



Introduction

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Course Information



Course Objective

- ▶ This course has a **system-based** focus
- ▶ Learn the theory of **machine learning** and **deep learning**
- ▶ Learn the practical aspects of building **machine learning** and **deep learning** algorithms using data parallel programming platforms, such as **Spark** and **TensorFlow**



Topics of Study

- ▶ Part 1: large scale **machine learning**

- ▶ Part 2: large scale **deep learning**



Topics of Study

- ▶ Part 1: large scale **machine learning**
 - Spark ML
 - Linear regression and logistic regression
 - Decision tree and ensemble models

- ▶ Part 2: large scale **deep learning**



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► Part 1: large scale **machine learning**

- Spark ML
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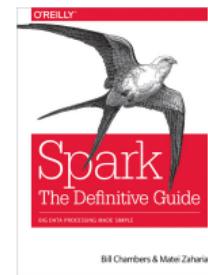
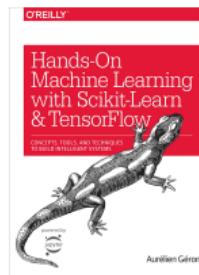
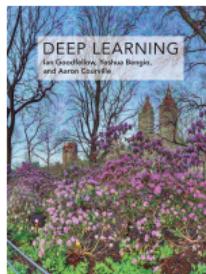
► Part 2: large scale **deep learning**

- TensorFlow
- Deep feedforward networks
- Convolutional neural networks (CNNs)
- Recurrent neural networks (RNNs)
- Autoencoders and Restricted Boltzmann machines (RBMs)



The Course Material

- ▶ Deep learning, I. Goodfellow et al., Cambridge: MIT press, 2016
- ▶ Hands-on machine learning with Scikit-Learn and TensorFlow, A. Geron, O'Reilly Media, 2017
- ▶ Spark - The Definitive Guide, M. Zaharia et al., O'Reilly Media, 2018.





The Course Grading

- ▶ Two lab assignments: 30%
- ▶ One final project: 20%
- ▶ Eight review questions: 20%
- ▶ The final exam: 30%



The Labs and Project

- ▶ Self-selected groups of two
- ▶ Labs
 - Include Scala/Python programming
 - Lab1: Regression using Spark ML
 - Lab2: Deep neural network and CNN using Tensorflow
- ▶ Project
 - Selection of a large dataset and method
 - RNNs, Autoencoders, or RBMs
 - Demonstrated as a demo and short report



The Course Web Page

<https://id2223kth.github.io>



The Course Overview

Sheepdog or Mop





Chihuahua or Muffin



@teenybiscuit

Barn Owl or Apple



Raw Chicken or Donald Trump





Artificial Intelligence Challenge

- ▶ Artificial intelligence (AI) can solve problems that can be described by a list of formal mathematical rules.



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- ▶ The challenge is to solve the tasks that are hard for people to describe formally.



Artificial Intelligence Challenge

- ▶ Artificial intelligence (AI) can solve problems that can be described by a list of formal mathematical rules.
- ▶ The challenge is to solve the tasks that are hard for people to describe formally.
- ▶ Let computers to learn from experience.



History of AI

Greek Myths

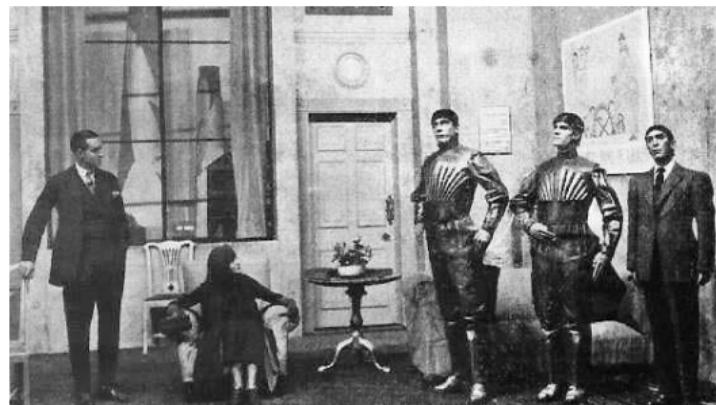
- ▶ Hephaestus, the god of blacksmith, created a metal automaton, called Talos.



[the left figure: <http://mythologian.net/hephaestus-the-blacksmith-of-gods>]
[the right figure: <http://elderscrolls.wikia.com/wiki/Talos>]

1920: Rossum's Universal Robots (R.U.R.)

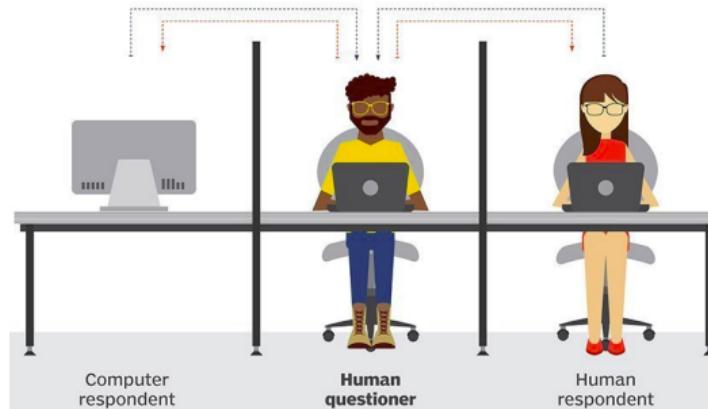
- ▶ A science fiction play by Karel Čapek, in 1920.
- ▶ A factory that creates artificial people named robots.



[<https://dev.to/lshultebraucks/a-short-history-of-artificial-intelligence-7hm>]

1950: Turing Test

- ▶ In 1950, **Turing** introduced the **Turing test**.
- ▶ An attempt to define **machine intelligence**.



[<https://searchenterpriseai.techtarget.com/definition/Turing-test>]



1956: The Dartmouth Workshop

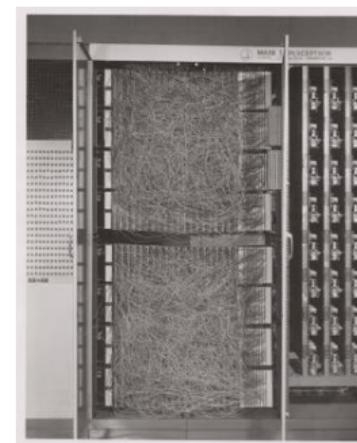
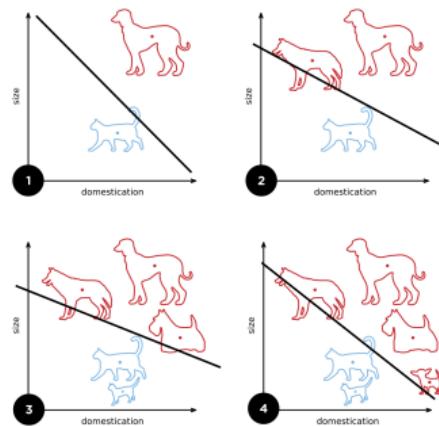
- ▶ Probably the **first workshop of AI**.
- ▶ Researchers from **CMU, MIT, IBM** met together and founded the **AI research**.



[<https://twitter.com/lordsaicom/status/898139880441696257>]

1958: Perceptron

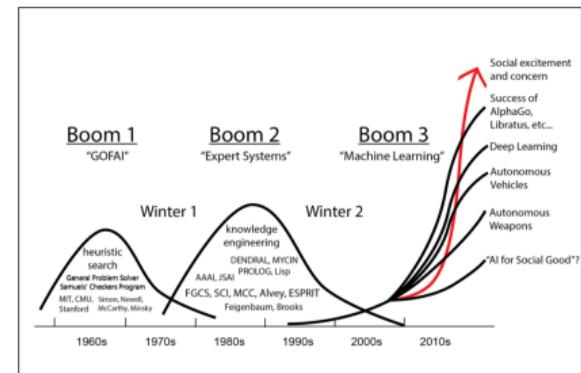
- ▶ A **supervised learning** algorithm for **binary classifiers**.
- ▶ Implemented in custom-built hardware as the **Mark 1 perceptron**.



[<https://en.wikipedia.org/wiki/Perceptron>]

1974–1980: The First AI Winter

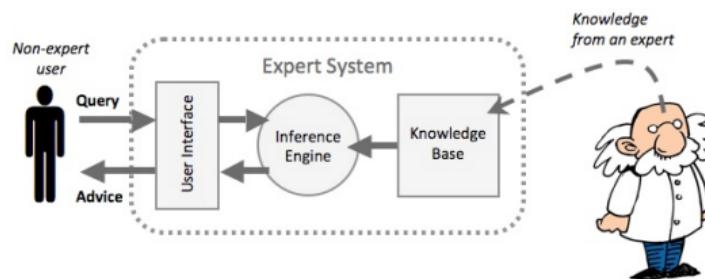
- ▶ The over **optimistic settings**, which were not occurred
- ▶ The **problems**:
 - Limited **computer power**
 - Lack of **data**
 - Intractability and the **combinatorial explosion**



[<http://www.technologystories.org/ai-evolution>]

1980's: Expert systems

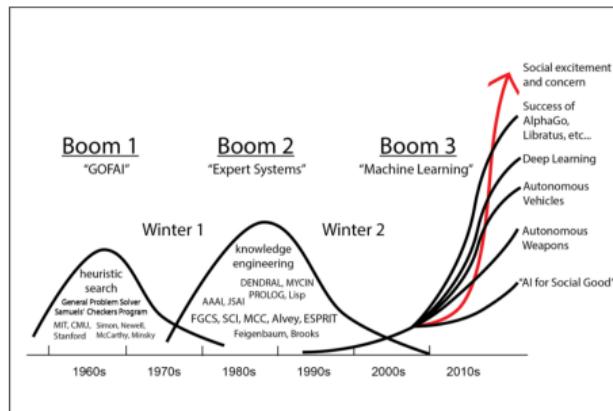
- ▶ The programs that solve problems in a **specific domain**.
- ▶ **Two** engines:
 - Knowledge engine: **represents** the **facts and rules** about a specific topic.
 - Inference engine: **applies** the **facts and rules** from the knowledge engine to new facts.



[https://www.igcseict.info/theory/7_2/expert]

1987–1993: The Second AI Winter

- ▶ After a series of financial setbacks.
- ▶ The fall of **expert systems** and hardware companies.



[<http://www.technologystories.org/ai-evolution>]



1997: IBM Deep Blue

- ▶ The first chess computer to beat a world chess champion Garry Kasparov.



[<http://marksist.org/icerik/Tarihte-Bugun/1757/11-Mayis-1997-Deep-Blue-adli-bilgisayar>]



2012: AlexNet - Image Recognition

- ▶ The [ImageNet competition](#) in image classification.
- ▶ The [AlexNet Convolutional Neural Network \(CNN\)](#) won the challenge by a large margin.

IM_{AG}ENET

2016: DeepMind AlphaGo

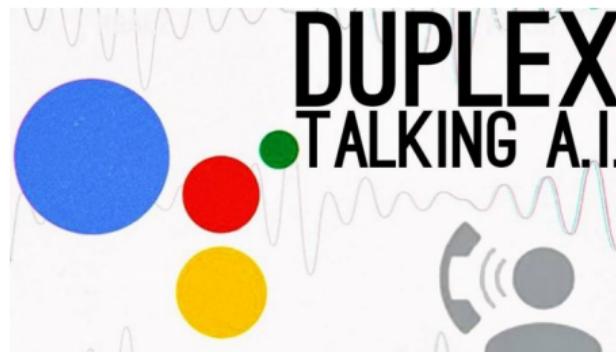
- ▶ DeepMind **AlphaGo** won **Lee Sedol**, one of the best players at Go.
- ▶ In 2017, DeepMind published **AlphaGo Zero**.
 - The **next generation** of AlphaGo.
 - It learned Go by playing **against itself**.



[<https://www.zdnet.com/article/google-alphago-caps-victory-by-winning-final-historic-go-match>]

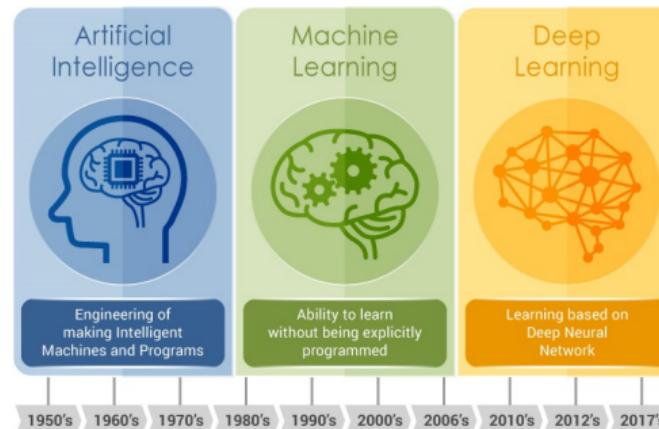
2018: Google Duplex

- ▶ An AI system for accomplishing real-world tasks over the phone.
- ▶ A Recurrent Neural Network (RNN) built using TensorFlow.



AI Generations

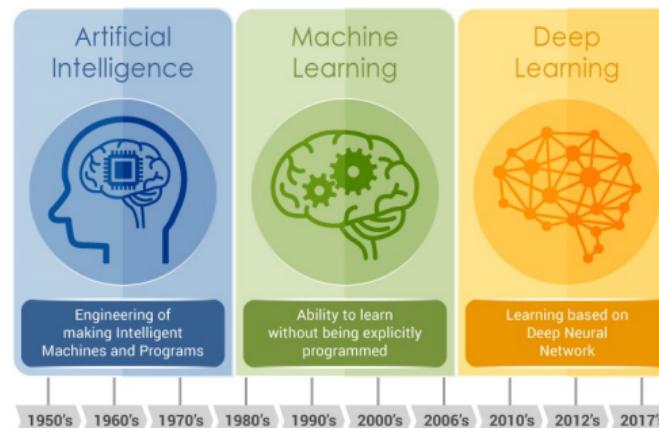
- ▶ Rule-based AI
- ▶ Machine learning
- ▶ Deep learning



[<https://bit.ly/2woLEzs>]

AI Generations - Rule-based AI

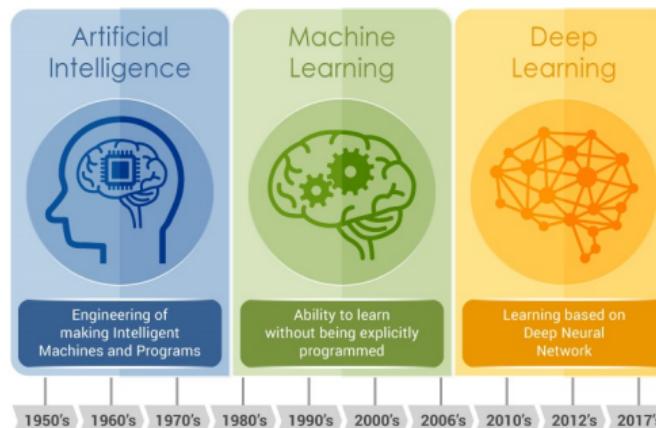
- ▶ Hard-code knowledge
- ▶ Computers reason using logical inference rules



[<https://bit.ly/2woLEzs>]

AI Generations - Machine Learning

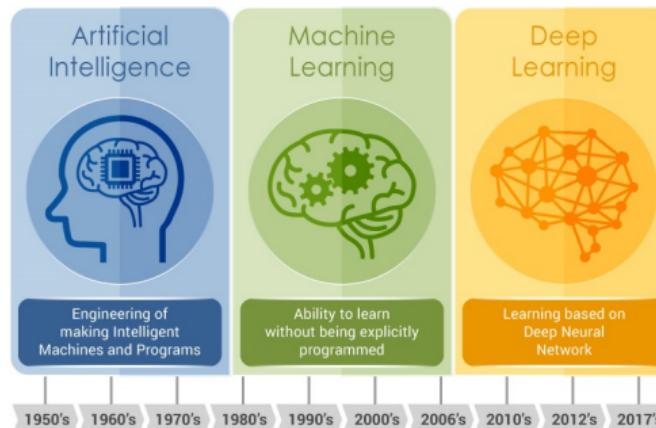
- ▶ If AI systems acquire **their own knowledge**
- ▶ **Learn from data without being explicitly programmed**



[<https://bit.ly/2woLEzs>]

AI Generations - Deep Learning

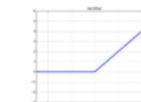
- ▶ For many tasks, it is **difficult to know what features** should be extracted
- ▶ Use **machine learning** to **discover** the mapping from **representation to output**



[<https://bit.ly/2woLEzs>]

Why Does Deep Learning Work Now?

- ▶ Huge **quantity** of data
- ▶ Tremendous increase in **computing power**
- ▶ Better **training** algorithms



Non-Linearity



Machine Learning and Deep Learning



Learning Algorithms

- ▶ A **ML algorithm** is an algorithm that is able to **learn** from data.
- ▶ What is **learning**?

Learning Algorithms

- ▶ A **ML algorithm** is an algorithm that is able to **learn from data**.
- ▶ What is **learning**?
- ▶ A computer program is said to **learn** from **experience E** with respect to some class of **tasks T** and **performance measure P**, if its performance at tasks in **T**, as measured by **P**, improves with experience **E**. (Tom M. Mitchell)



Learning Algorithms - Example 1

- ▶ A **spam filter** that can learn to flag **spam** given examples of **spam emails** and examples of **regular emails**.



[<https://bit.ly/2oiplYM>]

Learning Algorithms - Example 1

- ▶ A **spam filter** that can learn to flag **spam** given examples of **spam emails** and examples of **regular emails**.
- ▶ **Task T**: flag spam for new emails
- ▶ **Experience E**: the training data
- ▶ **Performance measure P**: the ratio of correctly classified emails



[<https://bit.ly/2oiplYM>]

Learning Algorithms - Example 2

- ▶ Given dataset of prices of 500 houses, how can we learn to **predict the prices** of other houses, as a **function of the size of their living areas**?



[<https://bit.ly/2MyiJUy>]

Learning Algorithms - Example 2

- ▶ Given dataset of prices of 500 houses, how can we learn to **predict the prices** of other houses, as a **function of the size of their living areas?**
- ▶ **Task T:** predict the price
- ▶ **Experience E:** the dataset of living areas and prices
- ▶ **Performance measure P:** the difference between the predicted price and the real price



[<https://bit.ly/2MyiJUy>]

Types of Machine Learning Algorithms

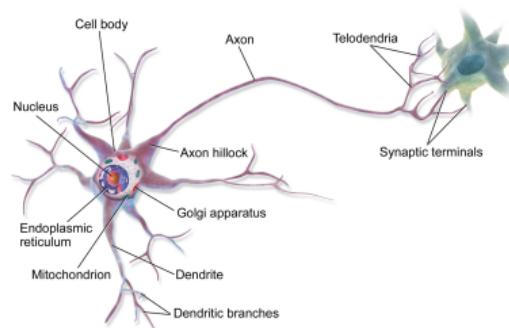
- ▶ Supervised learning
 - Input data is **labeled**, e.g., spam/not-spam or a stock price at a time.
 - Regression vs. classification

- ▶ Unsupervised learning
 - Input data is **unlabeled**.
 - Find **hidden structures** in data.



From Machine Learning to Deep Learning

- ▶ Deep Learning (DL) is part of ML methods based on learning data representations.
- ▶ Mimic the neural networks of our brain.



[A. Geron, O'Reilly Media, 2017]



Artificial Neural Networks

- ▶ Artificial Neural Network (ANN) is inspired by biological neurons.

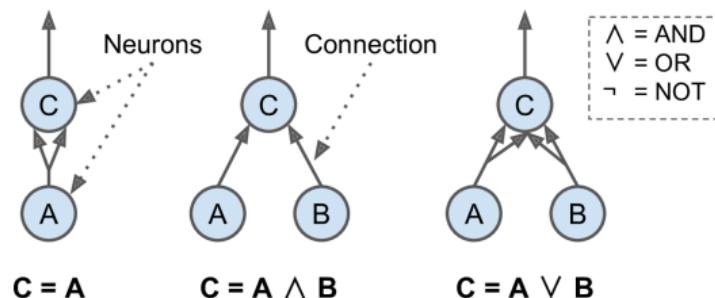


Artificial Neural Networks

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- ▶ One or more binary inputs and one binary output
- ▶ Activates its output when more than a certain number of its inputs are active.

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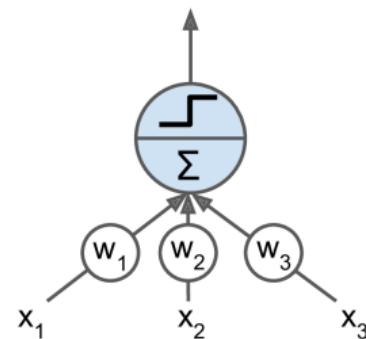
The Linear Threshold Unit (LTU)

- ▶ Inputs of a LTU are **numbers** (**not binary**).

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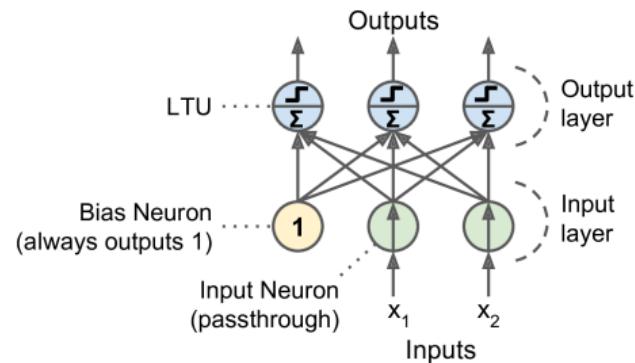
- ▶ Inputs of a LTU are **numbers** (not binary).
- ▶ Each **input connection** is associated with a **weight**.
- ▶ Computes a **weighted sum of its inputs** and applies a **step function** to that **sum**.

- ▶ $z = w_1x_1 + w_2x_2 + \dots + w_nx_n = \mathbf{w}^T\mathbf{x}$
- ▶ $\hat{y} = \text{step}(z) = \text{step}(\mathbf{w}^T\mathbf{x})$



The Perceptron

- ▶ The **perceptron** is a **single layer** of LTUs.
- ▶ The **input neurons** output whatever **input** they are fed.
- ▶ A **bias neuron**, which just **outputs 1** all the time.



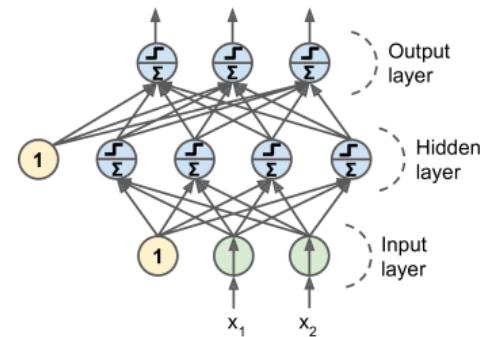


Deep Learning Models

- ▶ Deep Neural Network ([DNN](#))
- ▶ Convolutional Neural Network ([CNN](#))
- ▶ Recurrent Neural Network ([RNN](#))
- ▶ Autoencoders

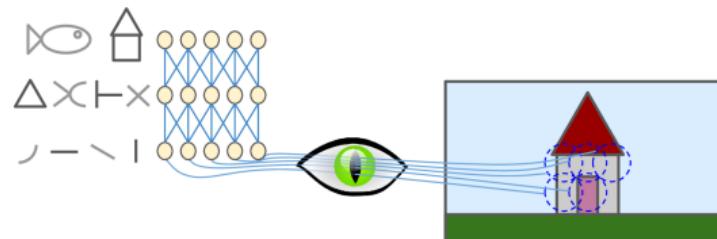
Deep Neural Networks

- ▶ Multi-Layer Perceptron (MLP)
 - One input layer
 - One or more layers of LTUs (hidden layers)
 - One final layer of LTUs (output layer)
- ▶ Deep Neural Network (DNN) is an ANN with two or more hidden layers.
- ▶ Backpropagation training algorithm



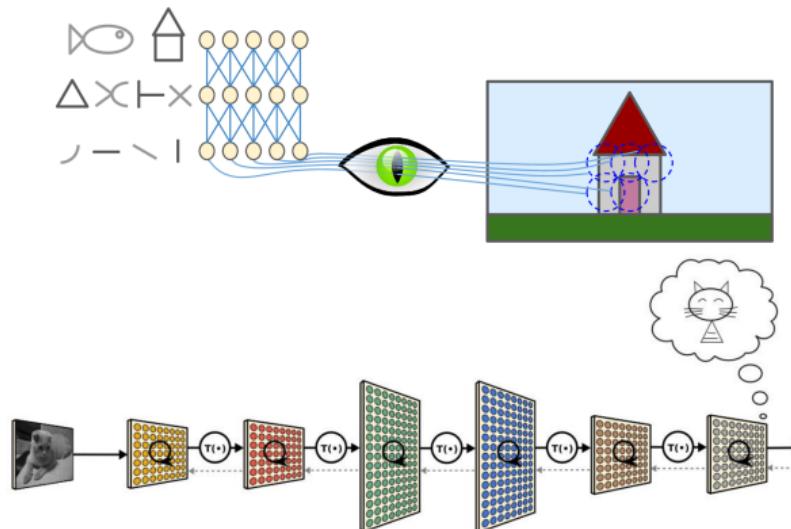
Convolutional Neural Networks

- ▶ Many neurons in the **visual cortex** react only to a **limited region** of the visual field.



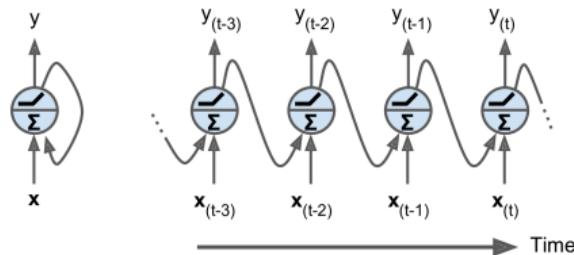
Convolutional Neural Networks

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- ▶ The **higher-level** neurons are based on the outputs of **neighboring lower-level** neurons



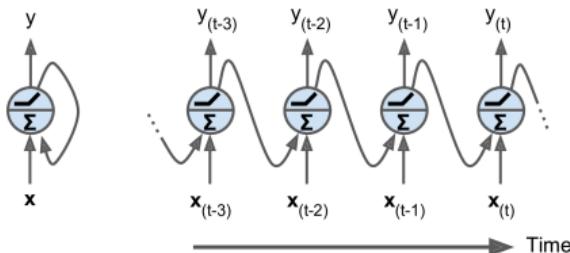
Recurrent Neural Networks

- ▶ The **output** depends on the **input** and the **previous computations**.



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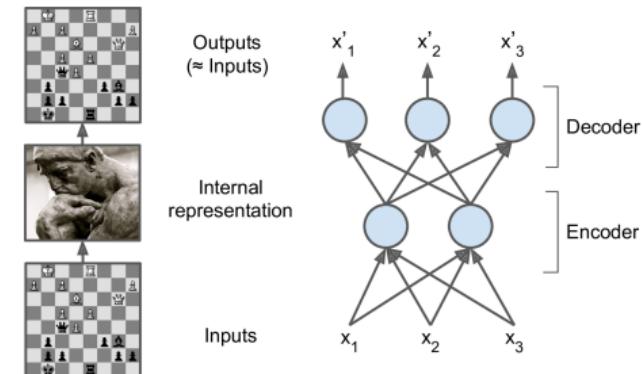


- ▶ Analyze **time series data**, e.g., stock market, and autonomous driving systems
- ▶ Work on sequences of **arbitrary lengths**, rather than on **fixed-sized inputs**



Autoencoders

- ▶ Learn **efficient representations** of the input data, **without any supervision**.
 - With a **lower dimensionality** than the input data
- ▶ **Generative model**: generate **new data** that looks very similar to the training data.
- ▶ Preserve **as much information as possible**



[A. Geron, O'Reilly Media, 2017]



Linear Algebra Review

Vector

- ▶ A **vector** is an **array of numbers**.
- ▶ Notation:
 - Denoted by **bold lowercase letters**, e.g., **x**.
 - **x_i** denotes the **i**th entry.

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$



Matrix and Tensor

- ▶ A **matrix** is a 2-D array of numbers.
- ▶ A **tensor** is an array with more than two axes.
- ▶ Notation:
 - Denoted by **bold uppercase letters**, e.g., **A**.
 - a_{ij} denotes the entry in i th row and j th column.
 - If **A** is $m \times n$, it has m rows and n columns.

$$\mathbf{A} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \dots & a_{1,n} \\ a_{2,1} & a_{2,2} & a_{2,3} & \dots & a_{2,n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m,1} & a_{m,2} & a_{m,3} & \dots & a_{m,n} \end{bmatrix}$$



Matrix Addition and Subtraction

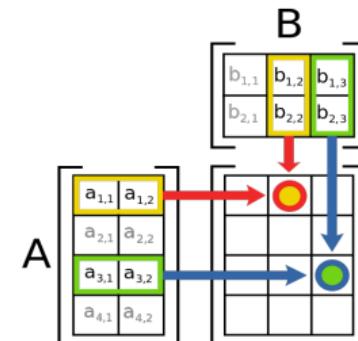
- The **matrices** must have the **same dimensions**.

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

Matrix Product

- ▶ The **matrix product** of matrices **A** and **B** is a third matrix **C**, where $\mathbf{C} = \mathbf{AB}$.
- ▶ If **A** is of shape $m \times n$ and **B** is of shape $n \times p$, then **C** is of shape $m \times p$.

$$c_{ij} = \sum_k a_{ik} b_{kj}$$



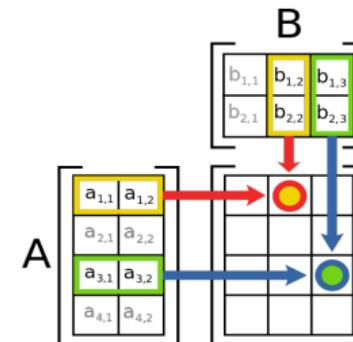
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- ▶ Properties
 - Associative: $(\mathbf{AB})\mathbf{C} = \mathbf{A}(\mathbf{BC})$
 - Not commutative: $\mathbf{AB} \neq \mathbf{BA}$



[https://en.wikipedia.org/wiki/Matrix_multiplication]



Matrix Transpose

- ▶ Swap the rows and columns of a matrix.

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \Rightarrow \mathbf{A}^T = \begin{bmatrix} a & c & e \\ b & d & f \end{bmatrix}$$



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- ▶ Properties

- $\mathbf{A}_{ij} = \mathbf{A}_{ji}^T$
- If \mathbf{A} is $m \times n$, then \mathbf{A}^T is $n \times m$
- $(\mathbf{A} + \mathbf{B})^T = \mathbf{A}^T + \mathbf{B}^T$
- $(\mathbf{AB})^T = \mathbf{B}^T \mathbf{A}^T$



Inverse of a Matrix

- If \mathbf{A} is a **square** matrix, its **inverse** is called \mathbf{A}^{-1} .

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$$

- Where \mathbf{I} , the **identity** matrix, is a **diagonal matrix** with all **1's** on the diagonal.

$$\mathbf{I}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \mathbf{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



L^p Norm for Vectors

- ▶ We can measure the **size of vectors** using a **norm** function.
- ▶ Norms are functions **mapping vectors to non-negative values**.
- ▶ L¹ norm

$$\|x\|_1 = \sum_i |x_i|$$



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- ▶ L² norm

$$\|\mathbf{x}\|_2 = \left(\sum_i |x_i|^2 \right)^{\frac{1}{2}} = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$$



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$$\|\mathbf{x}\|_p = \left(\sum_i |x_i|^p \right)^{\frac{1}{p}}$$



Probability Review



Random Variables

- ▶ Random variable: a **variable** that can take on **different values randomly**.
- ▶ Random variables may be **discrete** or **continuous**.
 - Discrete random variable: **finite or countably infinite number of states**
 - Continuous random variable: **real value**



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- ▶ Random variables may be **discrete** or **continuous**.
 - Discrete random variable: **finite or countably infinite number of states**
 - Continuous random variable: **real value**
- ▶ Notation:
 - Denoted by an **upper case letter**, e.g., **X**
 - Values of a random variable **X** are denoted by **lower case letters**, e.g., **x** and **y**.



Probability Distributions

- ▶ **Probability distribution:** how likely a **random variable** is to take on each of its **possible states**.
 - E.g., the **random variable** **X** denotes the outcome of a **coin toss**.
 - The **probability distribution** of **X** would take the value **0.5** for **X = head**, and **0.5** for **Y = tail** (assuming the coin is **fair**).



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- ▶ The way we **describe probability distributions** depends on whether the variables are **discrete** or **continuous**.



Discrete Variables

- ▶ Probability mass function (PMF): the probability distribution of a discrete random variable X .
- ▶ Notation: denoted by a lowercase p .
 - E.g., $p(x) = 1$ indicates that $X = x$ is certain
 - E.g., $p(x) = 0$ indicates that $X = x$ is impossible



Discrete Variables

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 - E.g., $p(x) = 1$ indicates that $X = x$ is certain
 - E.g., $p(x) = 0$ indicates that $X = x$ is impossible
- ▶ Properties:
 - The domain D of p must be the set of all possible states of X
 - $\forall x \in D(X), 0 \leq p(x) \leq 1$
 - $\sum_{x \in D(X)} p(x) = 1$



Independence

- ▶ Two random variables X and Y are **independent**, if their **probability distribution** can be expressed as their **products**.

$$\forall x \in D(X), y \in D(Y), p(X = x, Y = y) = p(X = x)p(Y = y)$$



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$$p(X = \text{head}, Y = 3) = p(X = \text{head})p(Y = 3) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$



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 - E.g., X and Y random variables for the first and the second labs, respectively.

$$p(Y = \text{lab2} \mid X = \text{lab1}) = \frac{p(Y = \text{lab2}, X = \text{lab1})}{p(X = \text{lab1})} = \frac{0.6}{0.8} = \frac{3}{4}$$



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- The **standard deviation**, shown by σ , is the **square root of the variance**.

Covariance (1/2)

- ▶ The **covariance** gives some sense of **how much two values are linearly related** to each other.

$$\text{Cov}(X, Y) = E[(X - E[X])(Y - E[Y])]$$

$$\text{Cov}(X, Y) = \sum_{(x,y)} p(x, y)(x - E[X])(y - E[Y])$$

Covariance (2/2)

			Y		
	p(X, Y)	1	2	3	p(X)
	1	1/4	1/4	0	1/2
X	2	0	1/4	1/4	1/2
	p(Y)	1/4	1/2	1/4	1

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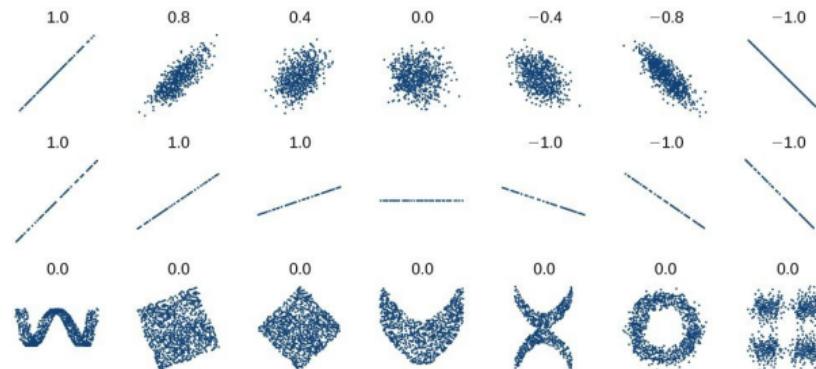
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$$\begin{aligned}
 &= \frac{1}{4} \left(1 - \frac{3}{2}\right)(1 - 2) + \frac{1}{4} \left(1 - \frac{3}{2}\right)(2 - 2) + 0 \left(1 - \frac{3}{2}\right)(3 - 2) + \\
 &= 0 \left(2 - \frac{3}{2}\right)(1 - 2) + \frac{1}{4} \left(2 - \frac{3}{2}\right)(2 - 2) + \frac{1}{4} \left(2 - \frac{3}{2}\right)(3 - 2) = \frac{1}{4}
 \end{aligned}$$

Correlation Coefficient

- The **Correlation coefficient** is a quantity that measures the **strength** of the **association** (or dependence) between two random variables, e.g., **X** and **Y**.

$$\rho(X, Y) = \frac{\text{Cov}(X, Y)}{\sigma(X)\sigma(Y)}$$





Probability and Likelihood (1/2)

- ▶ Let $X : \{x^{(1)}, x^{(2)}, \dots, x^{(m)}\}$ be a discrete random variable drawn independently from a distribution probability p depending on a parameter θ .



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- ▶ $p(X | \theta = \frac{2}{3})$ is the **probability** of X given $\theta = \frac{2}{3}$.
- ▶ $p(X = h | \theta)$ is the **likelihood** of θ given $X = h$.
- ▶ **Likelihood (L)**: a function of the **parameters (θ)** of a probability model, given **specific observed data**, e.g., $X = h$.

$$L(\theta | X) = p(X | \theta)$$



Probability and Likelihood (2/2)

- ▶ The **likelihood** differs from that of a **probability**.
- ▶ A **probability** $p(X | \theta)$ refers to the occurrence of **future events**.
- ▶ A **likelihood** $L(\theta | X)$ refers to **past events** with known outcomes.



Maximum Likelihood Estimator

- If samples in \mathbf{X} are **independent** we have:

$$\begin{aligned} L(\theta \mid \mathbf{X}) &= p(\mathbf{X} \mid \theta) = p(x^{(1)}, x^{(2)}, \dots, x^{(m)} \mid \theta) \\ &= p(x^{(1)} \mid \theta)p(x^{(2)} \mid \theta) \cdots p(x^{(m)} \mid \theta) = \prod_{i=1}^m p(x^{(i)} \mid \theta) \end{aligned}$$

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- The **maximum likelihood estimator (MLE)**: what is the **most likely value** of θ given the training set?

$$\hat{\theta}_{\text{MLE}} = \arg \max_{\theta} L(\theta \mid X) = \arg \max_{\theta} \prod_{i=1}^m p(x^{(i)} \mid \theta)$$



Maximum Likelihood Estimator - Example

- ▶ Six tosses of a coin, with the following model:
 - Possible outcomes: **h** with probability of θ , and **t** with probability $(1 - \theta)$.
 - Results of coin tosses are **independent of one another**.
- ▶ Data: **X** : {**h, t, t, t, h, t**}



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- ▶ The likelihood is

$$\begin{aligned}L(\theta | X) &= p(X | \theta) \\&= p(X = h | \theta)p(X = t | \theta)p(X = t | \theta)p(X = t | \theta)p(X = h | \theta)p(X = t | \theta) \\&= \theta(1 - \theta)(1 - \theta)(1 - \theta)\theta(1 - \theta) \\&= \theta^2(1 - \theta)^4\end{aligned}$$



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 - Possible outcomes: **h** with probability of θ , and **t** with probability $(1 - \theta)$.
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- ▶ $\hat{\theta}$ is the value of θ that maximizes the likelihood:

$$\hat{\theta}_{MLE} = \arg \max_{\theta} L(\theta | X) = \frac{2}{2 + 4}$$



Log-Likelihood

- ▶ The MLE product is prone to numerical underflow.

$$\hat{\theta}_{\text{MLE}} = \arg \max_{\theta} L(\theta \mid X) = \arg \max_{\theta} \prod_{i=1}^m p(x^{(i)} \mid \theta)$$



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$$\hat{\theta}_{\text{MLE}} = \arg \max_{\theta} L(\theta | X) = \arg \max_{\theta} \prod_{i=1}^m p(x^{(i)} | \theta)$$

- To overcome this problem we can use the logarithm of the likelihood.
 - It does not change its arg max, but transforms a product into a sum.

$$\hat{\theta}_{\text{MLE}} = \arg \max_{\theta} \sum_{i=1}^m \log p(x^{(i)} | \theta)$$



Negative Log-Likelihood

- ▶ Likelihood: $L(\theta \mid X) = \prod_{i=1}^m p(x^{(i)} \mid \theta)$



Negative Log-Likelihood

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- ▶ Negative log-likelihood is also called the **cross-entropy**



Cross-Entropy

- ▶ Cross-entropy: quantify the difference (error) between two probability distributions.
- ▶ How close is the predicted distribution to the true distribution?

$$H(p, q) = - \sum_x p(x) \log(q(x))$$

- ▶ Where p is the true distribution, and q the predicted distribution.



Cross-Entropy - Example

- ▶ Six tosses of a coin: $X : \{h, t, t, t, h, t\}$
- ▶ The true distribution p : $p(h) = \frac{2}{6}$ and $p(t) = \frac{4}{6}$
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- ▶ The predicted distribution q : h with probability of θ , and t with probability $(1 - \theta)$.
- ▶ Likelihood: $\theta^2(1 - \theta)^4$
- ▶ Negative log likelihood: $-\log(\theta^2(1 - \theta)^4) = -2\log(\theta) - 4\log(1 - \theta)$
- ▶ Cross entropy: $H(p, q) = -\sum_x p(x)\log(q(x))$
 $= -p(h)\log(q(h)) + p(t)\log(q(t)) = -\frac{2}{6}\log(\theta) - \frac{4}{6}\log(1 - \theta)$



Summary



Summary

- ▶ Logic-based AI, Machine Learning, Deep Learning
- ▶ Deep Learning models
 - Deep Feed Forward
 - Convolutional Neural Network (CNN)
 - Recurrent Neural Network (RNN)
 - Autoencoders
- ▶ Linear algebra and probability
 - Random variables
 - Probability distribution
 - Likelihood
 - Negative log-likelihood and cross-entropy



References

- ▶ Ian Goodfellow et al., Deep Learning (Ch. 1, 2, 3)



Questions?

Acknowledgements

Some of the pictures were copied from the book Hands-On Machine Learning with Scikit-Learn and TensorFlow, Aurelien Geron, O'Reilly Media, 2017.