



# Introduction

Amir H. Payberah  
[payberah@kth.se](mailto:payberah@kth.se)  
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# Course Information



## Course Objective

- ▶ This course has a **system-based** focus
- ▶ Learn the theory of **machine learning** and **deep learning**
- ▶ Learn the practical aspects of building **machine learning** and **deep learning** algorithms using data parallel programming platforms, such as **Spark** and **TensorFlow**



# Topics of Study

- ▶ Part 1: large scale **machine learning**
- ▶ Part 2: large scale **deep learning**



# Topics of Study

- ▶ Part 1: large scale **machine learning**
  - Spark ML
  - Linear regression and logistic regression
  - Decision tree and ensemble models
  
- ▶ Part 2: large scale **deep learning**



# Topics of Study

► Part 1: large scale **machine learning**

- Spark ML
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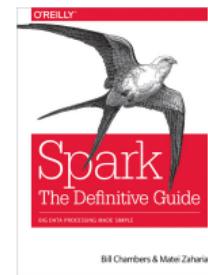
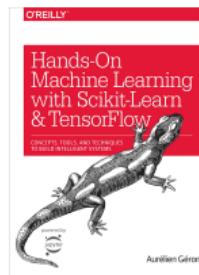
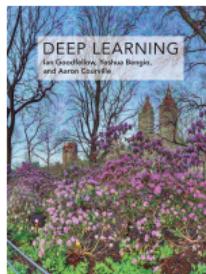
► Part 2: large scale **deep learning**

- TensorFlow
- Deep feedforward networks
- Convolutional neural networks (CNNs)
- Recurrent neural networks (RNNs)
- Autoencoders and Restricted Boltzmann machines (RBMs)



# The Course Material

- ▶ Deep learning, I. Goodfellow et al., Cambridge: MIT press, 2016
- ▶ Hands-on machine learning with Scikit-Learn and TensorFlow, A. Geron, O'Reilly Media, 2017
- ▶ Spark - The Definitive Guide, M. Zaharia et al., O'Reilly Media, 2018.





## The Course Grading

- ▶ Two lab assignments: 30%
- ▶ One final project: 20%
- ▶ Eight review questions: 20%
- ▶ The final exam: 30%



# The Labs and Project

- ▶ Self-selected groups of two
- ▶ Labs
  - Include Scala/Python programming
  - Lab1: Regression using Spark ML
  - Lab2: Deep neural network and CNN using Tensorflow
- ▶ Project
  - Selection of a large dataset and method
  - RNNs, Autoencoders, or RBMs
  - Demonstrated as a demo and short report



## The Course Web Page

<https://id2223kth.github.io>



# The Course Overview

# Sheepdog or Mop





# Chihuahua or Muffin



@teenybiscuit

# Barn Owl or Apple



@teenybiscuit

# Raw Chicken or Donald Trump





# Artificial Intelligence Challenge

- ▶ Artificial intelligence (AI) can solve problems that can be described by a list of formal mathematical rules.



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- ▶ The challenge is to solve the tasks that are hard for people to describe formally.



# Artificial Intelligence Challenge

- ▶ Artificial intelligence (AI) can solve problems that can be described by a list of formal mathematical rules.
- ▶ The challenge is to solve the tasks that are hard for people to describe formally.
- ▶ Let computers to learn from experience.



# History of AI

# Greek Myths

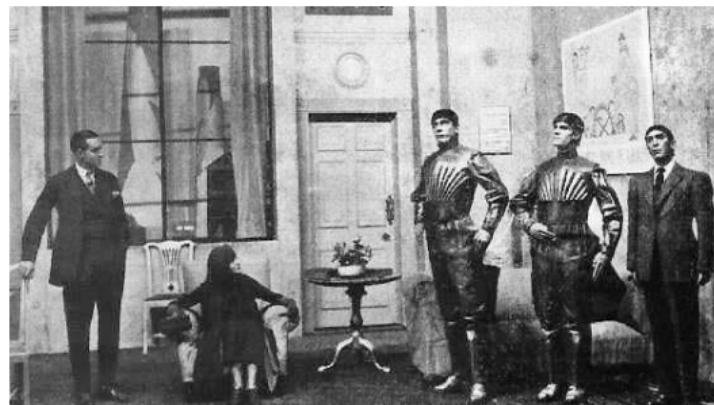
- ▶ Hephaestus, the god of blacksmith, created a metal automaton, called Talos.



[the left figure: <http://mythologian.net/hephaestus-the-blacksmith-of-gods>]  
[the right figure: <http://elderscrolls.wikia.com/wiki/Talos>]

## 1920: Rossum's Universal Robots (R.U.R.)

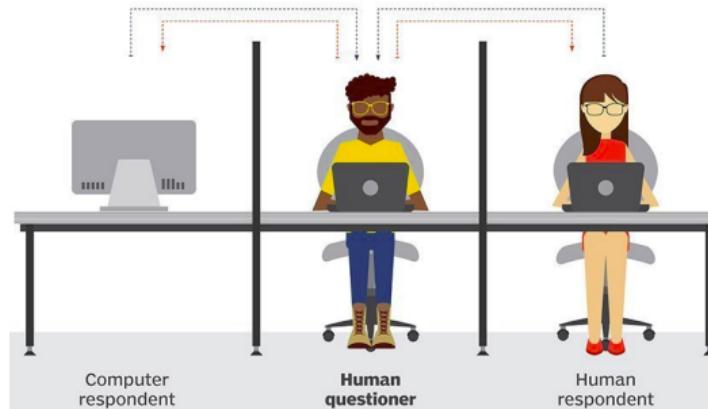
- ▶ A science fiction play by Karel Čapek, in 1920.
- ▶ A factory that creates artificial people named robots.



[<https://dev.to/lshultebraucks/a-short-history-of-artificial-intelligence-7hm>]

# 1950: Turing Test

- ▶ In 1950, **Turing** introduced the **Turing test**.
- ▶ An attempt to define **machine intelligence**.



[<https://searchenterpriseai.techtarget.com/definition/Turing-test>]



## 1956: The Dartmouth Workshop

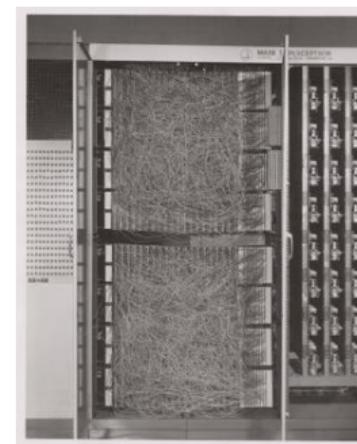
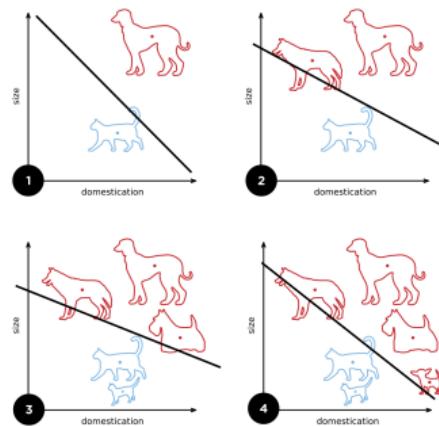
- ▶ Probably the **first workshop of AI**.
- ▶ Researchers from **CMU, MIT, IBM** met together and founded the **AI research**.



[<https://twitter.com/lordsaicom/status/898139880441696257>]

# 1958: Perceptron

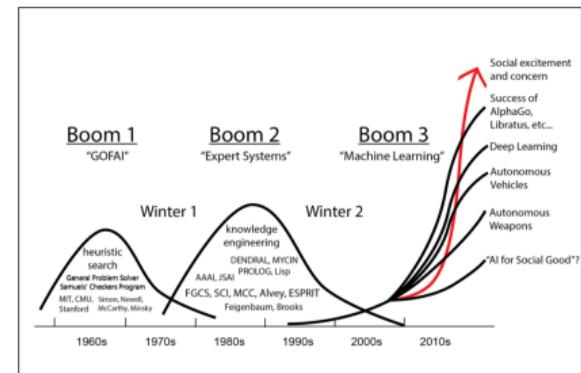
- ▶ A **supervised learning** algorithm for **binary classifiers**.
- ▶ Implemented in custom-built hardware as the **Mark 1 perceptron**.



[<https://en.wikipedia.org/wiki/Perceptron>]

# 1974–1980: The First AI Winter

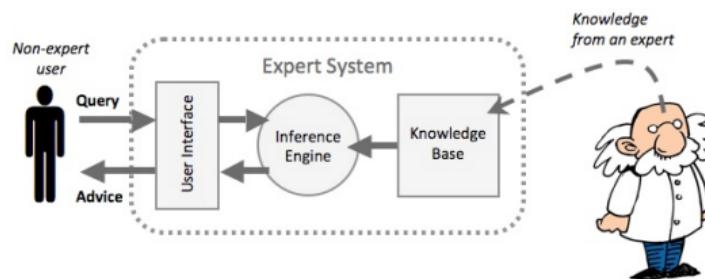
- ▶ The over **optimistic settings**, which were not occurred
- ▶ The **problems**:
  - Limited **computer power**
  - Lack of **data**
  - Intractability and the **combinatorial explosion**



[<http://www.technologystories.org/ai-evolution>]

# 1980's: Expert systems

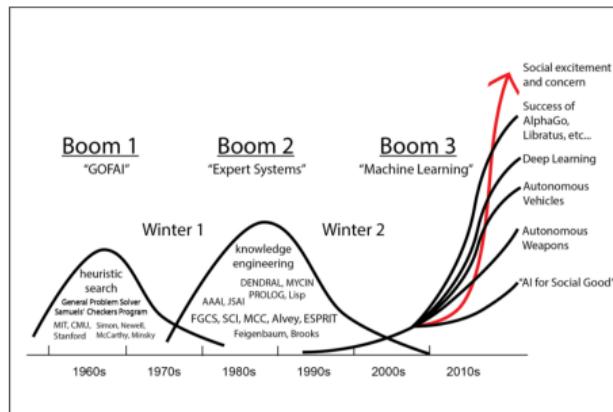
- ▶ The programs that solve problems in a **specific domain**.
- ▶ **Two** engines:
  - Knowledge engine: **represents** the **facts and rules** about a specific topic.
  - Inference engine: **applies** the **facts and rules** from the knowledge engine to new facts.



[[https://www.igcseict.info/theory/7\\_2/expert](https://www.igcseict.info/theory/7_2/expert)]

## 1987–1993: The Second AI Winter

- ▶ After a series of financial setbacks.
- ▶ The fall of **expert systems** and hardware companies.



[<http://www.technologystories.org/ai-evolution>]



## 1997: IBM Deep Blue

- ▶ The first chess computer to beat a world chess champion Garry Kasparov.



[<http://marksist.org/icerik/Tarihte-Bugun/1757/11-Mayis-1997-Deep-Blue-adli-bilgisayar>]



## 2012: AlexNet - Image Recognition

- ▶ The **ImageNet competition** in **image classification**.
- ▶ The **AlexNet Convolutional Neural Network (CNN)** won the challenge by a **large margin**.

IM<sub>AGE</sub>NET

## 2016: DeepMind AlphaGo

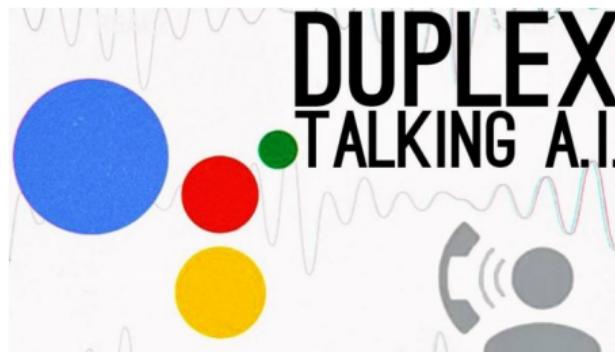
- ▶ DeepMind **AlphaGo** won **Lee Sedol**, one of the best players at Go.
- ▶ In 2017, DeepMind published **AlphaGo Zero**.
  - The **next generation** of AlphaGo.
  - It learned Go by playing **against itself**.



[<https://www.zdnet.com/article/google-alphago-caps-victory-by-winning-final-historic-go-match>]

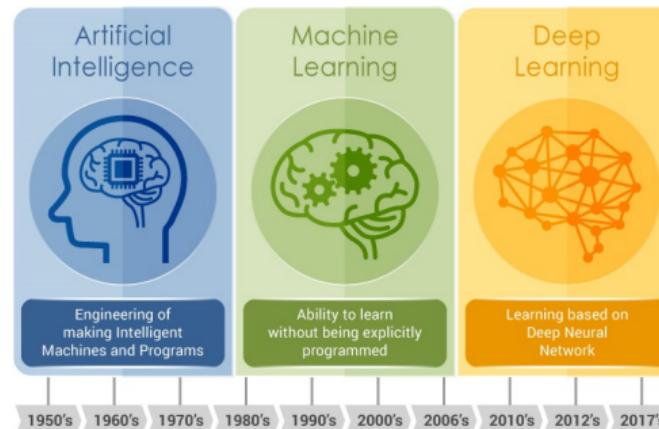
## 2018: Google Duplex

- ▶ An AI system for accomplishing real-world tasks over the phone.
- ▶ A Recurrent Neural Network (RNN) built using TensorFlow.



# AI Generations

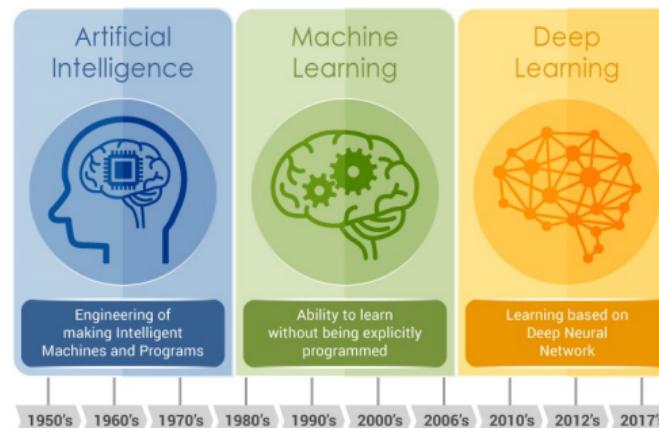
- ▶ Rule-based AI
- ▶ Machine learning
- ▶ Deep learning



[<https://bit.ly/2woLEzs>]

# AI Generations - Rule-based AI

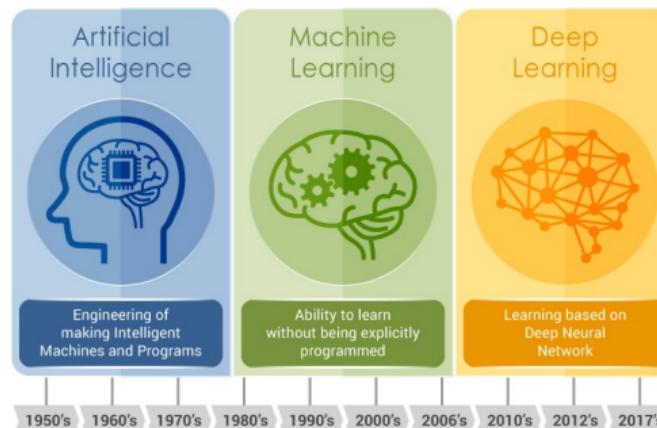
- ▶ Hard-code knowledge
- ▶ Computers reason using logical inference rules



[<https://bit.ly/2woLEzs>]

# AI Generations - Machine Learning

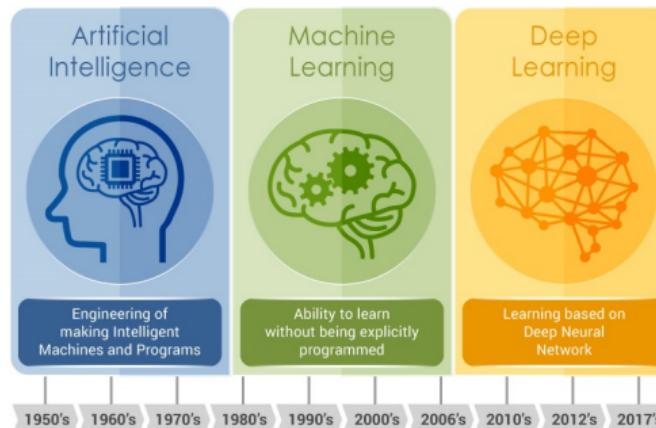
- ▶ If AI systems acquire **their own knowledge**
- ▶ **Learn from data without being explicitly programmed**



[<https://bit.ly/2woLEzs>]

# AI Generations - Deep Learning

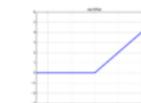
- ▶ For many tasks, it is **difficult to know what features** should be extracted
- ▶ Use **machine learning** to **discover** the mapping from **representation to output**



[<https://bit.ly/2woLEzs>]

# Why Does Deep Learning Work Now?

- ▶ Huge **quantity** of data
- ▶ Tremendous increase in **computing power**
- ▶ Better **training** algorithms



Non-Linearity



# Machine Learning and Deep Learning



# Learning Algorithms

- ▶ A **ML algorithm** is an algorithm that is able to **learn** from data.
- ▶ What is **learning**?

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- ▶ A **ML algorithm** is an algorithm that is able to **learn from data**.
- ▶ What is **learning**?
- ▶ A computer program is said to **learn** from **experience E** with respect to some class of **tasks T** and **performance measure P**, if its performance at tasks in **T**, as measured by **P**, improves with experience **E**. (Tom M. Mitchell)



# Learning Algorithms - Example 1

- ▶ A **spam filter** that can learn to flag **spam** given examples of **spam emails** and examples of **regular emails**.



[<https://bit.ly/2oiplYM>]

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- ▶ A **spam filter** that can learn to flag **spam** given examples of **spam emails** and examples of **regular emails**.
- ▶ **Task T**: flag spam for new emails
- ▶ **Experience E**: the training data
- ▶ **Performance measure P**: the ratio of correctly classified emails



[<https://bit.ly/2oiplYM>]

## Learning Algorithms - Example 2

- ▶ Given dataset of prices of 500 houses, how can we learn to **predict the prices** of other houses, as a **function of the size of their living areas**?



[<https://bit.ly/2MyiJUy>]

## Learning Algorithms - Example 2

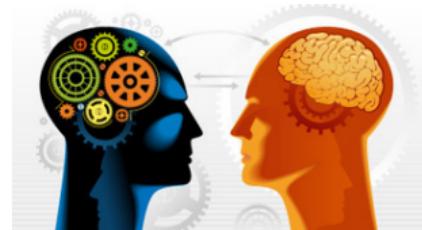
- ▶ Given dataset of prices of 500 houses, how can we learn to **predict the prices** of other houses, as a **function of the size of their living areas?**
- ▶ **Task T:** predict the price
- ▶ **Experience E:** the dataset of living areas and prices
- ▶ **Performance measure P:** the difference between the predicted price and the real price



[<https://bit.ly/2MyiJUy>]

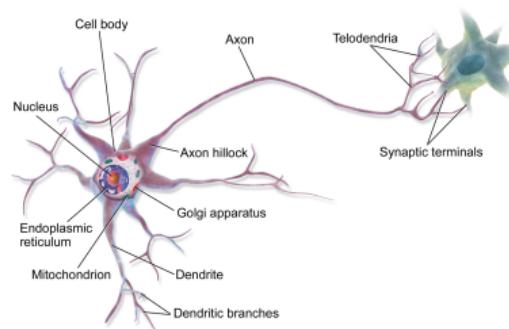
# Types of Machine Learning Algorithms

- ▶ Supervised learning
  - Input data is **labeled**, e.g., spam/not-spam or a stock price at a time.
  - Regression vs. classification
  
- ▶ Unsupervised learning
  - Input data is **unlabeled**.
  - Find **hidden structures** in data.



# From Machine Learning to Deep Learning

- ▶ Deep Learning (DL) is part of ML methods based on learning data representations.
- ▶ Mimic the neural networks of our brain.



[A. Geron, O'Reilly Media, 2017]



# Artificial Neural Networks

- ▶ Artificial Neural Network (ANN) is inspired by biological neurons.

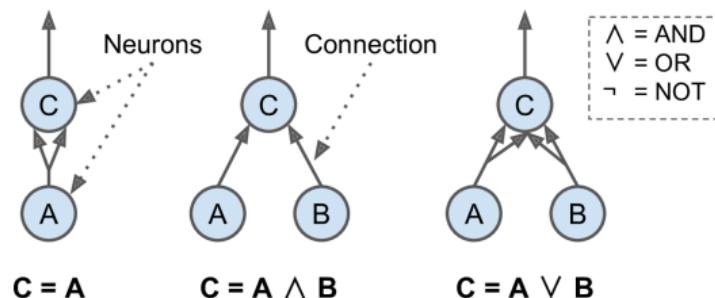


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- ▶ Activates its output when more than a certain number of its inputs are active.

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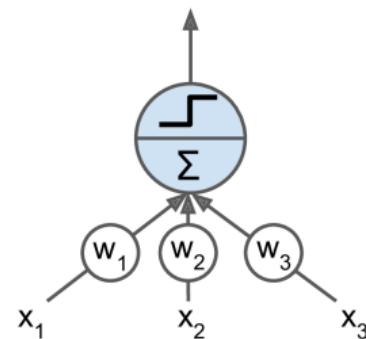
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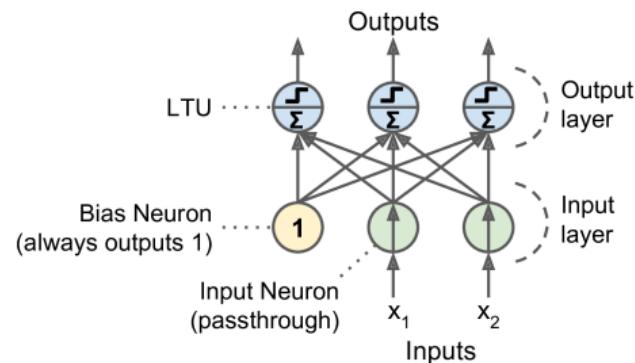
- ▶ Inputs of a LTU are **numbers** (not binary).
- ▶ Each **input connection** is associated with a **weight**.
- ▶ Computes a **weighted sum of its inputs** and applies a **step function** to that **sum**.

- ▶  $z = w_1x_1 + w_2x_2 + \dots + w_nx_n = \mathbf{w}^T\mathbf{x}$
- ▶  $\hat{y} = \text{step}(z) = \text{step}(\mathbf{w}^T\mathbf{x})$



# The Perceptron

- ▶ The **perceptron** is a **single layer** of LTUs.
- ▶ The **input neurons** output whatever **input** they are fed.
- ▶ A **bias neuron**, which just **outputs 1** all the time.



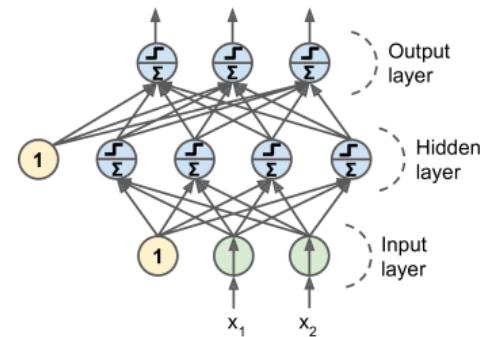


# Deep Learning Models

- ▶ Deep Neural Network ([DNN](#))
- ▶ Convolutional Neural Network ([CNN](#))
- ▶ Recurrent Neural Network ([RNN](#))
- ▶ Autoencoders

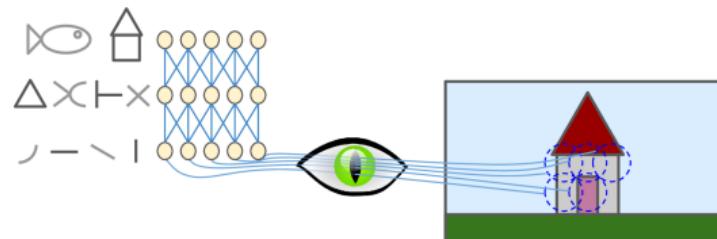
# Deep Neural Networks

- ▶ Multi-Layer Perceptron (MLP)
  - One input layer
  - One or more layers of LTUs (hidden layers)
  - One final layer of LTUs (output layer)
- ▶ Deep Neural Network (DNN) is an ANN with two or more hidden layers.
- ▶ Backpropagation training algorithm



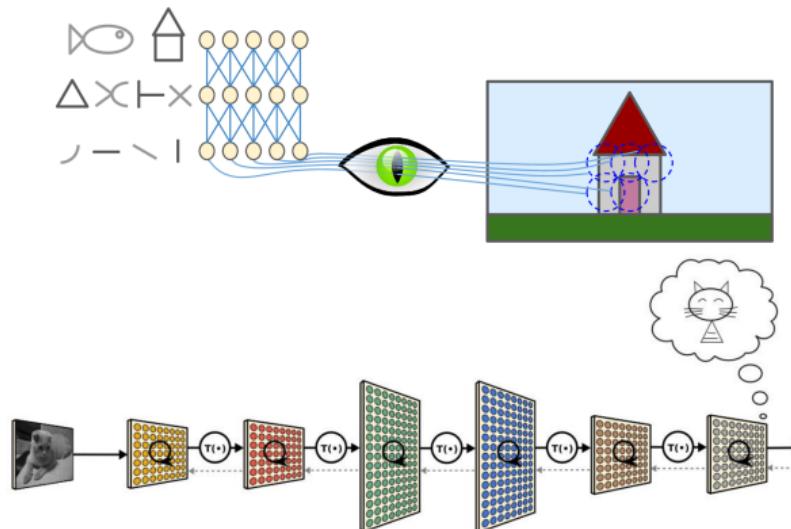
# Convolutional Neural Networks

- ▶ Many neurons in the **visual cortex** react only to a **limited region** of the visual field.



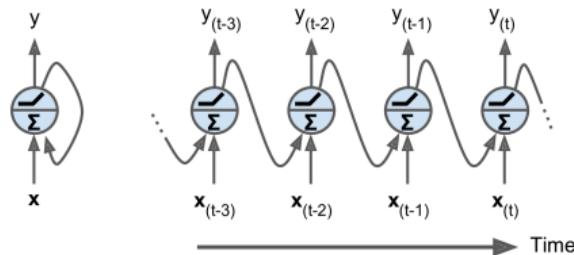
# Convolutional Neural Networks

- ▶ Many neurons in the **visual cortex** react only to a **limited region** of the visual field.
- ▶ The **higher-level** neurons are based on the outputs of **neighboring lower-level** neurons



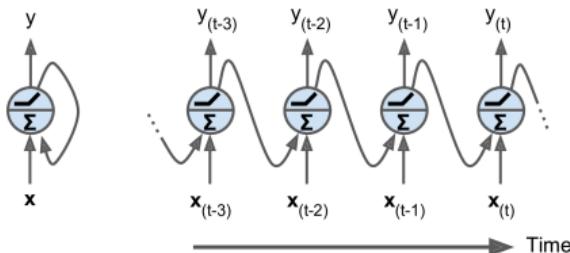
# Recurrent Neural Networks

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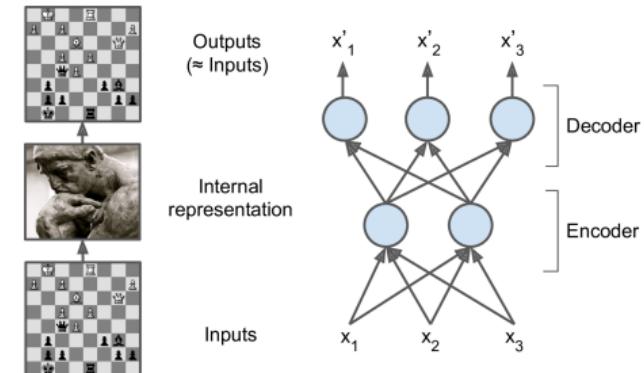


- ▶ Analyze **time series data**, e.g., stock market, and autonomous driving systems
- ▶ Work on sequences of **arbitrary lengths**, rather than on **fixed-sized inputs**



# Autoencoders

- ▶ Learn **efficient representations** of the input data, **without any supervision**.
  - With a **lower dimensionality** than the input data
- ▶ **Generative model**: generate **new data** that looks very similar to the training data.
- ▶ Preserve **as much information as possible**



[A. Geron, O'Reilly Media, 2017]



# Linear Algebra Review

# Vector

- ▶ A **vector** is an **array of numbers**.
- ▶ Notation:
  - Denoted by **bold lowercase letters**, e.g., **x**.
  - **x<sub>i</sub>** denotes the **i**th entry.

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$



# Matrix and Tensor

- ▶ A **matrix** is a 2-D array of numbers.
- ▶ A **tensor** is an array with more than two axes.
- ▶ Notation:
  - Denoted by **bold uppercase letters**, e.g., **A**.
  - $a_{ij}$  denotes the entry in  $i$ th row and  $j$ th column.
  - If **A** is  $m \times n$ , it has  $m$  rows and  $n$  columns.

$$\mathbf{A} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \dots & a_{1,n} \\ a_{2,1} & a_{2,2} & a_{2,3} & \dots & a_{2,n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m,1} & a_{m,2} & a_{m,3} & \dots & a_{m,n} \end{bmatrix}$$



# Matrix Addition and Subtraction

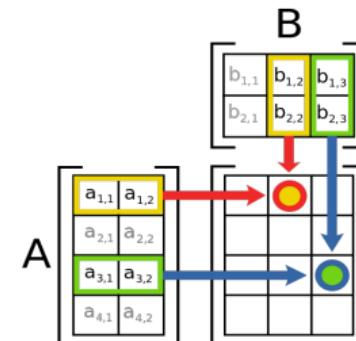
- The **matrices** must have the **same dimensions**.

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

# Matrix Product

- ▶ The **matrix product** of matrices **A** and **B** is a third matrix **C**, where  $\mathbf{C} = \mathbf{AB}$ .
- ▶ If **A** is of shape  $m \times n$  and **B** is of shape  $n \times p$ , then **C** is of shape  $m \times p$ .

$$c_{ij} = \sum_k a_{ik} b_{kj}$$



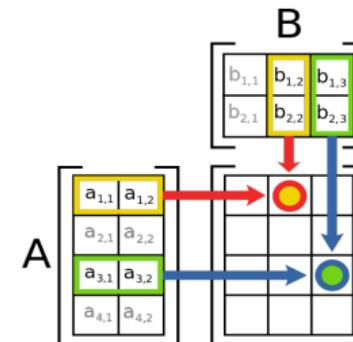
[[https://en.wikipedia.org/wiki/Matrix\\_multiplication](https://en.wikipedia.org/wiki/Matrix_multiplication)]

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- ▶ Properties
  - Associative:  $(\mathbf{AB})\mathbf{C} = \mathbf{A}(\mathbf{BC})$
  - Not commutative:  $\mathbf{AB} \neq \mathbf{BA}$



[[https://en.wikipedia.org/wiki/Matrix\\_multiplication](https://en.wikipedia.org/wiki/Matrix_multiplication)]



# Matrix Transpose

- ▶ Swap the rows and columns of a matrix.

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \Rightarrow \mathbf{A}^T = \begin{bmatrix} a & c & e \\ b & d & f \end{bmatrix}$$



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- ▶ Properties

- $\mathbf{A}_{ij} = \mathbf{A}_{ji}^T$
- If  $\mathbf{A}$  is  $m \times n$ , then  $\mathbf{A}^T$  is  $n \times m$
- $(\mathbf{A} + \mathbf{B})^T = \mathbf{A}^T + \mathbf{B}^T$
- $(\mathbf{AB})^T = \mathbf{B}^T \mathbf{A}^T$



## Inverse of a Matrix

- If  $\mathbf{A}$  is a **square** matrix, its **inverse** is called  $\mathbf{A}^{-1}$ .

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$$

- Where  $\mathbf{I}$ , the **identity** matrix, is a **diagonal matrix** with all **1's** on the diagonal.

$$\mathbf{I}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \mathbf{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



## L<sup>p</sup> Norm for Vectors

- ▶ We can measure the **size of vectors** using a **norm** function.
- ▶ Norms are functions **mapping vectors to non-negative values**.
- ▶ L<sup>1</sup> norm

$$\|x\|_1 = \sum_i |x_i|$$



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- ▶ L<sup>2</sup> norm

$$\|\mathbf{x}\|_2 = \left( \sum_i |x_i|^2 \right)^{\frac{1}{2}} = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$$



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$$\|\mathbf{x}\|_p = \left( \sum_i |x_i|^p \right)^{\frac{1}{p}}$$



# Probability Review



# Random Variables

- ▶ Random variable: a **variable** that can take on **different values randomly**.
- ▶ Random variables may be **discrete** or **continuous**.
  - Discrete random variable: **finite or countably infinite number of states**
  - Continuous random variable: **real value**



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  - Discrete random variable: **finite or countably infinite number of states**
  - Continuous random variable: **real value**
- ▶ Notation:
  - Denoted by an **upper case letter**, e.g., **X**
  - Values of a random variable **X** are denoted by **lower case letters**, e.g., **x** and **y**.



# Probability Distributions

- ▶ **Probability distribution:** how likely a **random variable** is to take on each of its **possible states**.
  - E.g., the **random variable** **X** denotes the outcome of a **coin toss**.
  - The **probability distribution** of **X** would take the value **0.5** for **X = head**, and **0.5** for **Y = tail** (assuming the coin is **fair**).



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- ▶ The way we **describe probability distributions** depends on whether the variables are **discrete** or **continuous**.



# Discrete Variables

- ▶ Probability mass function (PMF): the probability distribution of a discrete random variable  $X$ .
- ▶ Notation: denoted by a lowercase  $p$ .
  - E.g.,  $p(x) = 1$  indicates that  $X = x$  is certain
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- ▶ Properties:
  - The domain  $D$  of  $p$  must be the set of all possible states of  $X$
  - $\forall x \in D(X), 0 \leq p(x) \leq 1$
  - $\sum_{x \in D(X)} p(x) = 1$



## Independence

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$$p(X = \text{head}, Y = 3) = p(X = \text{head})p(Y = 3) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$



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  - E.g.,  $X$  and  $Y$  random variables for the first and the second labs, respectively.

$$p(Y = \text{lab2} \mid X = \text{lab1}) = \frac{p(Y = \text{lab2}, X = \text{lab1})}{p(X = \text{lab1})} = \frac{0.6}{0.8} = \frac{3}{4}$$



# Expectation

- ▶ The **expected value** of a random variable  $X$  with respect to a probability distribution  $p(x)$  is the **average** value that  $X$  takes on when it is drawn from  $p(x)$ .

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  - $\text{Var}(X) = 0.3(1 - 1.9)^2 + 0.5(2 - 1.9)^2 + 0.2(3 - 1.9)^2 = 0.49$
- The **standard deviation**, shown by  $\sigma$ , is the **square root of the variance**.

## Covariance (1/2)

- ▶ The **covariance** gives some sense of **how much two values are linearly related** to each other.

$$\text{Cov}(X, Y) = E[(X - E[X])(Y - E[Y])]$$

$$\text{Cov}(X, Y) = \sum_{(x,y)} p(x, y)(x - E[X])(y - E[Y])$$

## Covariance (2/2)

|   |         |     | Y   |     |      |
|---|---------|-----|-----|-----|------|
|   | p(X, Y) | 1   | 2   | 3   | p(X) |
|   | 1       | 1/4 | 1/4 | 0   | 1/2  |
| X | 2       | 0   | 1/4 | 1/4 | 1/2  |
|   | p(Y)    | 1/4 | 1/2 | 1/4 | 1    |

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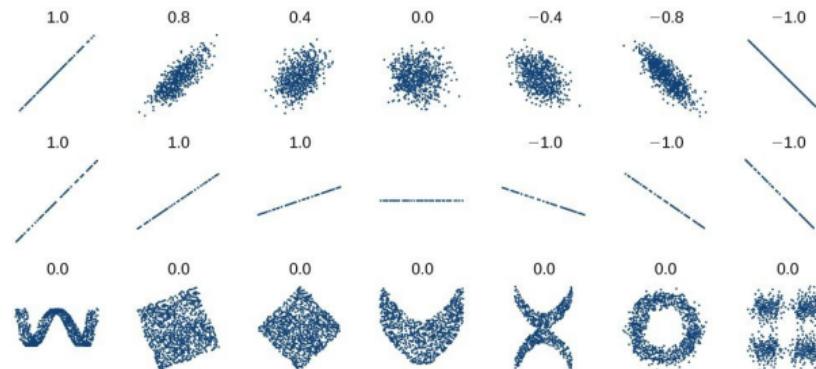
$$\text{Cov}(X, Y) = \sum \sum_{(x,y)} p(x,y)(x - E[X])(y - E[Y])$$

$$\begin{aligned}
&= \frac{1}{4} \left(1 - \frac{3}{2}\right)(1 - 2) + \frac{1}{4} \left(1 - \frac{3}{2}\right)(2 - 2) + 0 \left(1 - \frac{3}{2}\right)(3 - 2) + \\
&= 0 \left(2 - \frac{3}{2}\right)(1 - 2) + \frac{1}{4} \left(2 - \frac{3}{2}\right)(2 - 2) + \frac{1}{4} \left(2 - \frac{3}{2}\right)(3 - 2) = \frac{1}{4}
\end{aligned}$$

# Correlation Coefficient

- The **Correlation coefficient** is a quantity that measures the **strength** of the **association** (or dependence) between two random variables, e.g., **X** and **Y**.

$$\rho(X, Y) = \frac{\text{Cov}(X, Y)}{\sigma(X)\sigma(Y)}$$





## Probability and Likelihood (1/2)

- ▶ Let  $X : \{x^{(1)}, x^{(2)}, \dots, x^{(m)}\}$  be a discrete random variable drawn independently from a distribution probability  $p$  depending on a parameter  $\theta$ .



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- ▶  $p(X = h | \theta)$  is the **likelihood** of  $\theta$  given  $X = h$ .
- ▶ **Likelihood ( $L$ )**: a function of the **parameters ( $\theta$ )** of a probability model, given **specific observed data**, e.g.,  $X = h$ .

$$L(\theta | X) = p(X | \theta)$$



## Probability and Likelihood (2/2)

- ▶ The **likelihood** differs from that of a **probability**.
- ▶ A **probability**  $p(X | \theta)$  refers to the occurrence of **future events**.
- ▶ A **likelihood**  $L(\theta | X)$  refers to **past events** with known outcomes.



# Maximum Likelihood Estimator

- If samples in  $\mathbf{X}$  are **independent** we have:

$$\begin{aligned} L(\theta \mid \mathbf{X}) &= p(\mathbf{X} \mid \theta) = p(x^{(1)}, x^{(2)}, \dots, x^{(m)} \mid \theta) \\ &= p(x^{(1)} \mid \theta)p(x^{(2)} \mid \theta) \cdots p(x^{(m)} \mid \theta) = \prod_{i=1}^m p(x^{(i)} \mid \theta) \end{aligned}$$

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- The **maximum likelihood estimator (MLE)**: what is the **most likely value** of  $\theta$  given the training set?

$$\hat{\theta}_{\text{MLE}} = \arg \max_{\theta} L(\theta \mid X) = \arg \max_{\theta} \prod_{i=1}^m p(x^{(i)} \mid \theta)$$



## Maximum Likelihood Estimator - Example

- ▶ Six tosses of a coin, with the following model:
  - Possible outcomes: **h** with probability of  $\theta$ , and **t** with probability  $(1 - \theta)$ .
  - Results of coin tosses are independent of one another.
- ▶ Data: **X** : {**h, t, t, t, h, t**}



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- ▶ The likelihood is

$$\begin{aligned}L(\theta | X) &= p(X | \theta) \\&= p(X = h | \theta)p(X = t | \theta)p(X = t | \theta)p(X = t | \theta)p(X = h | \theta)p(X = t | \theta) \\&= \theta(1 - \theta)(1 - \theta)(1 - \theta)\theta(1 - \theta) \\&= \theta^2(1 - \theta)^4\end{aligned}$$



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- ▶  $\hat{\theta}$  is the value of  $\theta$  that maximizes the likelihood:

$$\hat{\theta}_{MLE} = \arg \max_{\theta} L(\theta | X) = \frac{2}{2 + 4}$$



## Log-Likelihood

- ▶ The MLE product is prone to numerical underflow.

$$\hat{\theta}_{\text{MLE}} = \arg \max_{\theta} L(\theta \mid X) = \arg \max_{\theta} \prod_{i=1}^m p(x^{(i)} \mid \theta)$$



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- To overcome this problem we can use the logarithm of the likelihood.
  - It does not change its arg max, but transforms a product into a sum.

$$\hat{\theta}_{\text{MLE}} = \arg \max_{\theta} \sum_{i=1}^m \log p(x^{(i)} \mid \theta)$$



# Negative Log-Likelihood

- ▶ Likelihood:  $L(\theta \mid X) = \prod_{i=1}^m p(x^{(i)} \mid \theta)$



## Negative Log-Likelihood

- ▶ Likelihood:  $L(\theta | X) = \prod_{i=1}^m p(x^{(i)} | \theta)$
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- ▶ Negative log-likelihood is also called the **cross-entropy**



## Cross-Entropy

- ▶ Cross-entropy: quantify the difference (error) between two probability distributions.
- ▶ How close is the predicted distribution to the true distribution?

$$H(p, q) = - \sum_x p(x) \log(q(x))$$

- ▶ Where  $p$  is the true distribution, and  $q$  the predicted distribution.



## Cross-Entropy - Example

- ▶ Six tosses of a coin:  $X : \{h, t, t, t, h, t\}$
- ▶ The true distribution  $p$ :  $p(h) = \frac{2}{6}$  and  $p(t) = \frac{4}{6}$
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- ▶ The predicted distribution  $q$ :  $h$  with probability of  $\theta$ , and  $t$  with probability  $(1 - \theta)$ .
- ▶ Likelihood:  $\theta^2(1 - \theta)^4$
- ▶ Negative log likelihood:  $-\log(\theta^2(1 - \theta)^4) = -2\log(\theta) - 4\log(1 - \theta)$
- ▶ Cross entropy:  $H(p, q) = -\sum_x p(x)\log(q(x))$   
 $= -p(h)\log(q(h)) + p(t)\log(q(t)) = -\frac{2}{6}\log(\theta) - \frac{4}{6}\log(1 - \theta)$



# Summary



# Summary

- ▶ Logic-based AI, Machine Learning, Deep Learning
- ▶ Deep Learning models
  - Deep Feed Forward
  - Convolutional Neural Network (CNN)
  - Recurrent Neural Network (RNN)
  - Autoencoders
- ▶ Linear algebra and probability
  - Random variables
  - Probability distribution
  - Likelihood
  - Negative log-likelihood and cross-entropy



## References

- ▶ Ian Goodfellow et al., Deep Learning (Ch. 1, 2, 3)



# Questions?

## Acknowledgements

Some of the pictures were copied from the book Hands-On Machine Learning with Scikit-Learn and TensorFlow, Aurelien Geron, O'Reilly Media, 2017.