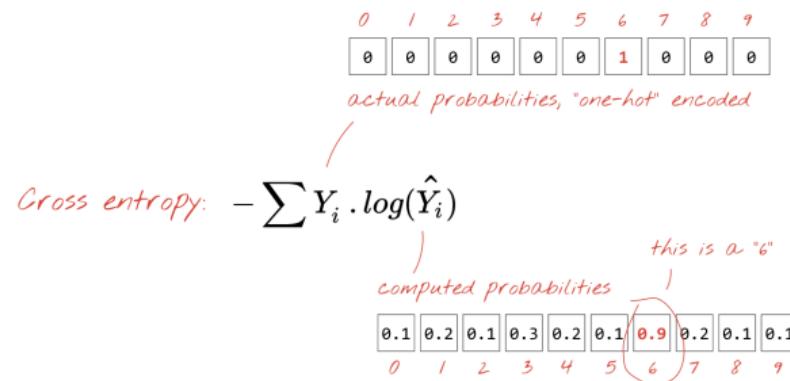
- ▶ Each neuron must now add its **bias**.
- ▶ Apply the **softmax activation function** for each instance $\mathbf{x}^{(i)}$.

- ▶ For each input instance $\mathbf{x}^{(i)}$: $\mathbf{L}_i = \begin{bmatrix} L_{i,0} \\ L_{i,1} \\ \vdots \\ L_{i,9} \end{bmatrix}$
- ▶ $\hat{\mathbf{y}}_i = \text{softmax}(\mathbf{L}_i + \mathbf{b})$



How Good the Predictions Are?

- ▶ Define the cost function $J(\mathbf{W})$ as the **cross-entropy** of what the network tells us $\hat{\mathbf{y}}_i$ and what we know to be the truth \mathbf{y}_i , for each instance $\mathbf{x}^{(i)}$.
- ▶ Compute the **partial derivatives** of the cross-entropy relatively to all the **weights** and all the **biases**, $\nabla_{\mathbf{W}} J(\mathbf{W})$.
- ▶ Update weights and biases by a **fraction of the gradient** $\mathbf{W}^{(\text{next})} = \mathbf{W} - \eta \nabla_{\mathbf{W}} J(\mathbf{W})$

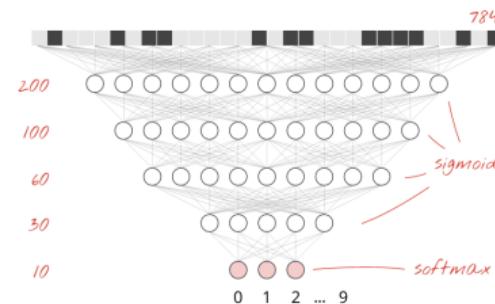


Adding More Layers

- ▶ Add more layers to **improve the accuracy**.
- ▶ On **intermediate layers** we will use the the **sigmoid** activation function.
- ▶ We keep **softmax** as the activation function on the **last layer**.



[<https://github.com/GoogleCloudPlatform/tensorflow-without-a-phd>]



Some Improvement

- ▶ Better activation function optimizer, e.g., using $\text{ReLU}(z) = \max(0, z)$.
- ▶ Overcome Network overfitting, e.g., using dropout.
- ▶ Network initialization. e.g., using He initialization.
- ▶ Better better optimizer, e.g., using Adam optimizer.

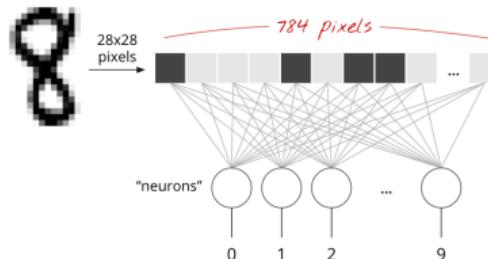


[<https://github.com/GoogleCloudPlatform/tensorflow-without-a-phd>]



Vanilla Deep Neural Networks Challenges (1/2)

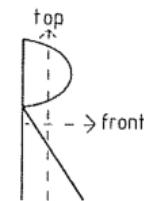
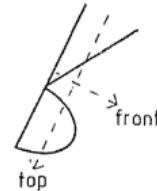
- ▶ Pixels of each image was flattened into a single vector (really **bad idea**).



- ▶ Vanilla deep neural networks **do not scale**.
 - In MNIST, images are black-and-white **28x28** pixel images: $28 \times 28 = 784$ weights.
- ▶ Handwritten digits are **made of shapes** and we **discarded the shape** information when we flattened the pixels.

Vanilla Deep Neural Networks Challenges (2/2)

- ▶ Difficult to **recognize** objects.
- ▶ **Rotation**
- ▶ **Lighting**: objects may **look different** depending on the level of **external lighting**.
- ▶ **Deformation**: objects can be deformed in a variety of **non-affine ways**.
- ▶ **Scale variation**: visual classes often exhibit **variation** in their size.
- ▶ **Viewpoint invariance**.





Tackle the Challenges

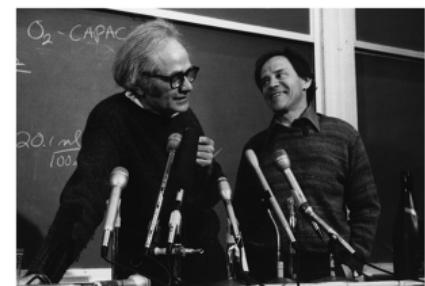
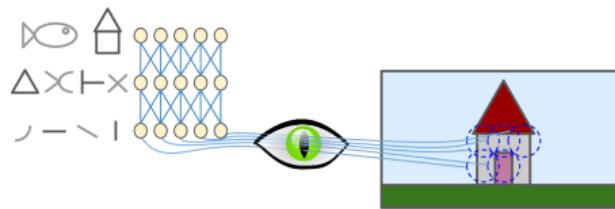
- ▶ Convolutional neural networks (CNN) can tackle the vanilla model challenges.
- ▶ CNN is a type of neural network that can take advantage of shape information.
- ▶ It applies a series of filters to the raw pixel data of an image to extract and learn higher-level features, which the model can then use for classification.



Filters and Convolution Operations

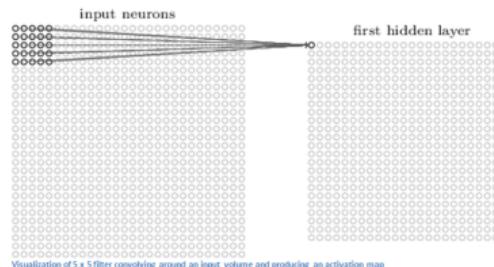
Brain Visual Cortex Inspired CNNs

- ▶ In 1959, David H. Hubel and Torsten Wiesel.
- ▶ Many **neurons in the visual cortex** have a **small local receptive field**.
- ▶ They **react** only to visual stimuli located in a **limited region of the visual field**.



Receptive Fields and Filters

- ▶ Imagine a **flashlight** that is shining over the top left of the image.
- ▶ The **region that it is shining over** is called the **receptive field**.
- ▶ This **flashlight** is called a **filter**.
- ▶ A filter is a **set of weights**.
- ▶ A **filter** is a **feature detector**, e.g., straight edges, simple colors, and curves.



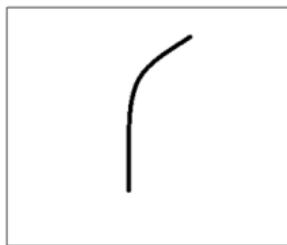
[<https://adshpande3.github.io/A-Beginner's-Guide-To-Understanding-Convolutional-Neural-Networks>]



Filters Example (1/3)

0	0	0	0	0	30	0	0
0	0	0	0	30	0	0	0
0	0	0	30	0	0	0	0
0	0	0	30	0	0	0	0
0	0	0	30	0	0	0	0
0	0	0	30	0	0	0	0
0	0	0	0	0	0	0	0

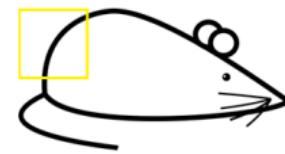
Pixel representation of filter



Visualization of a curve detector filter



Original image



Visualization of the filter on the image

[<https://adेशपांडे3.github.io/A-Beginner's-Guide-To-Understanding-Convolutional-Neural-Networks>]

Filters Example (2/3)



Visualization of the receptive field

0	0	0	0	0	0	30	0
0	0	0	0	50	50	50	0
0	0	0	20	50	0	0	0
0	0	0	50	50	0	0	0
0	0	0	50	50	0	0	0
0	0	0	50	50	0	0	0
0	0	0	50	50	0	0	0
0	0	0	50	50	0	0	0

Pixel representation of the receptive field

*

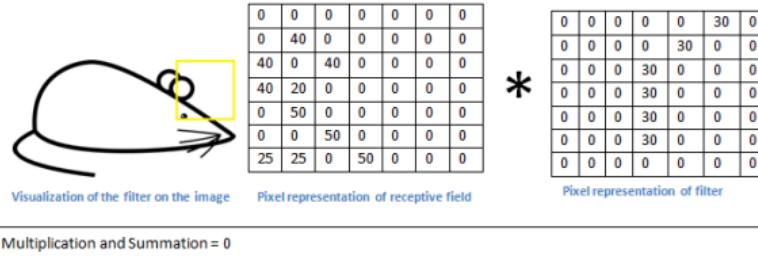
0	0	0	0	0	30	0
0	0	0	0	30	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	30	0	0	0
0	0	0	0	0	0	0

Pixel representation of filter

$$\text{Multiplication and Summation} = (50 \cdot 30) + (50 \cdot 30) + (50 \cdot 30) + (20 \cdot 30) + (50 \cdot 30) = 6600 \text{ (A large number!)}$$

[<https://adephande3.github.io/A-Beginner's-Guide-To-Understanding-Convolutional-Neural-Networks>]

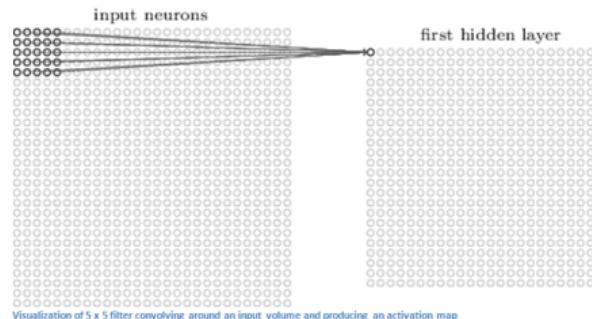
Filters Example (3/3)



[<https://adephante3.github.io/A-Beginner's-Guide-To-Understanding-Convolutional-Neural-Networks>]

Convolution Operation

- ▶ Convolution takes a **filter** and multiplying it over the entire area of an input image.
- ▶ Imagine this **flashlight (filter)** sliding across all the areas of the input image.



[<https://adephante3.github.io/A-Beginner's-Guide-To-Understanding-Convolutional-Neural-Networks>]



Convolution Operation - More Formal Definition

- ▶ Convolution is a mathematical operation on two functions x and h .
 - You can think of x as the input image, and h as a filter (kernel) on the input image.
- ▶ For a 1D convolution we can define it as below:

$$y(k) = \sum_{n=0}^{N-1} h(n) \cdot x(k-n)$$

- ▶ N is the number of elements in h .
- ▶ We are sliding the filter h over the input image x .



Convolution Operation - 1D Example (1/2)

- ▶ Suppose our input 1D image is x , and filter h are as follows:

$$x = \boxed{10 \quad 50 \quad 60 \quad 10 \quad 20 \quad 40 \quad 30}$$

$$h = \boxed{1/3 \quad 1/3 \quad 1/3}$$

- ▶ Let's call the output image y .
- ▶ What is the value of $y(3)$?

$$y(k) = \sum_{n=0}^{N-1} h(n) \cdot x(k-n)$$

Convolution Operation - 1D Example (2/2)

- To compute $y(3)$, we slide the filter so that it is centered around $x(3)$.

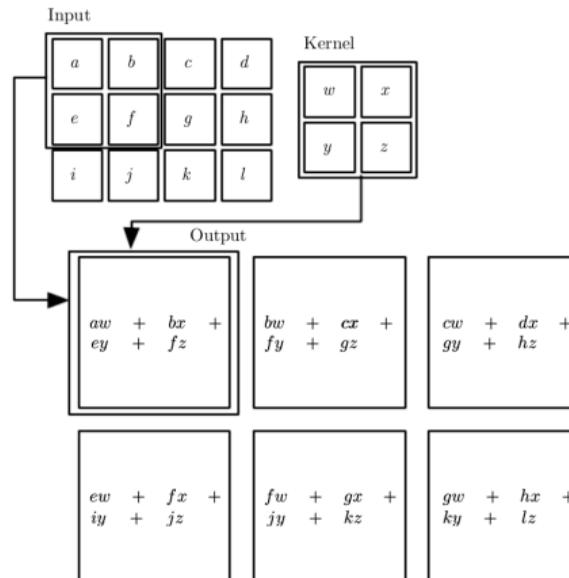
10	50	60	10	20	30	40
0	1/3	1/3	1/3	0	0	0

$$y(3) = \frac{1}{3}50 + \frac{1}{3}60 + \frac{1}{3}10 = 40$$

- We can compute the other values of y as well.

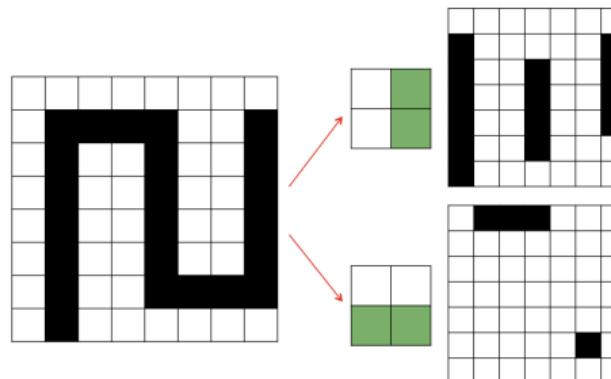
$$y = [20 \ 40 \ 40 \ 30 \ 20 \ 30 \ 23.333]$$

Convolution Operation - 2D Example (1/2)



Convolution Operation - 2D Example (2/2)

- ▶ Detect **vertical** and **horizontal lines** in an image.
- ▶ **Slide the filters** across the entirety of the image.
- ▶ The **result** is our **feature map**: indicates where we've found the **feature** we're looking for in the original image.

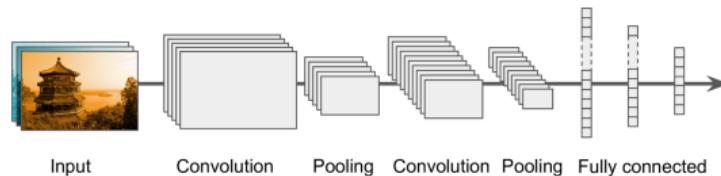




Convolutional Neural Network (CNN)

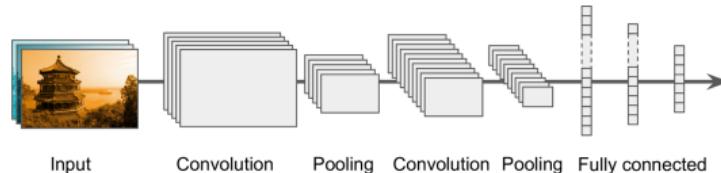
CNN Components (1/2)

- ▶ **Convolutional layers:** apply a specified number of **convolution filters** to the image.
- ▶ **Pooling layers:** **downsample the image** data extracted by the convolutional layers to **reduce the dimensionality** of the feature map in order to decrease processing time.
- ▶ **Dense layers:** a **fully connected layer** that performs **classification** on the features extracted by the convolutional layers and downsampled by the pooling layers.

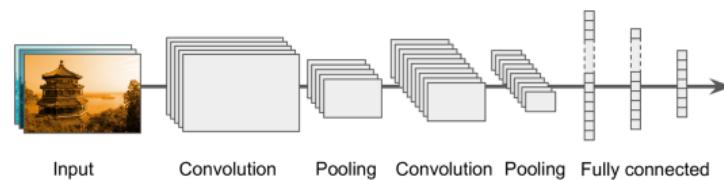


CNN Components (2/2)

- ▶ A **CNN** is composed of a **stack of convolutional modules**.
- ▶ Each **module** consists of a **convolutional layer** followed by a **pooling layer**.
- ▶ The **last module** is followed by **one or more dense layers** that perform **classification**.
- ▶ The **final dense layer** contains a **single node** for each target class in the model, with a **softmax** activation function.

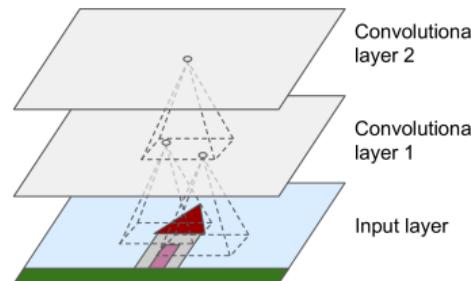


Convolutional Layer



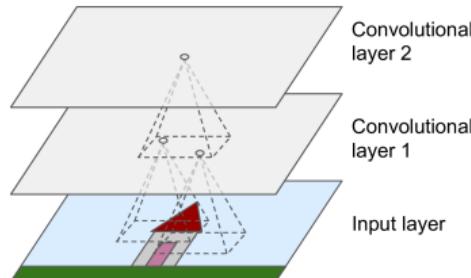
Convolutional Layer (1/4)

- ▶ Sparse interactions
- ▶ Each neuron in the convolutional layers are **only** connected to pixels in their **receptive fields** (not every single pixel).



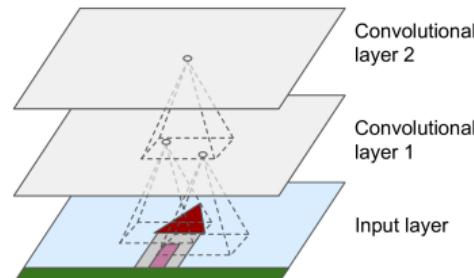
Convolutional Layer (2/4)

- ▶ Each neuron applies **filters** on its **receptive field**.
 - Calculates a **weighted sum** of the input pixels in the receptive fields
- ▶ Adds a **bias**, and feeds the result through its **activation function** to the next layer.
- ▶ The **output** of this layer is a **feature map (activation map)**



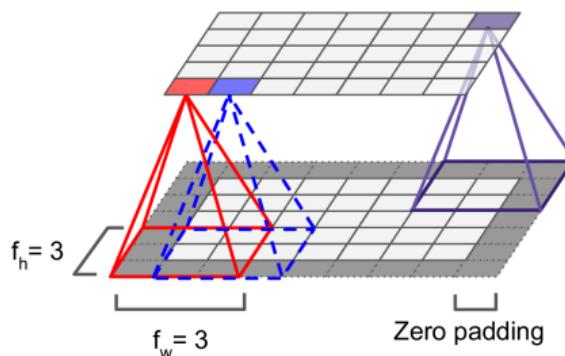
Convolutional Layer (3/4)

- ▶ Parameter sharing
- ▶ All neurons of a convolutional layer reuses the same weights.
- ▶ They apply the same filter in different positions.
- ▶ Whereas in a fully-connected network, each neuron had its own set of weights.



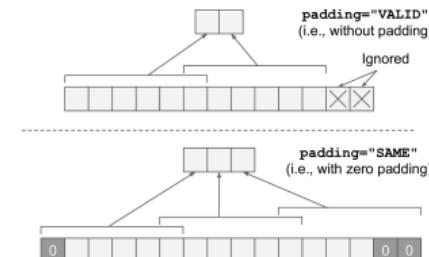
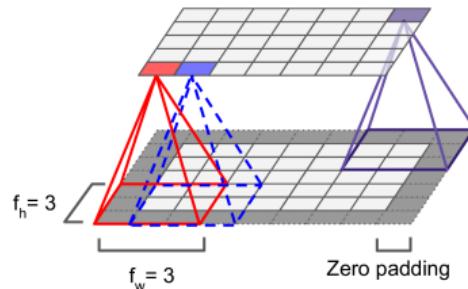
Convolutional Layer (4/4)

- ▶ Assume the **filter size (kernel size)** is $f_w \times f_h$.
 - f_h and f_w are the **height and width of the receptive field**, respectively.
- ▶ A neuron in **row i** and **column j** of a given layer is connected to the outputs of the neurons in the **previous layer** in **rows i to $i + f_h - 1$** , and **columns j to $j + f_w - 1$** .



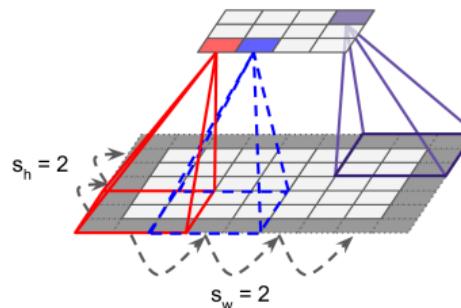
Padding

- ▶ What will happen if you apply a **5x5 filters** to a **32x32 input volume**?
 - The output volume would be **28x28**.
 - The spatial **dimensions decrease**.
- ▶ **Zero padding**: in order for a layer to have the **same height and width** as the previous layer, it is common to **add zeros around the inputs**.
- ▶ In **Tensorflow**, padding can be either **SAME** or **VALID** to have zero padding or not.

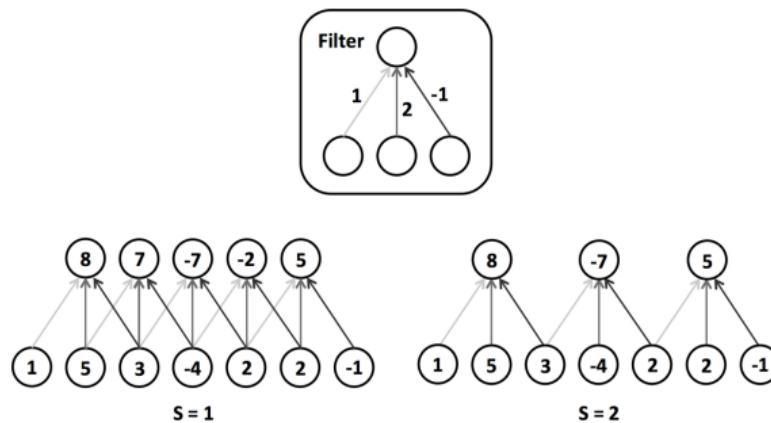


Stride (1/2)

- ▶ The **distance** between two consecutive receptive fields is called the **stride**.
- ▶ The stride controls **how the filter convolves** around the input volume.
- ▶ Assume s_h and s_w are the **vertical and horizontal strides**, then, a neuron located in **row i** and **column j** in a layer is connected to the outputs of the neurons in the **previous layer** located in **rows $i \times s_h$** to $i \times s_h + f_h - 1$, and **columns $j \times s_w$** to $j \times s_w + f_w - 1$.

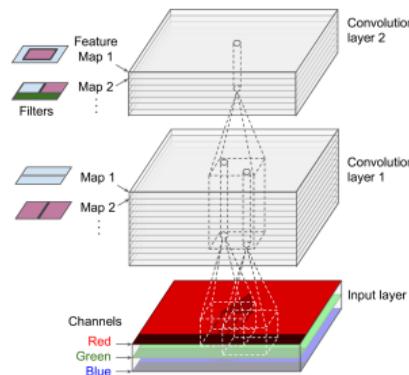


Stride (2/2)



Stacking Multiple Feature Maps

- ▶ Up to now, we represented each convolutional layer with a **single feature map**.
- ▶ Each convolutional layer can be composed of **several feature maps** of equal sizes.
- ▶ Input images are also composed of **multiple sublayers**: **one per color channel**.
- ▶ A **convolutional layer simultaneously applies multiple filters** to its inputs.

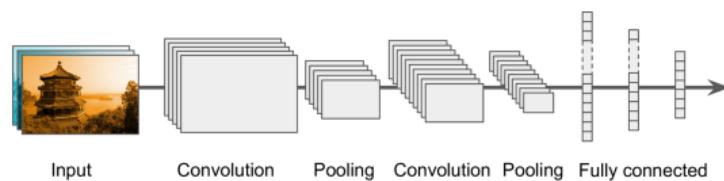




Activation Function

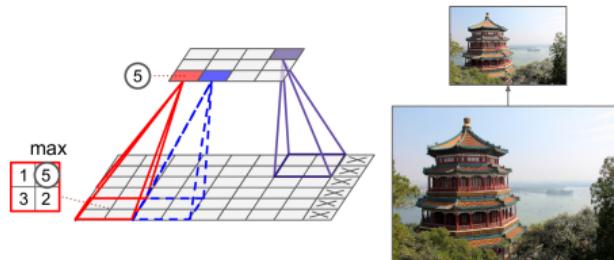
- ▶ After calculating a **weighted sum** of the input pixels in the **receptive fields**, and adding **biases**, each neuron feeds the result through its **ReLU activation function** to the next layer.
- ▶ The purpose of this activation function is to add **non-linearity** to a system.

Pooling Layer



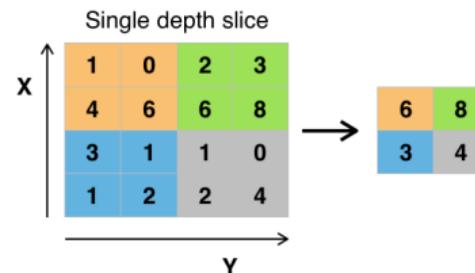
Pooling Layer (1/2)

- ▶ After the activation functions, we can apply a **pooling layer**.
- ▶ Its goal is to **subsample (shrink)** the input image.
 - To **reduce** the computational load, the memory usage, and the number of parameters.



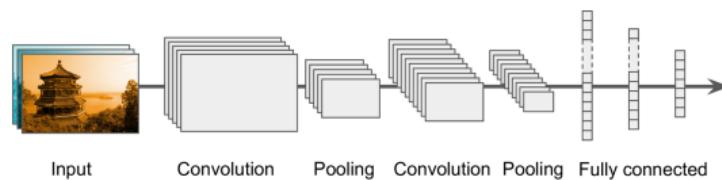
Pooling Layer (2/2)

- ▶ Each **neuron** in a pooling layer is connected to the outputs of a **receptive field** in the previous layer.
- ▶ A pooling neuron has **no weights**.
- ▶ It **aggregates the inputs** using an aggregation function such as the **max or mean**.



Example of Maxpool with a 2x2 filter and a stride of 2

Fully Connected Layer





Fully Connected Layer

- ▶ This layer takes an input from the **last convolution module**, and outputs an **N** dimensional vector.
 - **N** is the **number of classes** that the program has to choose from.
- ▶ For example, if you wanted a **digit classification** program, **N** would be 10.
- ▶ Each number in this **N** dimensional vector represents the **probability of a certain class**.



Flattening

- ▶ We need to **convert the output** of the convolutional part of the CNN into a **1D feature vector**.
- ▶ This operation is called **flattening**.
- ▶ It gets the **output of the convolutional layers**, **flattens** all its structure to create a **single long feature vector** to be used by the **dense layer** for the final classification.



Example

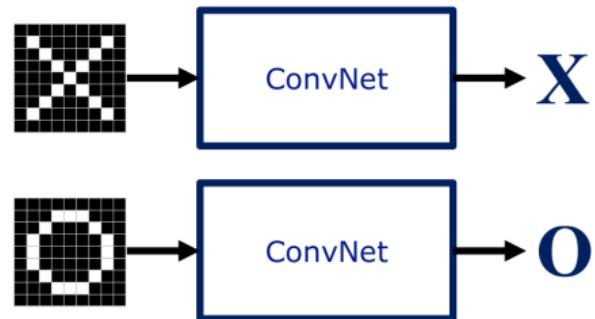


A Toy ConvNet: X's and O's

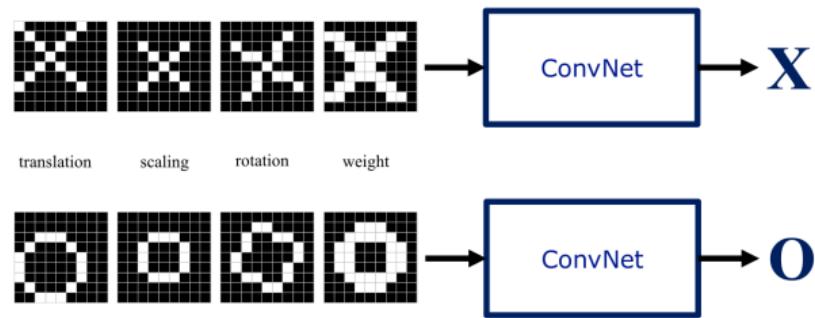
A two-dimensional
array of pixels



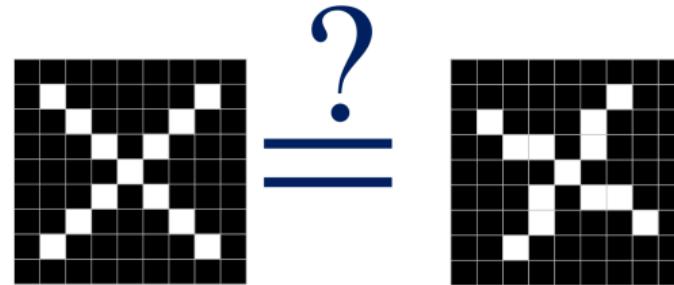
For Example



Trickier Cases



Deciding is Hard



What Computers See



-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	1	1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	1	-1
-1	1	-1	-1	-1	1	-1	-1	-1
-1	-1	-1	-1	-1	-1	1	-1	-1

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	-1	-1	-1	-1	-1	1	-1	-1
-1	1	-1	-1	-1	1	-1	-1	-1
-1	-1	1	1	-1	1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	1	-1
-1	-1	-1	1	-1	1	-1	1	-1
-1	1	-1	-1	-1	1	-1	-1	-1
-1	-1	-1	-1	-1	-1	1	-1	-1

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	X	-1	-1	-1	-1	X	X	-1
-1	X	X	-1	-1	X	X	-1	-1
-1	-1	X	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	1	-1	1	X	-1	-1
-1	-1	X	X	-1	-1	X	X	-1
-1	X	X	-1	-1	-1	X	-1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1

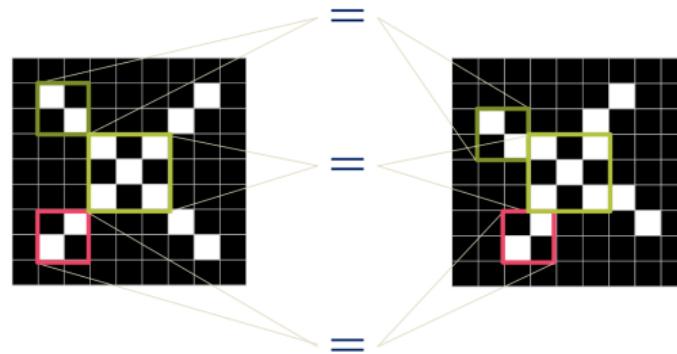
Computers are Literal

-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1	-1
-1	-1	1	-1	-1	-1	1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1	-1
-1	-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	-1	-1	-1	1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1	-1
-1	-1	-1	-1	-1	-1	-1	-1	1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1	1



-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	-1	-1	-1	-1	-1	-1	1	-1	-1
-1	1	-1	-1	-1	-1	1	-1	-1	-1
-1	-1	1	-1	-1	-1	1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1	-1
-1	-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	-1	1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	1	-1	-1	-1
-1	-1	-1	-1	-1	-1	-1	1	-1	-1
-1	-1	-1	-1	-1	-1	-1	-1	1	-1

ConvNets Match Pieces of the Image



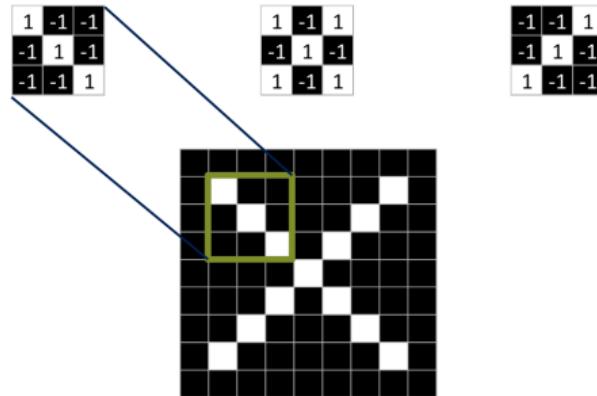
Filters Match Pieces of the Image

$$\begin{array}{|c|c|c|} \hline 1 & -1 & -1 \\ \hline -1 & 1 & -1 \\ \hline -1 & -1 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 1 & -1 & 1 \\ \hline -1 & 1 & -1 \\ \hline 1 & -1 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline -1 & -1 & 1 \\ \hline -1 & 1 & -1 \\ \hline 1 & -1 & -1 \\ \hline \end{array}$$

Filters Match Pieces of the Image

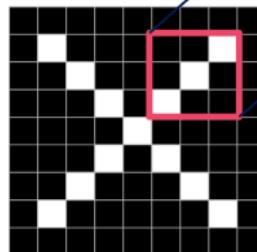


Filters Match Pieces of the Image

$$\begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & -1 \end{bmatrix}$$

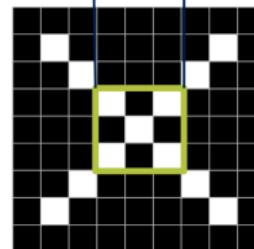


Filters Match Pieces of the Image

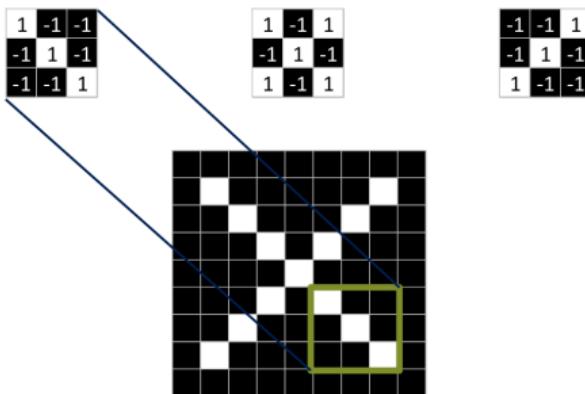
$$\begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & -1 \end{bmatrix}$$



Filters Match Pieces of the Image

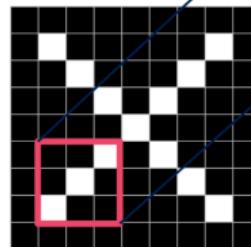


Filters Match Pieces of the Image

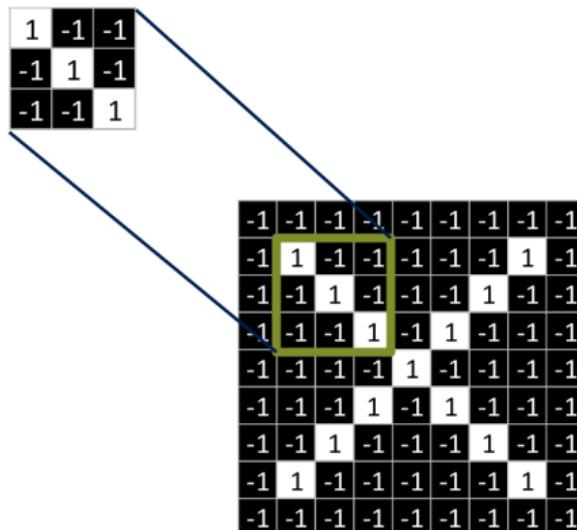
$$\begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

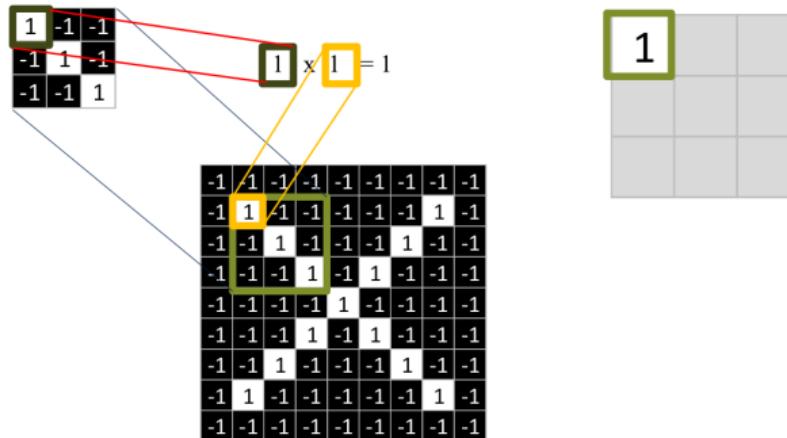
$$\begin{bmatrix} -1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & -1 \end{bmatrix}$$



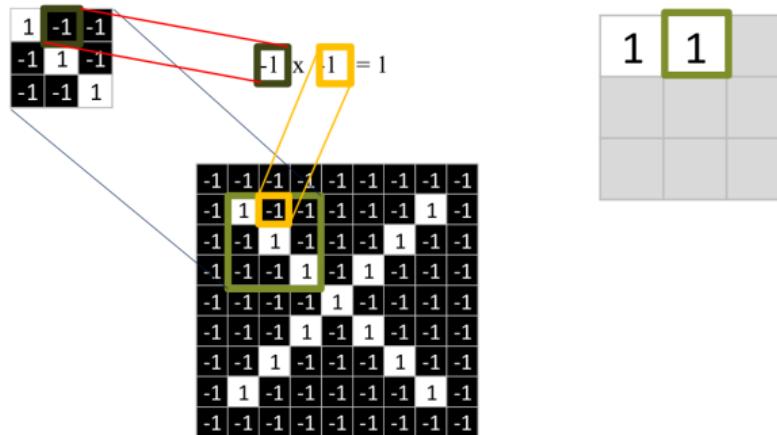
Filtering: The Math Behind the Match



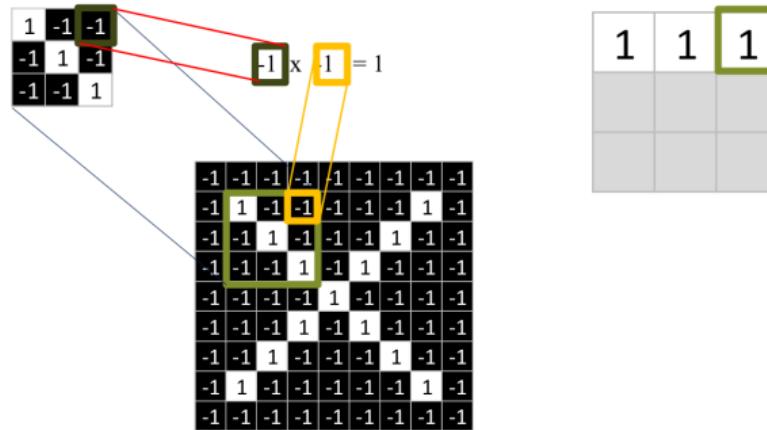
Filtering: The Math Behind the Match



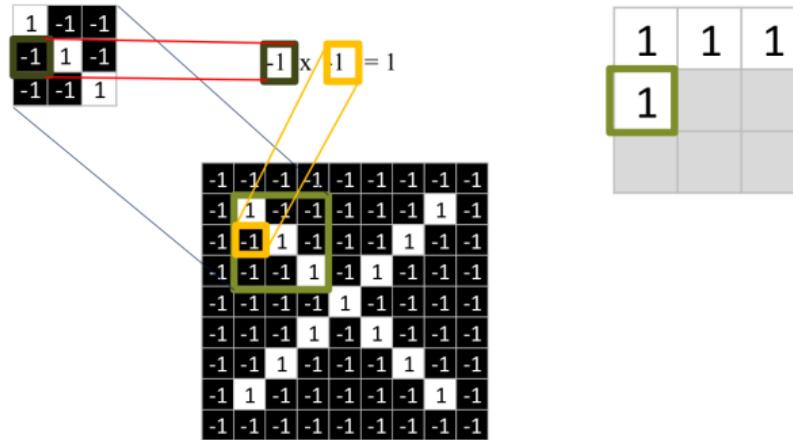
Filtering: The Math Behind the Match



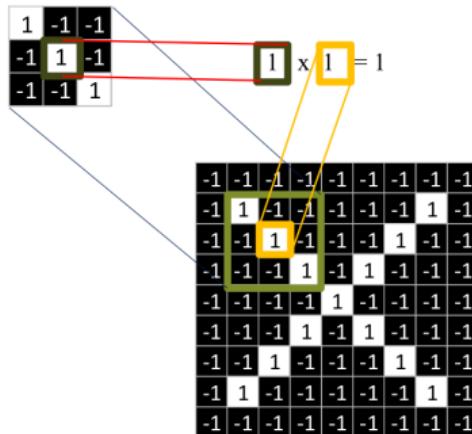
Filtering: The Math Behind the Match



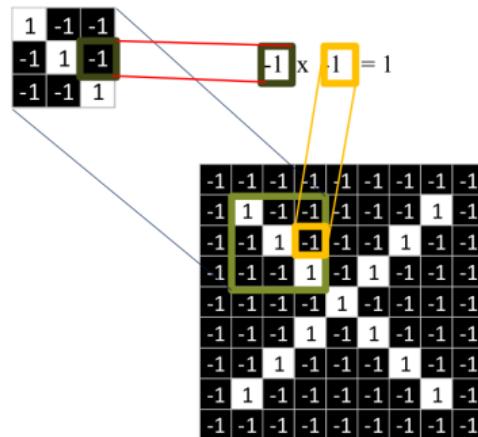
Filtering: The Math Behind the Match



Filtering: The Math Behind the Match

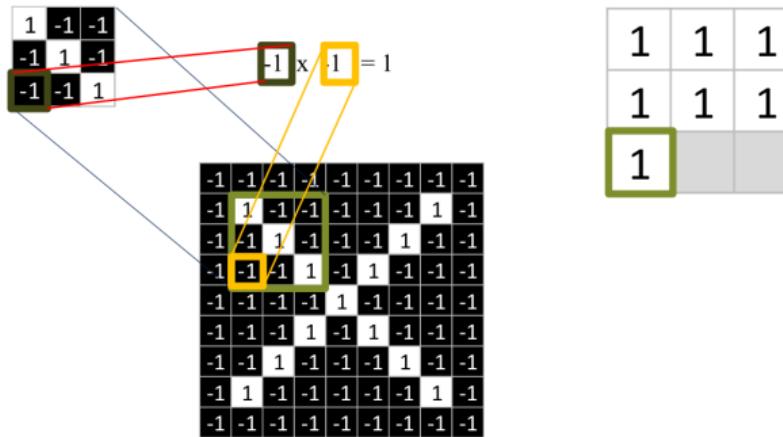


Filtering: The Math Behind the Match

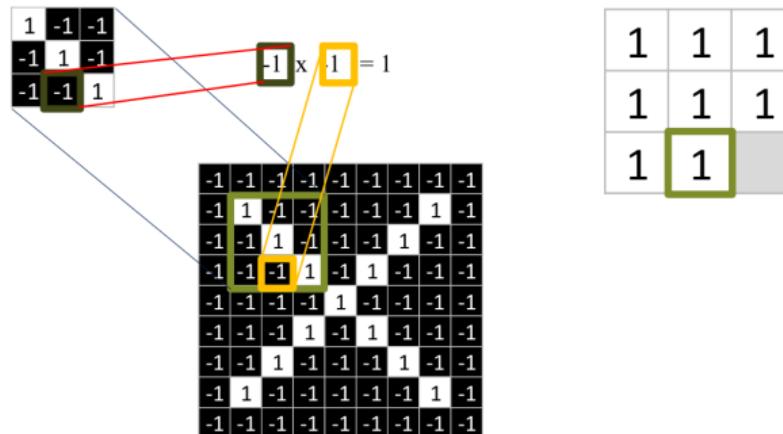


1	1	1
1	1	1
1	1	1

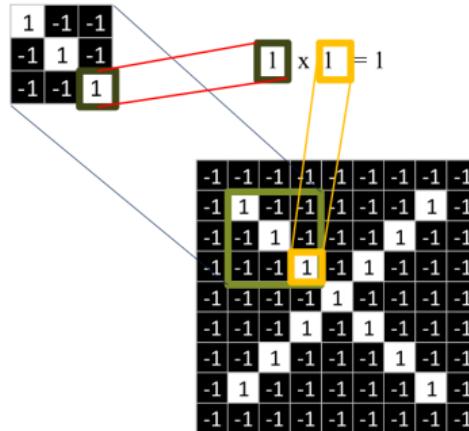
Filtering: The Math Behind the Match



Filtering: The Math Behind the Match



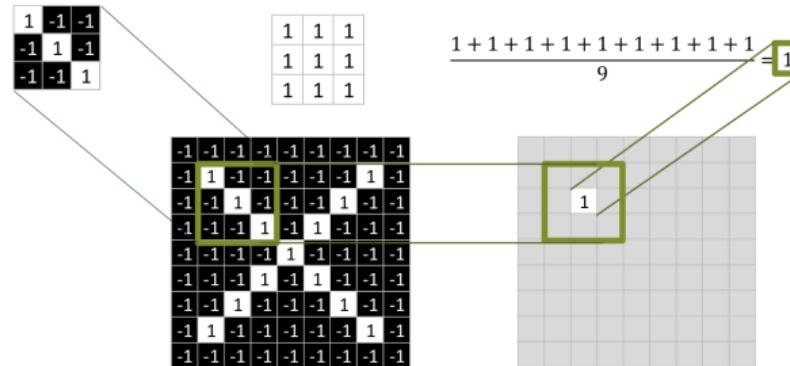
Filtering: The Math Behind the Match



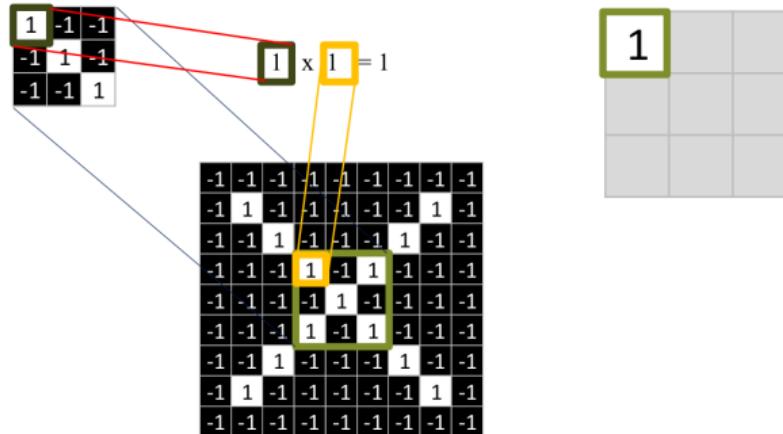
1	1	1
1	1	1
1	1	1



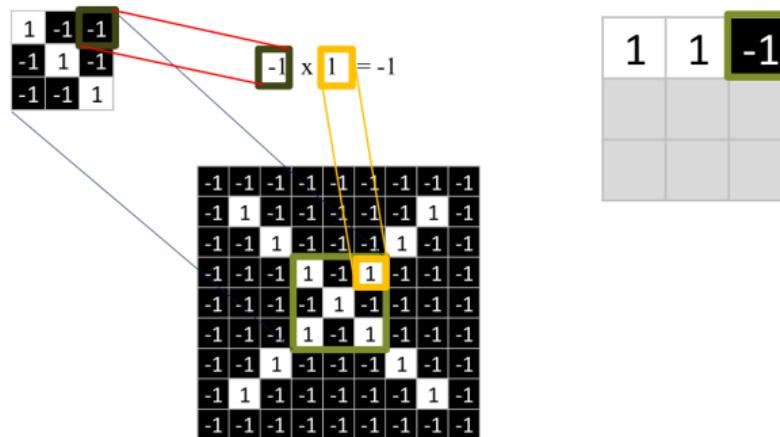
Filtering: The Math Behind the Match



Filtering: The Math Behind the Match

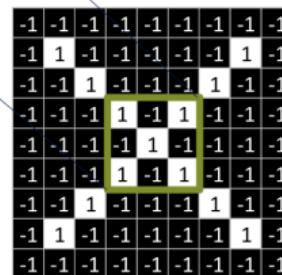


Filtering: The Math Behind the Match



Filtering: The Math Behind the Match

$$\begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$



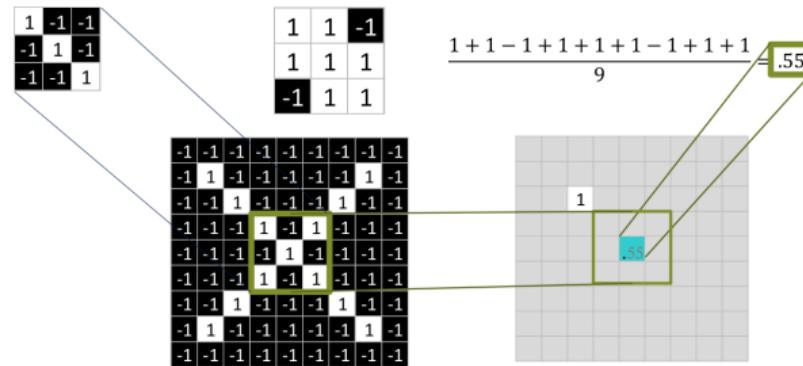
A diagram illustrating the application of a 3x3 kernel to a larger input matrix. The input matrix is a 13x13 grid of alternating 1s and -1s. A 3x3 subgrid in the center of the input is highlighted with a green border. Above the input matrix is a 3x3 kernel matrix:

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

An arrow points from the top-left cell of the kernel to the corresponding cell in the highlighted 3x3 subgrid of the input matrix.

$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

Filtering: The Math Behind the Match



Convolution: Trying Every Possible Match

$$\begin{array}{|c|c|c|c|c|c|c|c|c|} \hline -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ \hline -1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 \\ \hline -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 \\ \hline -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 \\ \hline -1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & -1 \\ \hline -1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & -1 \\ \hline -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 \\ \hline -1 & -1 & 1 & -1 & -1 & 1 & -1 & -1 & -1 \\ \hline -1 & 1 & -1 & -1 & -1 & -1 & 1 & -1 & -1 \\ \hline -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ \hline \end{array} \times \begin{array}{|c|c|c|} \hline 1 & -1 & -1 \\ \hline -1 & 1 & -1 \\ \hline -1 & -1 & 1 \\ \hline \end{array} = \begin{array}{|c|c|c|c|c|c|c|c|c|} \hline 0.77 & -0.11 & 0.11 & 0.33 & 0.55 & -0.11 & 0.33 \\ \hline -0.11 & 1.00 & -0.11 & 0.33 & -0.11 & 0.11 & -0.11 \\ \hline 0.11 & -0.11 & 1.00 & -0.33 & 0.11 & -0.11 & 0.55 \\ \hline 0.33 & 0.33 & -0.33 & 0.55 & -0.33 & 0.33 & 0.33 \\ \hline 0.55 & -0.11 & 0.11 & -0.33 & 1.00 & -0.11 & 0.11 \\ \hline -0.11 & 0.11 & -0.11 & 0.33 & -0.11 & 1.00 & -0.11 \\ \hline 0.33 & -0.11 & 0.55 & 0.33 & 0.11 & -0.11 & 0.77 \\ \hline \end{array}$$

Three Filters Here, So Three Images Out

$$\begin{bmatrix} -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 & -1 & 1 & -1 \\ -1 & 1 & -1 & -1 & 1 & 1 & -1 \\ -1 & -1 & 1 & 1 & -1 & 1 & -1 \\ -1 & -1 & -1 & 1 & -1 & -1 & -1 \\ -1 & -1 & 1 & 1 & 1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 1 & 1 & -1 \\ -1 & 1 & -1 & -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix}$$



$$\begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

=

$$\begin{bmatrix} 0.77 & -0.11 & 0.11 & 0.33 & 0.55 & -0.11 & 0.33 \\ -0.11 & 1.00 & -0.11 & 0.33 & -0.11 & 0.11 & -0.11 \\ 0.11 & -0.11 & 1.00 & 0.33 & 0.33 & -0.11 & 0.33 \\ 0.33 & 0.11 & -0.33 & 0.55 & -0.33 & 0.33 & 0.33 \\ 0.55 & -0.11 & 0.11 & 0.33 & 1.00 & -0.11 & 0.11 \\ -0.11 & 0.11 & -0.11 & -0.33 & -0.11 & 1.00 & -0.11 \\ 0.33 & -0.11 & 0.55 & 0.33 & 0.33 & -0.11 & 0.77 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 & -1 & 1 & -1 \\ -1 & 1 & -1 & -1 & 1 & 1 & -1 \\ -1 & -1 & 1 & -1 & 1 & -1 & -1 \\ -1 & -1 & -1 & 1 & -1 & -1 & -1 \\ -1 & -1 & 1 & 1 & 1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & 1 & -1 \\ -1 & 1 & -1 & -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix}$$



$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

=

$$\begin{bmatrix} 0.33 & -0.55 & 0.11 & 0.11 & -0.55 & 0.33 \\ -0.55 & 0.55 & -0.55 & 0.33 & -0.55 & 0.55 & -0.55 \\ 0.11 & -0.55 & 0.55 & 0.77 & 0.55 & -0.55 & 0.11 \\ -0.11 & 0.33 & -0.77 & 1.00 & 0.77 & 0.33 & -0.33 \\ 0.11 & -0.55 & 0.55 & -0.77 & 0.55 & -0.55 & 0.11 \\ -0.55 & 0.55 & -0.55 & 0.11 & -0.55 & 0.55 & -0.55 \\ 0.33 & -0.55 & 0.11 & 0.11 & -0.55 & 0.33 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 & -1 & 1 & -1 \\ -1 & 1 & -1 & -1 & 1 & 1 & -1 \\ -1 & -1 & 1 & 1 & 1 & -1 & -1 \\ -1 & -1 & -1 & 1 & -1 & 1 & -1 \\ -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ -1 & 1 & -1 & -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix}$$



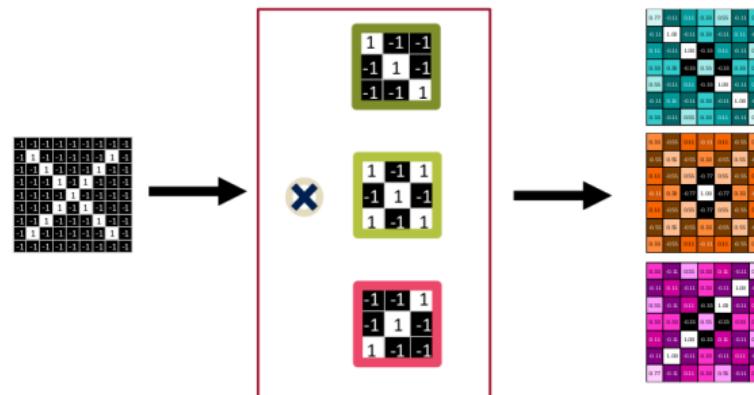
$$\begin{bmatrix} -1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & -1 \end{bmatrix}$$

=

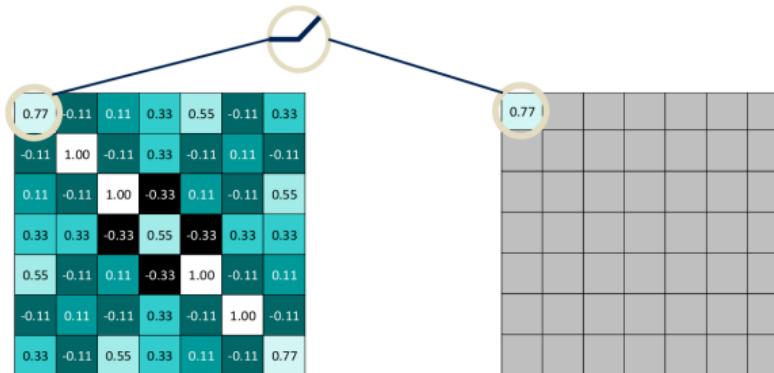
$$\begin{bmatrix} 0.33 & -0.11 & 0.55 & 0.33 & 0.11 & -0.11 & 0.77 \\ -0.11 & 0.11 & -0.11 & 0.33 & 0.11 & 1.00 & 0.11 \\ 0.55 & -0.11 & 0.11 & 0.33 & 1.00 & -0.11 & 0.11 \\ 0.33 & 0.11 & -0.33 & 0.55 & 0.33 & 0.33 & 0.33 \\ 0.11 & -0.11 & 1.00 & 0.33 & 0.11 & -0.11 & 0.55 \\ -0.11 & 1.00 & -0.11 & 0.33 & 0.11 & 0.11 & -0.11 \\ 0.77 & -0.11 & 0.11 & 0.33 & 0.55 & -0.11 & 0.33 \end{bmatrix}$$

Convolution Layer

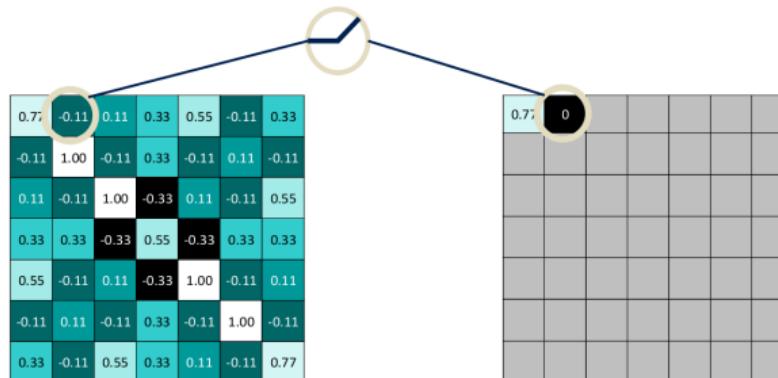
- One image becomes a stack of filtered images.



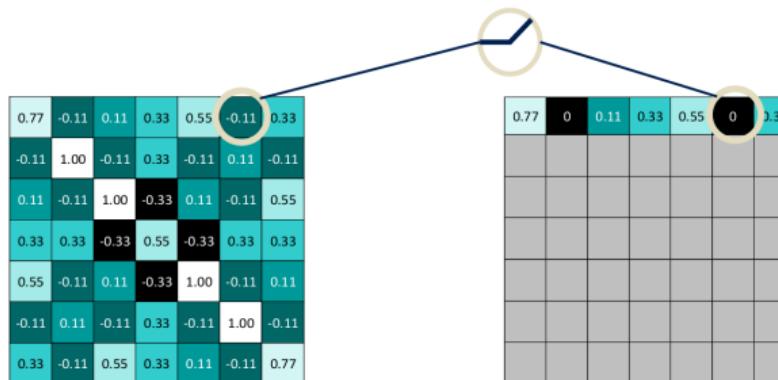
Rectified Linear Units (ReLUs)



Rectified Linear Units (ReLUs)



Rectified Linear Units (ReLUs)



Rectified Linear Units (ReLUs)

0.77	-0.11	0.11	0.33	0.55	-0.11	0.33
-0.11	1.00	-0.11	0.33	-0.11	0.11	-0.11
0.11	-0.11	1.00	-0.33	0.11	-0.11	0.55
0.33	0.33	-0.33	0.55	-0.33	0.33	0.33
0.55	-0.11	0.11	-0.33	1.00	-0.11	0.11
-0.11	0.11	-0.11	0.33	-0.11	1.00	-0.11
0.33	-0.11	0.55	0.33	0.11	-0.11	0.77

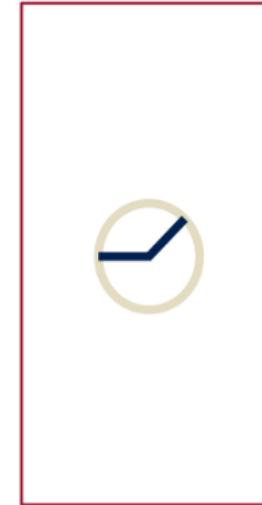


0.77	0	0.11	0.33	0.55	0	0.33
0	1.00	0	0.33	0	0.11	0
0.11	0	1.00	0	0.11	0	0.55
0.33	0.33	0	0.55	0	0.33	0.33
0.55	0	0.11	0	1.00	0	0.11
0	0.11	0	0.33	0	1.00	0
0.33	0	0.55	0.33	0.11	0	0.77

ReLU Layer

- ▶ A stack of images becomes a stack of images with **no negative values**.

0.77	-0.15	0.11	0.33	-0.05	-0.03	0.31
-0.11	1.09	-0.15	-0.31	-0.11	0.11	-0.11
0.11	-0.11	1.09	0.33	0.11	-0.15	0.35
0.01	0.15	-0.31	-0.51	0.11	0.11	0.31
0.35	-0.11	0.11	0.33	1.09	-0.11	0.31
0.11	0.11	-0.11	0.33	-0.11	1.09	-0.11
0.33	-0.11	0.55	-0.31	0.11	-0.11	0.27
0.33	-0.15	0.11	0.33	0.11	-0.25	0.31
0.36	-0.15	0.55	0.33	-0.15	0.39	0.36
0.33	-0.15	0.55	0.77	0.35	-0.15	0.31
0.11	0.11	-0.77	1.09	0.77	0.31	0.11
0.11	-0.15	0.55	0.77	0.35	-0.15	0.11
0.35	0.55	-0.31	0.33	-0.15	0.35	0.35
0.33	-0.15	0.11	0.33	0.11	-0.25	0.31
0.33	-0.11	0.55	0.33	0.11	-0.11	0.27
0.11	0.11	-0.33	-0.31	0.11	1.09	0.11
0.35	-0.11	0.11	-0.31	1.09	-0.11	0.11
0.33	0.75	-0.33	-0.31	0.33	0.11	0.33
0.33	-0.11	1.09	0.33	0.11	-0.11	0.35
0.11	1.09	0.11	0.33	0.11	0.11	0.11
0.33	-0.11	0.11	0.33	0.11	-0.11	0.33

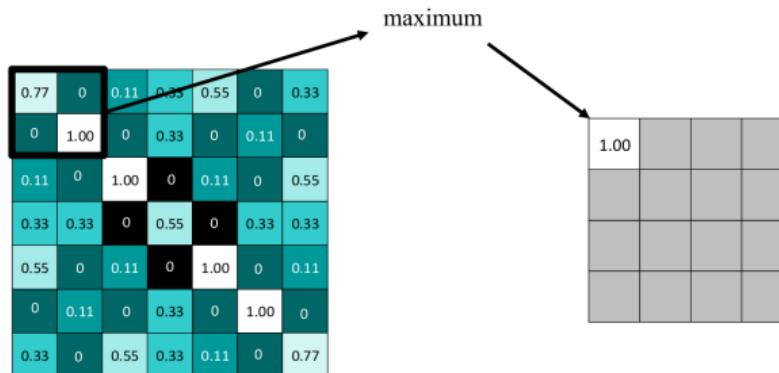


0.17	0	0.11	0.31	0.35	0	0.39
0	1.09	0	0.33	0	0.11	0
0.11	0	1.09	0	0.11	0	0.35
0.11	0.11	0	0.33	0	0.39	0.39
0.08	0	0.11	0	1.09	0	0.11
0	0.11	0	0.33	0	1.09	0
0.11	0	0.55	0.31	0.11	0	0.27
0.11	0	0.11	0	0.33	0	0.39
0.31	0	0.11	0	0.35	0	0.39

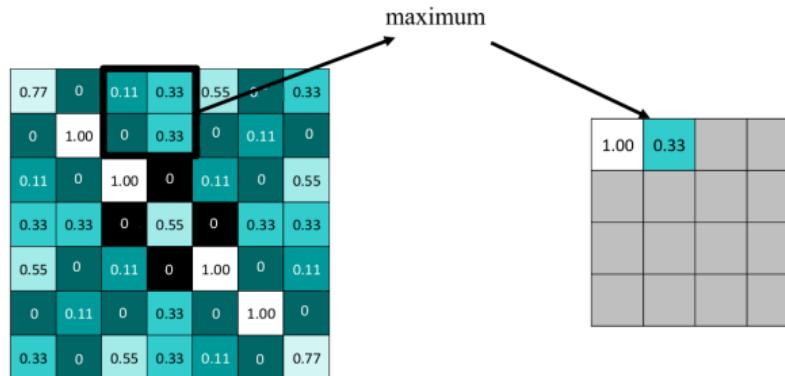
0.11	0	0.11	0	0.33	0	0.39
0	0.55	0	0.33	0	0.39	0
0.11	0	0.55	0	0.33	0	0.39
0.11	0	0.33	0	1.09	0	0.31
0.11	0	0.33	0	0.33	0	0.11
0	0.55	0	0.33	0	0.33	0
0.11	0	0.11	0	0.33	0	0.39

0.11	0	0.11	0.31	0.33	0	0.39
0	0.11	0	0.33	0	1.09	0
0.11	0	0.11	0	1.09	0	0.11
0.11	0.11	0	0.33	0	0.33	0.39
0.11	0	0.33	0	0.33	0	0.11
0	0.11	0	0.33	0	0.33	0
0.11	0	0.11	0.31	0.33	0	0.39

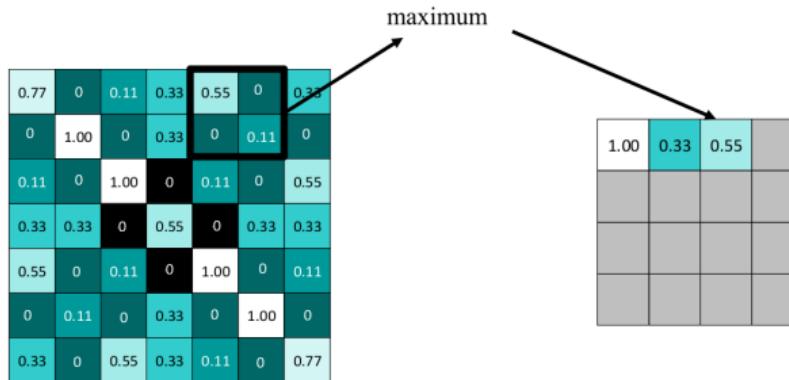
Pooling: Shrinking the Image Stack



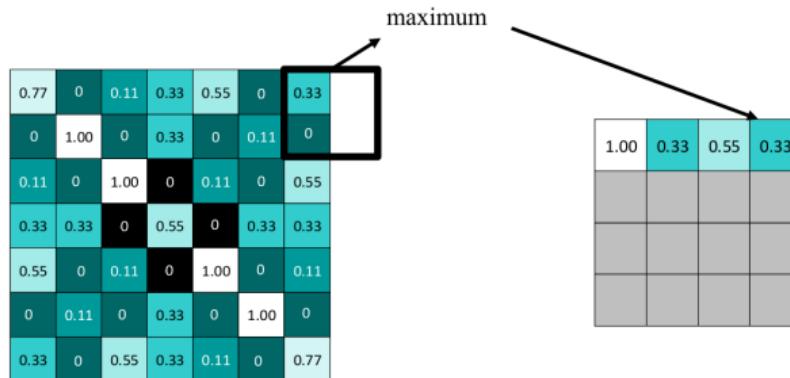
Pooling: Shrinking the Image Stack



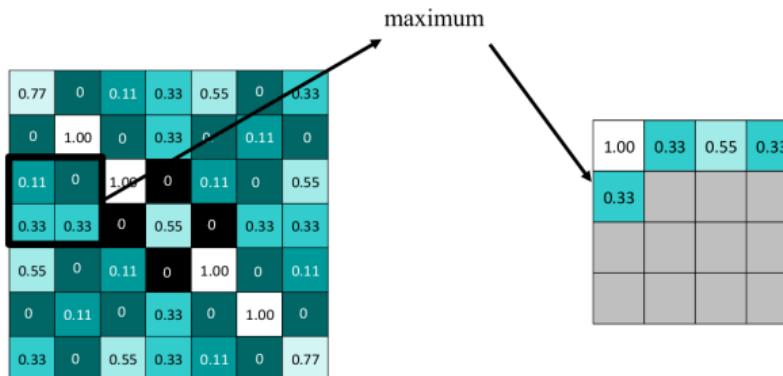
Pooling: Shrinking the Image Stack



Pooling: Shrinking the Image Stack



Pooling: Shrinking the Image Stack



Pooling: Shrinking the Image Stack

0.77	0	0.11	0.33	0.55	0	0.33
0	1.00	0	0.33	0	0.11	0
0.11	0	1.00	0	0.11	0	0.55
0.33	0.33	0	0.55	0	0.33	0.33
0.55	0	0.11	0	1.00	0	0.11
0	0.11	0	0.33	0	1.00	0
0.33	0	0.55	0.33	0.11	0	0.77

max pooling

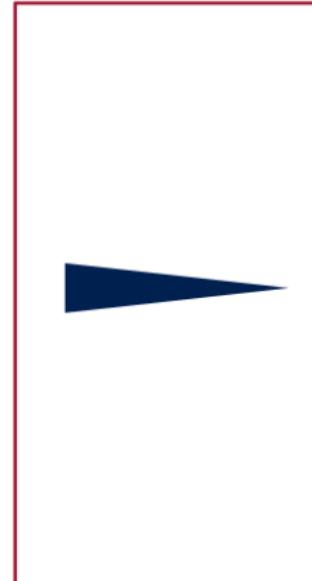
1.00	0.33	0.55	0.33
0.33	1.00	0.33	0.55
0.55	0.33	1.00	0.11
0.33	0.55	0.11	0.77

Repeat For All the Filtered Images

0.77	0	0.33	0.33	0.33	0	0.33
0	1.00	0	0.33	0	0.33	0
0.33	0	1.00	0	0.33	0	0.33
0.33	0.33	0	0.33	0	0.33	0.33
0.33	0	0.33	0	1.00	0	0.33
0.33	0.33	0	0.33	0	1.00	0
0.33	0	0.33	0.33	0.33	0	0.77

0.33	0	0.33	0	0.33	0	0.33
0	0.55	0	0.33	0	0.55	0
0.33	0	0.33	0	0.33	0	0.33
0	0.33	0	1.00	0	0.33	0
0.33	0	0.33	0	0.33	0	0.33
0	0.55	0	0.33	0	0.55	0
0.33	0	0.33	0	0.33	0	0.33

0.33	0	0.33	0.33	0.33	0	0.77
0	0.33	0	0.33	0	1.00	0
0.33	0	0.33	0	1.00	0	0.33
0.33	0.33	0	0.33	0	0.33	0.33
0.33	0	1.00	0	0.33	0	0.33
0	1.00	0	0.33	0	0.33	0
0.33	0	0.33	0.33	0.33	0	0.33



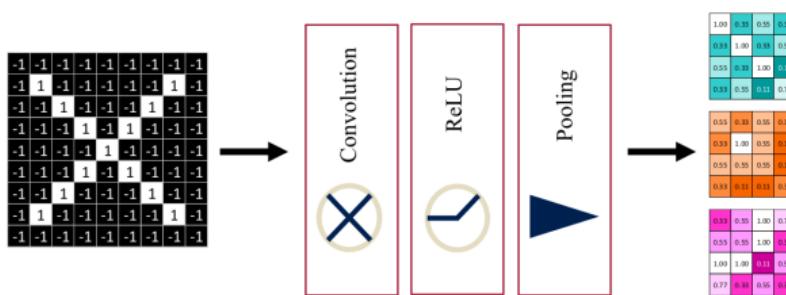
1.00	0.33	0.55	0.33
0.33	1.00	0.33	0.55
0.55	0.33	1.00	0.11
0.33	0.55	0.11	0.77

0.55	0.33	0.55	0.33
0.33	1.00	0.55	0.11
0.55	0.55	0.55	0.11
0.33	0.11	0.11	0.33

0.33	0.55	1.00	0.77
0.55	0.55	1.00	0.33
1.00	1.00	0.11	0.55
0.77	0.33	0.55	0.33

Layers Get Stacked

- ▶ The output of one becomes the input of the next.

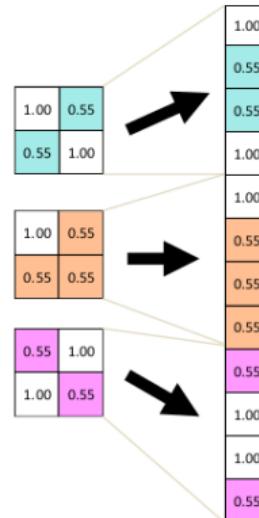


Deep Stacking



Fully Connected Layer

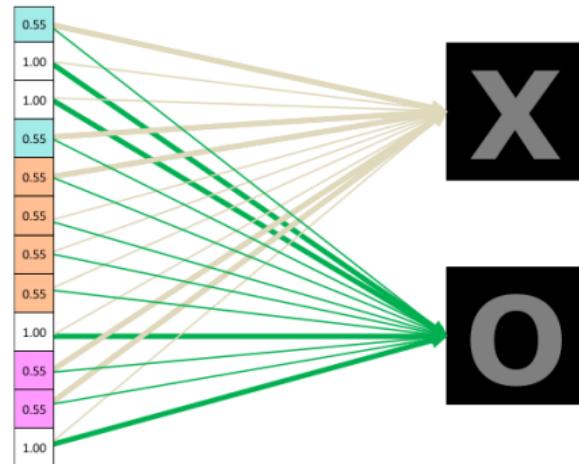
- ▶ Flattening the outputs before giving them to the **fully connected layer**.



Fully Connected Layer



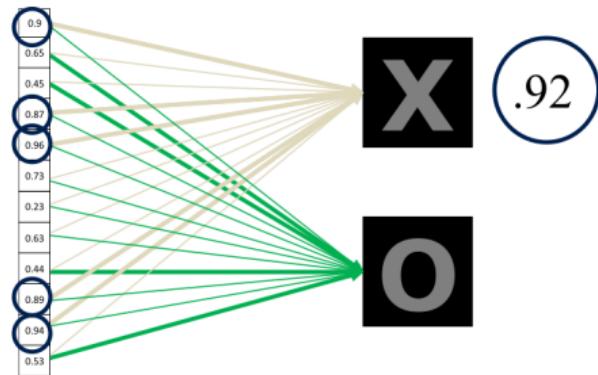
Fully Connected Layer



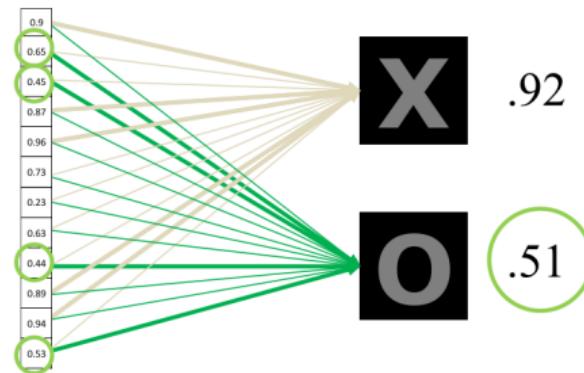
Fully Connected Layer



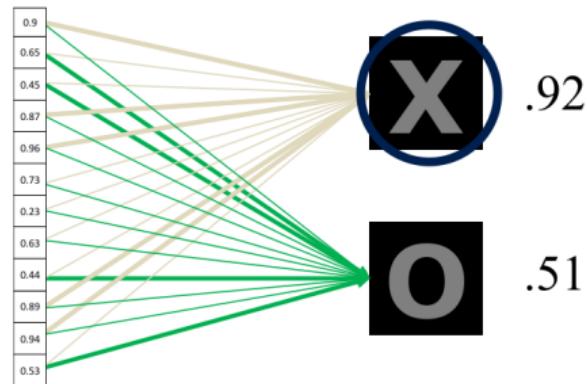
Fully Connected Layer



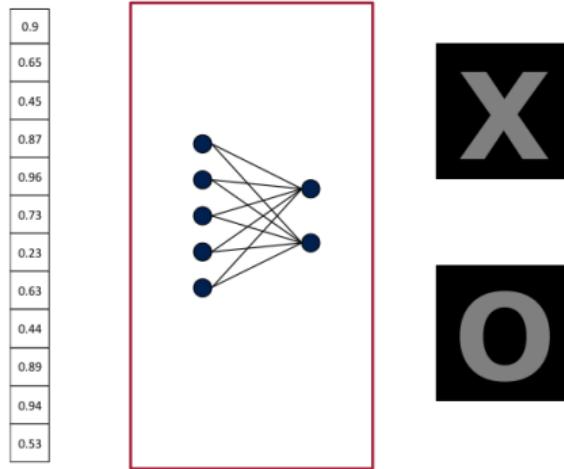
Fully Connected Layer



Fully Connected Layer

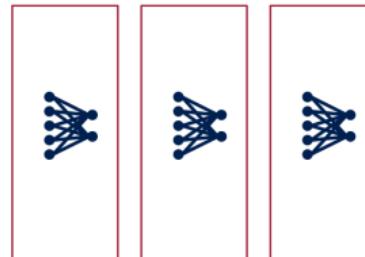


Fully Connected Layer

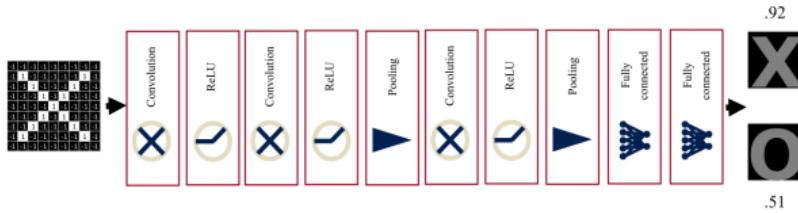


Fully Connected Layer

0.9
0.65
0.45
0.87
0.96
0.73
0.23
0.61
0.44
0.89
0.94
0.53



Putting It All Together







CNN in TensorFlow



CNN in TensorFlow (1/8)

- ▶ A CNN for the MNIST dataset with the following network.
- ▶ Conv. layer 1: computes 32 feature maps using a 5x5 filter with ReLU activation.
- ▶ Pooling layer 1: max pooling layer with a 2x2 filter and stride of 2.
- ▶ Conv. layer 2: computes 64 feature maps using a 5x5 filter.
- ▶ Pooling layer 2: max pooling layer with a 2x2 filter and stride of 2.
- ▶ Dense layer: densely connected layer with 1024 neurons.
- ▶ Logits layer



CNN in TensorFlow (2/8)

- ▶ Conv. layer 1: computes 32 feature maps using a 5x5 filter with ReLU activation.
- ▶ Input tensor shape: [batch_size, 28, 28, 1]
- ▶ Output tensor shape: [batch_size, 28, 28, 32]
- ▶ Padding same is added to preserve width and height.

```
# MNIST images are 28x28 pixels, and have one color channel
X = tf.placeholder(tf.float32, [None, 28, 28, 1])
y_true = tf.placeholder(tf.float32, [None, 10])

conv1 = tf.layers.conv2d(inputs=X, filters=32, kernel_size=[5, 5], padding="same",
activation=tf.nn.relu)
```



CNN in TensorFlow (3/8)

- ▶ Pooling layer 1: max pooling layer with a 2x2 filter and stride of 2.
- ▶ Input tensor shape: [batch_size, 28, 28, 32]
- ▶ Output tensor shape: [batch_size, 14, 14, 32]

```
pool1 = tf.layers.max_pooling2d(inputs=conv1, pool_size=[2, 2], strides=2)
```



CNN in TensorFlow (4/8)

- ▶ Conv. layer 2: computes 64 feature maps using a 5x5 filter.
- ▶ Input tensor shape: [batch_size, 14, 14, 32]
- ▶ Output tensor shape: [batch_size, 14, 14, 64]
- ▶ Padding same is added to preserve width and height.

```
conv2 = tf.layers.conv2d(inputs=pool1, filters=64, kernel_size=[5, 5], padding="same",
activation=tf.nn.relu)
```



CNN in TensorFlow (5/8)

- ▶ Pooling layer 2: max pooling layer with a 2x2 filter and stride of 2.
- ▶ Input tensor shape: [batch_size, 14, 14, 64]
- ▶ Output tensor shape: [batch_size, 7, 7, 64]

```
pool2 = tf.layers.max_pooling2d(inputs=conv2, pool_size=[2, 2], strides=2)
```



CNN in TensorFlow (6/8)

- ▶ **Flatten** tensor into a batch of vectors.
 - Input tensor shape: `[batch_size, 7, 7, 64]`
 - Output tensor shape: `[batch_size, 7 * 7 * 64]`

```
pool2_flat = tf.reshape(pool2, [-1, 7 * 7 * 64])
```

- ▶ **Dense layer:** densely connected layer with **1024 neurons**.
 - Input tensor shape: `[batch_size, 7 * 7 * 64]`
 - Output tensor shape: `[batch_size, 1024]`

```
dense = tf.layers.dense(inputs=pool2_flat, units=1024, activation=tf.nn.relu)
```



CNN in TensorFlow (7/8)

- ▶ Add **dropout** operation; 0.6 probability that element will be kept

```
dropout = tf.layers.dropout(inputs=dense, rate=0.4)
```

- ▶ **Logits layer**

- Input tensor shape: `[batch_size, 1024]`
- Output tensor shape: `[batch_size, 10]`

```
logits = tf.layers.dense(inputs=dropout, units=10)
```



CNN in TensorFlow (8/8)

```
# define the cost and accuracy functions
cross_entropy = tf.nn.softmax_cross_entropy_with_logits(logits=logits, labels=y_true)
cross_entropy = tf.reduce_mean(cross_entropy) * 100

# define the optimizer
lr = 0.003
optimizer = tf.train.AdamOptimizer(lr)
train_step = optimizer.minimize(cross_entropy)

# execute the model
init = tf.global_variables_initializer()

n_epochs = 2000
with tf.Session() as sess:
    sess.run(init)

    for i in range(n_epochs):
        batch_X, batch_y = mnist.train.next_batch(100)
        sess.run(train_step, feed_dict={X: batch_X, y_true: batch_y})
```





Training CNNs

Training CNN (1/4)

- ▶ Let's see how to use **backpropagation** on a **single convolutional layer**.
- ▶ Assume we have an input X of size **3×3** and a **single filter W** of size **2×2** .
- ▶ **No padding** and **stride = 1**.
- ▶ It generates an **output H** of size **2×2** .



Training CNN (2/4)

► Forward pass



$$h_{11} = W_{11}X_{11} + W_{12}X_{12} + W_{21}X_{21} + W_{22}X_{22}$$

$$h_{12} = W_{11}X_{12} + W_{12}X_{13} + W_{21}X_{22} + W_{22}X_{23}$$

$$h_{21} = W_{11}X_{21} + W_{12}X_{22} + W_{21}X_{31} + W_{22}X_{32}$$

$$h_{22} = W_{11}X_{22} + W_{12}X_{23} + W_{21}X_{32} + W_{22}X_{33}$$

Training CNN (3/4)

- ▶ Backward pass
- ▶ E is the error: $E = E_{h_{11}} + E_{h_{12}} + E_{h_{21}} + E_{h_{22}}$

X_{11}	X_{12}	X_{13}
X_{21}	X_{22}	X_{23}
X_{31}	X_{32}	X_{33}

w_{11}	w_{12}
w_{21}	w_{22}

h_{11}	h_{12}
h_{21}	h_{22}

$$\frac{\partial E}{\partial W_{11}} = \frac{\partial E_{h_{11}}}{\partial h_{11}} \frac{\partial h_{11}}{\partial W_{11}} + \frac{\partial E_{h_{12}}}{\partial h_{12}} \frac{\partial h_{12}}{\partial W_{11}} + \frac{\partial E_{h_{21}}}{\partial h_{21}} \frac{\partial h_{21}}{\partial W_{11}} + \frac{\partial E_{h_{22}}}{\partial h_{22}} \frac{\partial h_{22}}{\partial W_{11}}$$

$$\frac{\partial E}{\partial W_{12}} = \frac{\partial E_{h_{11}}}{\partial h_{11}} \frac{\partial h_{11}}{\partial W_{12}} + \frac{\partial E_{h_{12}}}{\partial h_{12}} \frac{\partial h_{12}}{\partial W_{12}} + \frac{\partial E_{h_{21}}}{\partial h_{21}} \frac{\partial h_{21}}{\partial W_{12}} + \frac{\partial E_{h_{22}}}{\partial h_{22}} \frac{\partial h_{22}}{\partial W_{12}}$$

$$\frac{\partial E}{\partial W_{21}} = \frac{\partial E_{h_{11}}}{\partial h_{11}} \frac{\partial h_{11}}{\partial W_{21}} + \frac{\partial E_{h_{12}}}{\partial h_{12}} \frac{\partial h_{12}}{\partial W_{21}} + \frac{\partial E_{h_{21}}}{\partial h_{21}} \frac{\partial h_{21}}{\partial W_{21}} + \frac{\partial E_{h_{22}}}{\partial h_{22}} \frac{\partial h_{22}}{\partial W_{21}}$$

$$\frac{\partial E}{\partial W_{22}} = \frac{\partial E_{h_{11}}}{\partial h_{11}} \frac{\partial h_{11}}{\partial W_{22}} + \frac{\partial E_{h_{12}}}{\partial h_{12}} \frac{\partial h_{12}}{\partial W_{22}} + \frac{\partial E_{h_{21}}}{\partial h_{21}} \frac{\partial h_{21}}{\partial W_{22}} + \frac{\partial E_{h_{22}}}{\partial h_{22}} \frac{\partial h_{22}}{\partial W_{22}}$$

Training CNN (4/4)

- ▶ Update the weights W



$$W_{11}^{(\text{next})} = W_{11} - \eta \frac{\partial E}{\partial W_{11}}$$

$$W_{12}^{(\text{next})} = W_{12} - \eta \frac{\partial E}{\partial W_{12}}$$

$$W_{21}^{(\text{next})} = W_{21} - \eta \frac{\partial E}{\partial W_{21}}$$

$$W_{22}^{(\text{next})} = W_{22} - \eta \frac{\partial E}{\partial W_{22}}$$



Summary



Summary

- ▶ Receptive fields and filters
- ▶ Convolution operation
- ▶ Padding and strides
- ▶ Pooling layer
- ▶ Flattening, dropout, dense



Reference

- ▶ Tensorflow and Deep Learning without a PhD
<https://codelabs.developers.google.com/codelabs/cloud-tensorflow-mnist>
- ▶ Ian Goodfellow et al., Deep Learning (Ch. 9)
- ▶ Aurélien Géron, Hands-On Machine Learning (Ch. 13)



Questions?