



Training Deep Feedforwards Networks

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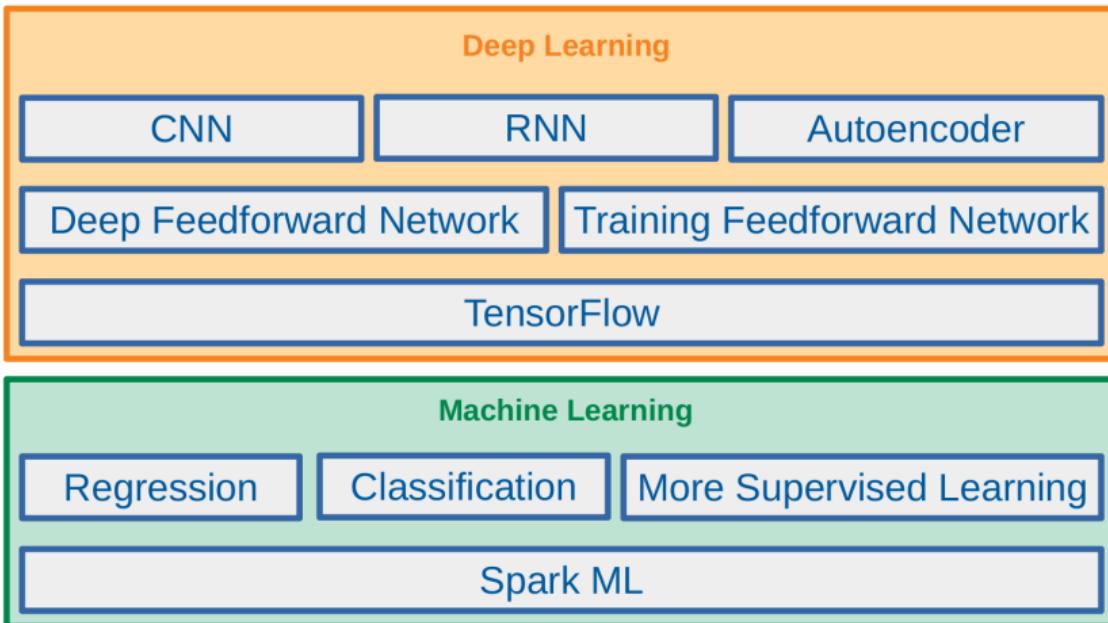


The Course Web Page

<https://id2223kth.github.io>

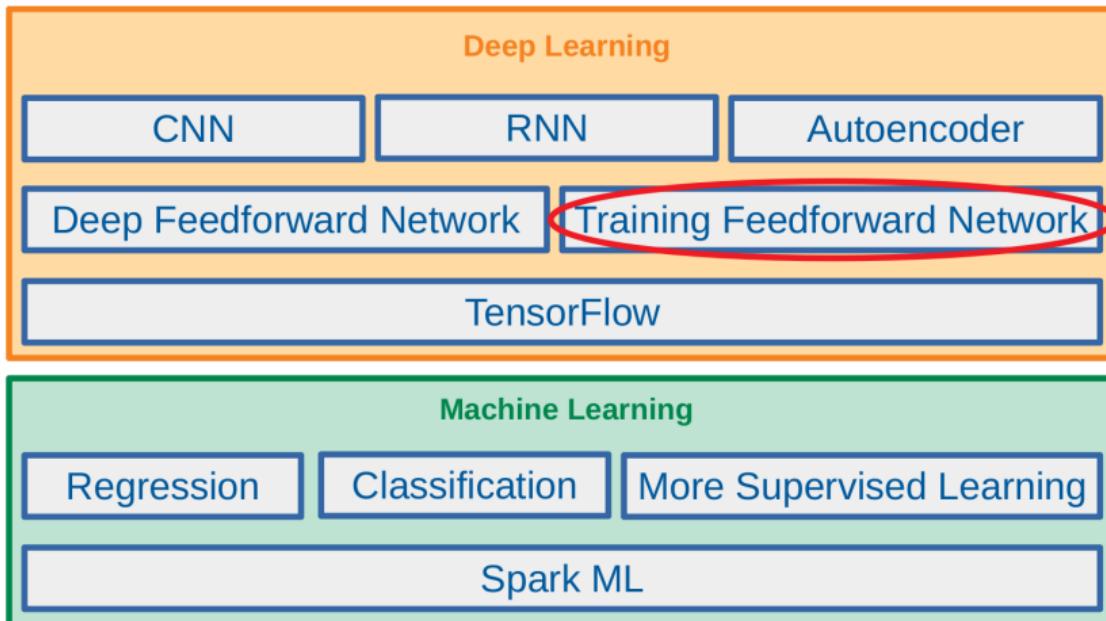


Where Are We?





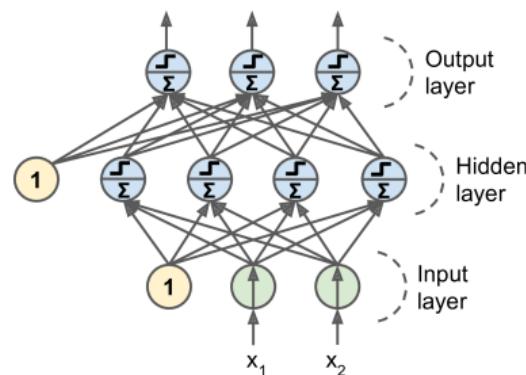
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Feedforward Neural Network Architecture

- ▶ A **feedforward neural network** is composed of:

- One **input layer**
- One or more **hidden layers**
- One final **output layer**



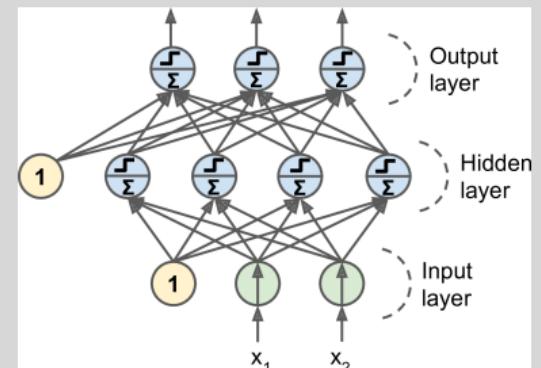
Feedforward Network in TensorFlow (1/2)

- ▶ `n_neurons_h`: number of neurons in the hidden layer.
- ▶ `n_neurons_out`: number of neurons in the output layer.
- ▶ `n_features`: number of features.

```
n_neurons_h = 4
n_neurons_out = 3
n_features = 2

# placeholder
X = tf.placeholder(tf.float32, shape=(None, n_features),
    name="X")
y_true = tf.placeholder(tf.int64, shape=(None),
    name="y")

# make the network
hidden = tf.layers.dense(X, n_neurons_h, name="hidden",
    activation=tf.sigmoid)
logit = tf.layers.dense(hidden, n_neurons_out, name="output") # logit = Wh + b
y_hat = tf.sigmoid(logit)
```





Feedforward Network in TensorFlow (2/2)

- ▶ Define the **cost** and the optimization, and **execute** the network.

```
# define the cost
cross_entropy = tf.nn.sigmoid_cross_entropy_with_logits(logit, y_true)
cost = tf.reduce_mean(cross_entropy)

# train the model
learning_rate = 0.1
optimizer = tf.train.GradientDescentOptimizer(learning_rate)
training_op = optimizer.minimize(cost)

# execute the model
init = tf.global_variables_initializer()

n_epochs = 100
with tf.Session() as sess:
    init.run()
    for epoch in range(n_epochs):
        sess.run(training_op, feed_dict={X: training_X, y_true: training_y})
```



Challenges of Training Feedforward Neural Networks

- ▶ Challenges ...





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- ▶ Overfitting: risk of overfitting a model with large number of parameters.



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Challenges of Training Feedforward Neural Networks

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- ▶ Overfitting: risk of overfitting a model with large number of parameters.
- ▶ Vanishing/exploding gradients: hard to train lower layers.
- ▶ Training speed: slow training with large networks.



Overfitting



High Degree of Freedom and Overfitting Problem

- ▶ With **large number of parameters**, a network has a **high degree of freedom**.
- ▶ It can **fit** a huge variety of **complex datasets**.



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- ▶ This **flexibility** also means that it is **prone to overfitting on training set**.



High Degree of Freedom and Overfitting Problem

- ▶ With **large number of parameters**, a network has a **high degree of freedom**.
- ▶ It can **fit** a huge variety of **complex datasets**.
- ▶ This **flexibility** also means that it is **prone to overfitting** on training set.
- ▶ **Regularization**: a way to **reduce** the risk of **overfitting**.
- ▶ It **reduces** the **degree of freedom** a model.



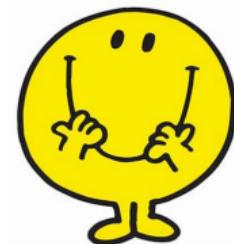
Avoiding Overfitting Through Regularization

- ▶ Early stopping
- ▶ ℓ_1 and ℓ_2 regularization
- ▶ Max-norm regularization
- ▶ Dropout
- ▶ Data augmentation



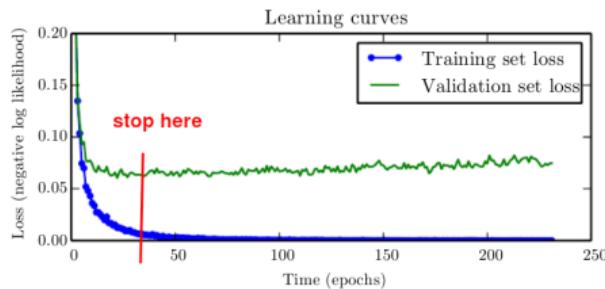
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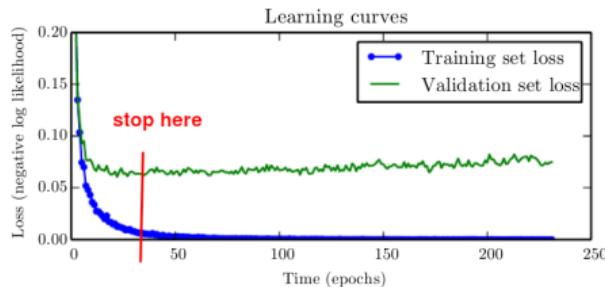
Early Stopping

- ▶ As the training steps go by, its prediction error on the training/validation set naturally goes down.



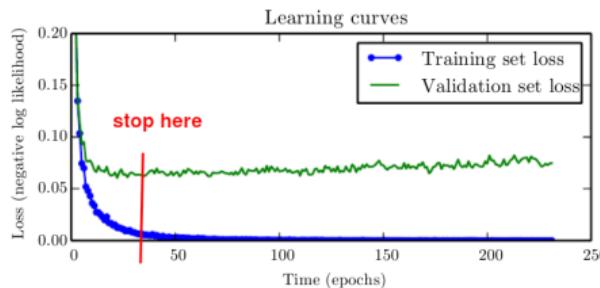
Early Stopping

- ▶ As the **training steps go by**, its prediction error on the **training/validation set** naturally goes down.
- ▶ After a while the **validation error stops decreasing** and **starts to go back up**.
 - The model has started to **overfit the training data**.



Early Stopping

- ▶ As the **training steps go by**, its **prediction error** on the **training/validation set** naturally **goes down**.
- ▶ After a while the **validation error** **stops decreasing** and **starts to go back up**.
 - The model has started to **overfit** the **training data**.
- ▶ In the **early stopping**, we **stop training** when the **validation error** reaches a **minimum**.



Avoiding Overfitting Through Regularization

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ℓ_1 and ℓ_2 Regularization (1/4)

- ▶ Penalize **large values** of weights w_j .

$$\tilde{J}(\mathbf{w}) = J(\mathbf{w}) + \lambda R(\mathbf{w})$$

- ▶ Two questions:
 1. How should we define $R(\mathbf{w})$?
 2. How do we determine λ ?



/1 and /2 Regularization (2/4)

- **/1 regression:** $R(\mathbf{w}) = \lambda \sum_{i=1}^n |w_i|$ is added to the cost function.

$$\tilde{J}(\mathbf{w}) = J(\mathbf{w}) + \lambda \sum_{i=1}^n |w_i|$$

- **/2 regression:** $R(\mathbf{w}) = \lambda \sum_{i=1}^n w_i^2$ is added to the cost function.

$$\tilde{J}(\mathbf{w}) = J(\mathbf{w}) + \lambda \sum_{i=1}^n w_i^2$$



ℓ_1 and ℓ_2 Regularization (3/4)

- ▶ Manually implement it in TensorFlow.

```
# make the network
hidden = tf.layers.dense(X, n_neurons_h, activation=tf.sigmoid, name="hidden")
logit = tf.layers.dense(hidden, n_neurons_out, name="output")
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logit = tf.layers.dense(hidden, n_neurons_out, name="output")

# extract the weights of layers
W1 = tf.get_default_graph().get_tensor_by_name("hidden/kernel:0")
W2 = tf.get_default_graph().get_tensor_by_name("output/kernel:0")
```



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# l1 regularization
reg_cost = tf.reduce_sum(tf.abs(W1)) + tf.reduce_sum(tf.abs(W2))
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# l1 regularization
reg_cost = tf.reduce_sum(tf.abs(W1)) + tf.reduce_sum(tf.abs(W2))

# define the cost
cross_entropy = tf.nn.sigmoid_cross_entropy_with_logits(logit, y_true)
base_cost = tf.reduce_mean(cross_entropy)

l1_param = 0.001
cost = base_cost + l1_param * reg_cost

# the rest is as before
```



/1 and /2 Regularization (5/5)

- ▶ Alternatively, we can pass a **regularization function** to the `tf.layers.dense()`.

```
# make the network
l1_param = 0.001 # l1 regularization hyperparameter

hidden = tf.layers.dense(X, n_neurons_h, activation=tf.sigmoid, name="hidden",
    kernel_regularizer=tf.contrib.layers.l1_regularizer(l1_param))
logit = tf.layers.dense(hidden, n_neurons_out, name="output",
    kernel_regularizer=tf.contrib.layers.l1_regularizer(l1_param))
```



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logit = tf.layers.dense(hidden, n_neurons_out, name="output",
    kernel_regularizer=tf.contrib.layers.l1_regularizer(l1_param))
```

```
# define the cost
cross_entropy = tf.nn.sigmoid_cross_entropy_with_logits(logit, y_true)

base_cost = tf.reduce_mean(cross_entropy)
reg_cost = tf.losses.get_regularization_loss()

cost = base_cost + reg_cost

# the rest is as before
```

Avoiding Overfitting Through Regularization

- ▶ Early stopping
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Max-Norm Regularization (1/3)

- ▶ Max-norm regularization: constrains the weights w_j of the incoming connections for each neuron j .
 - Prevents them from getting too large.



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- ▶ After each training step, clip \mathbf{w}_j as below:

$$\mathbf{w}_j \leftarrow \mathbf{w}_j \frac{\mathbf{r}}{\|\mathbf{w}_j\|_2}$$



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- ▶ After each training step, clip \mathbf{w}_j as below:

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- ▶ We have $\|\mathbf{w}_j\|_2 \leq \mathbf{r}$.
 - \mathbf{r} is the max-norm hyperparameter
 - $\|\mathbf{w}_j\|_2 = (\sum_i w_{i,j}^2)^{\frac{1}{2}} = \sqrt{w_{1,j}^2 + w_{2,j}^2 + \dots + w_{n,j}^2}$



Max-Norm Regularization (2/3)

```
# make the network
hidden = tf.layers.dense(X, n_neurons_h, activation=tf.sigmoid, name="hidden")
logit = tf.layers.dense(hidden, n_neurons_out, name="output")
```



Max-Norm Regularization (2/3)

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# define the cost
cross_entropy = tf.nn.sigmoid_cross_entropy_with_logits(logit, y_true)
cost = tf.reduce_mean(cross_entropy)

# define the optimizer
optimizer = tf.train.GradientDescentOptimizer(learning_rate)
training_op = optimizer.minimize(cost)
```



Max-Norm Regularization (3/3)

- ▶ Use `tf.clip_by_norm`.

```
# max-norm regularization - hidden layer
threshold = 1.0

weights = tf.get_default_graph().get_tensor_by_name("hidden/kernel:0")
clipped_weights = tf.clip_by_norm(weights, clip_norm=threshold, axes=1)
clip_weights = weights.assign(clipped_weights)
```



Max-Norm Regularization (3/3)

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weights = tf.get_default_graph().get_tensor_by_name("hidden/kernel:0")
clipped_weights = tf.clip_by_norm(weights, clip_norm=threshold, axes=1)
clip_weights = weights.assign(clipped_weights)

# executing the model
init = tf.global_variables_initializer()

with tf.Session() as sess:
    init.run()
    for epoch in range(n_epochs):
        sess.run(training_op, feed_dict={X: training_X, y_true: training_y})
        clip_weights.eval()
```

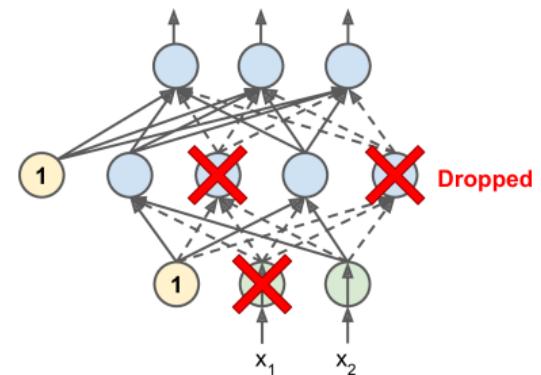
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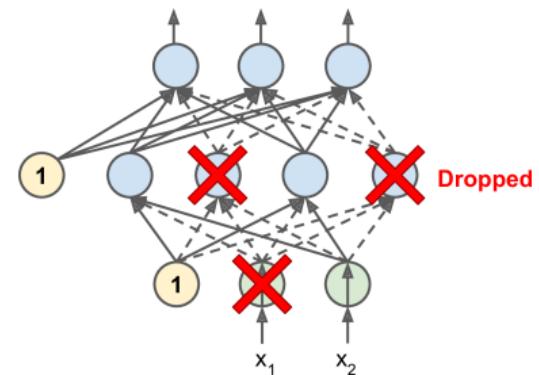
Dropout (1/2)

- ▶ At each **training step**, each neuron drops out temporarily with a **probability p** .



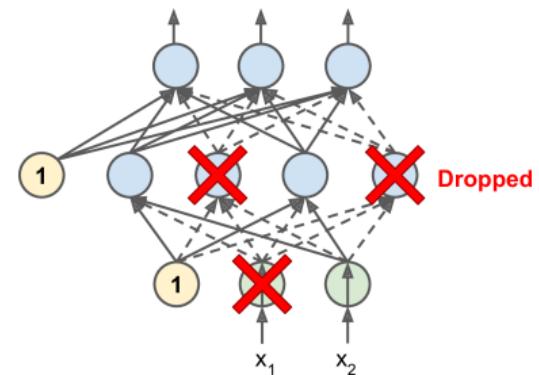
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- ▶ At each **training step**, each neuron drops out temporarily with a **probability p** .
 - The **hyperparameter p** is called the **dropout rate**.



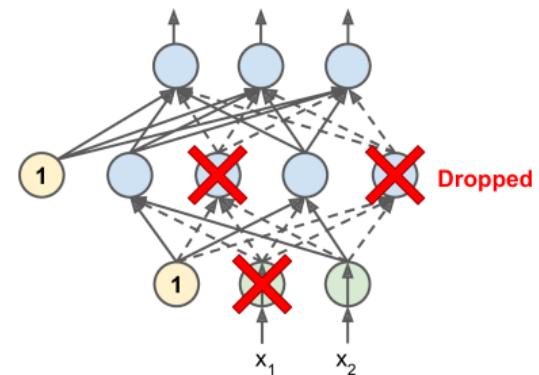
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 - A neuron will be **entirely ignored** during **this training step**.



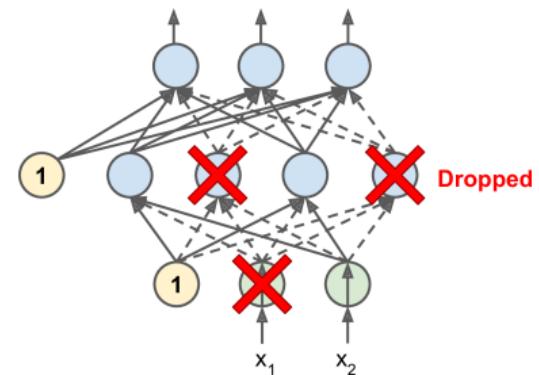
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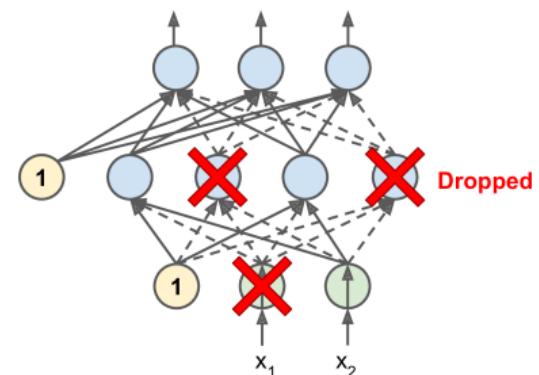
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 - Exclude the **output neurons**.



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 - The **hyperparameter p** is called the **dropout rate**.
 - A neuron will be **entirely ignored** during **this training step**.
 - It may be **active** during the **next step**.
 - Exclude the **output neurons**.
- ▶ After **training**, neurons don't get dropped anymore.





Dropout (2/2)

- ▶ Use `tf.layers.dropout`: specify the **dropout rate** rather than the **keep probability**.

```
# make the network
dropout_rate = 0.5 # == 1 - keep_prob
training = tf.placeholder_with_default(False, shape=(), name="training")

X_drop = tf.layers.dropout(X, dropout_rate, training=training)
hidden = tf.layers.dense(X_drop, n_neurons_h, activation=tf.sigmoid, name="hidden")
hidden_drop = tf.layers.dropout(hidden, dropout_rate, training=training)
logit = tf.layers.dense(hidden_drop, n_neurons_out, name="output")
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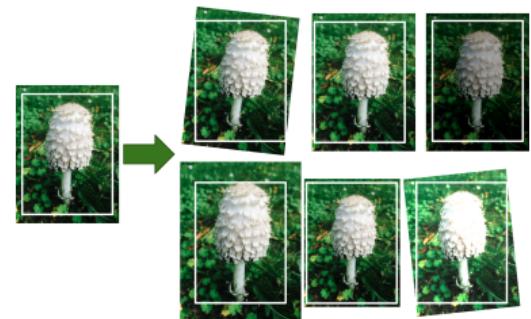
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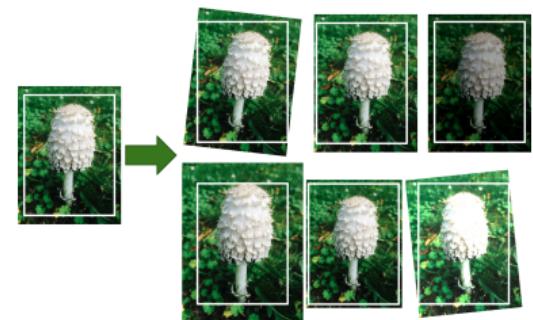
Data Augmentation

- ▶ One way to make a model **generalize better** is to **train it on more data**.
- ▶ This will **reduce overfitting**.



Data Augmentation

- ▶ One way to make a model **generalize better** is to **train it on more data**.
- ▶ This will **reduce overfitting**.
- ▶ Create **fake data** and add it to the **training set**.
 - E.g., in an **image classification** we can slightly shift, rotate and resize an image.
 - Add the resulting pictures to the **training set**.



Vanishing/Exploding Gradients





Vanishing/Exploding Gradients Problem (1/4)

- ▶ The **backpropagation** goes from **output** to **input** layer, and propagates the **error gradient** on the way.

$$\mathbf{w}^{(\text{next})} = \mathbf{w} - \eta \frac{\partial J(\mathbf{w})}{\partial \mathbf{w}}$$



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- ▶ Gradients often get **smaller and smaller** as the algorithm progresses **down to the lower layers**.
- ▶ As a result, the gradient descent update leaves the **lower layer connection weights** virtually **unchanged**.



Vanishing/Exploding Gradients Problem (1/4)

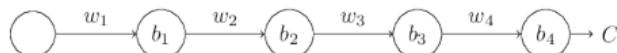
- ▶ The backpropagation goes from output to input layer, and propagates the error gradient on the way.

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- ▶ Gradients often get smaller and smaller as the algorithm progresses down to the lower layers.
- ▶ As a result, the gradient descent update leaves the lower layer connection weights virtually unchanged.
- ▶ This is called the vanishing gradients problem.

Vanishing/Exploding Gradients Problem (2/4)

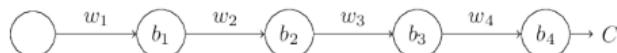
- ▶ Assume a network with just a single neuron in each layer.



- w_1, w_2, \dots are the weights
- b_1, b_2, \dots are the biases
- C is the cost function

Vanishing/Exploding Gradients Problem (2/4)

- ▶ Assume a network with just a single neuron in each layer.



- w_1, w_2, \dots are the **weights**
- b_1, b_2, \dots are the **biases**
- C is the **cost function**

- ▶ The output a_j from the j th neuron is $\sigma(z_j)$.

- σ is the **sigmoid** activation function
- $z_j = w_j a_{j-1} + b_j$
- E.g., $a_4 = \sigma(z_4) = \text{sigmoid}(w_4 a_3 + b_4)$

Vanishing/Exploding Gradients Problem (3/4)

- Let's compute the **gradient** associated to the **first hidden neuron** ($\frac{\partial C}{\partial b_1}$).



$$\frac{\partial C}{\partial b_1} = \frac{\partial C}{\partial a_4} \times \frac{\partial a_4}{\partial z_4} \times \frac{\partial z_4}{\partial a_3} \times \frac{\partial a_3}{\partial z_3} \times \frac{\partial z_3}{\partial a_2} \times \frac{\partial a_2}{\partial z_2} \times \frac{\partial z_2}{\partial a_1} \times \frac{\partial a_1}{\partial z_1} \times \frac{\partial z_1}{\partial b_1}$$

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$$\frac{\partial C}{\partial b_1} = \frac{\partial C}{\partial a_4} \times \frac{\partial a_4}{\partial z_4} \times \frac{\partial w_4 a_3 + b_4}{\partial a_3} \times \frac{\partial a_3}{\partial z_3} \times \frac{\partial w_3 a_2 + b_3}{\partial a_2} \times \frac{\partial a_2}{\partial z_2} \times \frac{\partial w_2 a_1 + b_2}{\partial a_1} \times \frac{\partial a_1}{\partial z_1} \times \frac{\partial w_1 a_0 + b_1}{\partial b_1}$$

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$$\frac{\partial C}{\partial b_1} = \frac{\partial C}{\partial a_4} \times \frac{\partial a_4}{\partial z_4} \times \frac{\partial w_4 a_3 + b_4}{\partial a_3} \times \frac{\partial a_3}{\partial z_3} \times \frac{\partial w_3 a_2 + b_3}{\partial a_2} \times \frac{\partial a_2}{\partial z_2} \times \frac{\partial w_2 a_1 + b_2}{\partial a_1} \times \frac{\partial a_1}{\partial z_1} \times \frac{\partial w_1 a_0 + b_1}{\partial b_1}$$

$$\frac{\partial C}{\partial b_1} = \frac{\partial C}{\partial a_4} \times \sigma'(z_4) \times w_4 \times \sigma'(z_3) \times w_3 \times \sigma'(z_2) \times w_2 \times \sigma'(z_1) \times 1$$

Vanishing/Exploding Gradients Problem (4/4)

- ▶ Now, consider $\frac{\partial C}{\partial b_3}$.



$$\frac{\partial C}{\partial b_3} = \frac{\partial C}{\partial a_4} \times \sigma'(z_4) \times w_4 \times \sigma'(z_3)$$

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Vanishing/Exploding Gradients Problem (4/4)

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$$\frac{\partial C}{\partial b_1} = \frac{\partial C}{\partial a_4} \times \sigma'(z_4) \times w_4 \times \sigma'(z_3) \times w_3 \times \sigma'(z_2) \times w_2 \times \sigma'(z_1) \times 1$$

- ▶ Assume $w_3 \sigma'(z_2) < \frac{1}{4}$ and $w_2 \sigma'(z_1) < \frac{1}{4}$
 - The gradient $\frac{\partial C}{\partial b_1}$ be a factor of 16 (or more) smaller than $\frac{\partial C}{\partial b_3}$.
 - This is the essential origin of the vanishing gradient problem.

Overcoming the Vanishing Gradient

- ▶ Parameter initialization strategies
- ▶ Nonsaturating activation function
- ▶ Batch normalization
- ▶ Gradient clipping



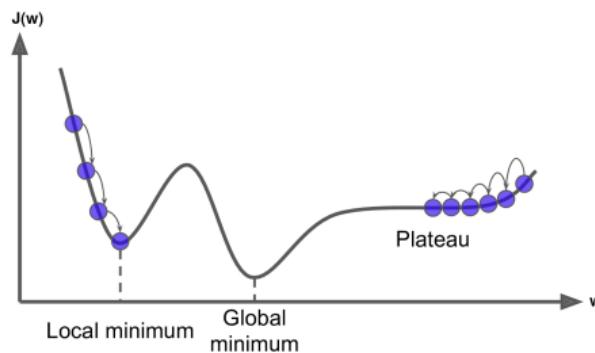
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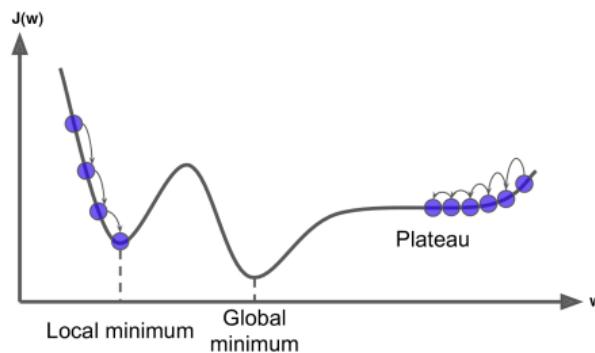
Parameter Initialization Strategies (1/5)

- The non-linearity of a neural network causes the cost functions to become non-convex.



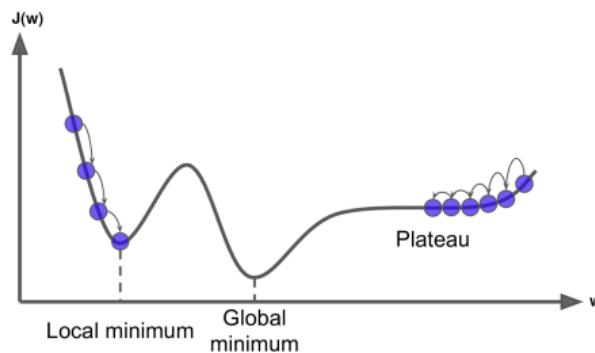
Parameter Initialization Strategies (1/5)

- ▶ The **non-linearity** of a neural network causes the **cost functions** to become **non-convex**.
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Parameter Initialization Strategies (1/5)

- ▶ The **non-linearity** of a neural network causes the **cost functions** to become **non-convex**.
- ▶ The stochastic gradient descent on **non-convex cost functions** performs is **sensitive** to the values of the **initial parameters**.
- ▶ Designing initialization strategies is a **difficult task**.





Parameter Initialization Strategies (2/5)

- ▶ The **initial parameters** need to **break symmetry** between **different units**.



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- ▶ **Two hidden units** with the **same activation function** connected to the **same inputs**, must have **different** initial parameters.



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 - The goal of having each unit **compute a different function**.



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- ▶ The **initial parameters** need to **break symmetry** between **different units**.
- ▶ **Two hidden units** with the **same activation function** connected to the **same inputs**, must have **different** initial parameters.
 - The goal of having each unit **compute a different function**.
- ▶ It motivates **random initialization** of the parameters.
 - Typically, we set the **biases** to **constants**, and initialize only the **weights randomly**.



Parameter Initialization Strategies (3/5)

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Parameter Initialization Strategies (3/5)

- ▶ We need the signals to flow properly in **both** directions.
- ▶ The **Xavier initialization** proposed that:
 - The **variance of the outputs** of each layer to be **equal** to the **variance of its inputs**.
 - The **gradients** to have **equal variance before and after** flowing through a layer in the reverse direction.



Parameter Initialization Strategies (4/5)

- ▶ Based on the **Xavier initialization**, the weights are **initialized** using **normal distribution** with **mean 0** and the following **standard deviation**.



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 - For the **sigmoid** activation function:

$$\sigma = \sqrt{\frac{2}{n_{\text{inputs}} + n_{\text{outputs}}}}$$

- n_{inputs} and n_{outputs} are the **number of input and output connections** for the layer whose weights are being initialized.



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- For the **ReLU** activation function:

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- **n_{inputs}** and **n_{outputs}** are the **number of input and output connections** for the layer whose weights are being initialized.



Parameter Initialization Strategies (5/5)

- ▶ Use `tf.variance_scaling_initializer()`

```
# make the network
he_init = tf.variance_scaling_initializer()

hidden = tf.layers.dense(X, n_neurons_h, activation=tf.sigmoid, name="hidden",
    kernel_initializer=he_init)

logit = tf.layers.dense(hidden, n_neurons_out, name="output")
```

Overcoming the Vanishing Gradient

- ▶ Parameter initialization strategies
- ▶ Nonsaturating activation function
- ▶ Batch normalization
- ▶ Gradient clipping

by Roger Hargreaves





Nonsaturating Activation Functions (1/4)

- ▶ $\text{ReLU}(z) = \max(0, z)$
- ▶ The **dying ReLUs** problem.

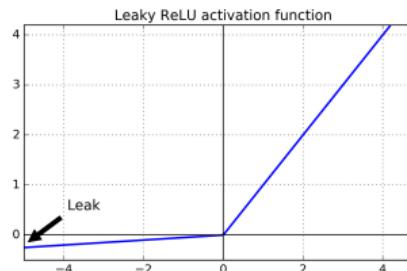


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- ▶ The **dying ReLUs** problem.
 - During **training**, some neurons **stop outputting anything other than 0**.
 - E.g., when the **weighted sum of the neuron's inputs is negative**, it starts outputting 0.
- ▶ Use **leaky ReLU** instead: $\text{LeakyReLU}_\alpha(z) = \max(\alpha z, z)$.
 - α is the **slope** of the function for $z < 0$.





Nonsaturating Activation Functions (2/4)

► Randomized Leaky ReLU (RReLU)

- α is picked **randomly** during training, and it is **fixed** during testing.



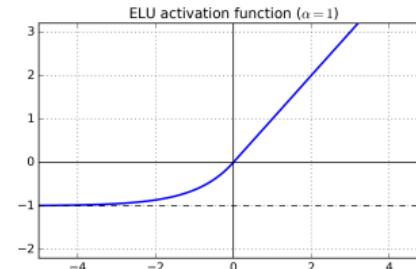
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 - Learn α **during training** (instead of being a hyperparameter).

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- ▶ Parametric Leaky ReLU (PReLU)
 - Learn α **during training** (instead of being a hyperparameter).
- ▶ Exponential Linear Unit (ELU)

$$\text{ELU}_\alpha(z) = \begin{cases} \alpha(\exp(z) - 1) & \text{if } z < 0 \\ z & \text{if } z \geq 0 \end{cases}$$



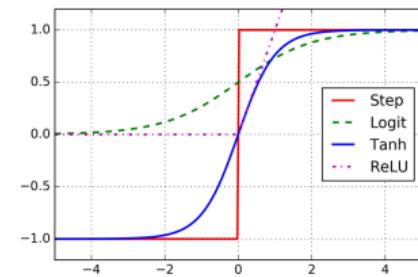
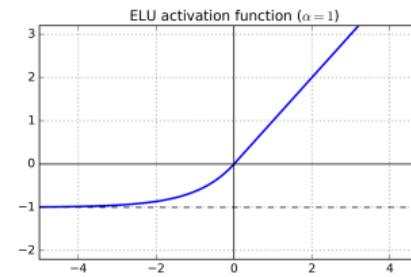
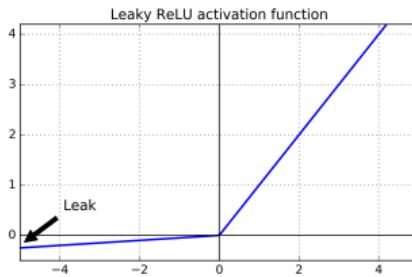


Nonsaturating Activation Functions (3/4)

- ▶ Which activation function should we use?

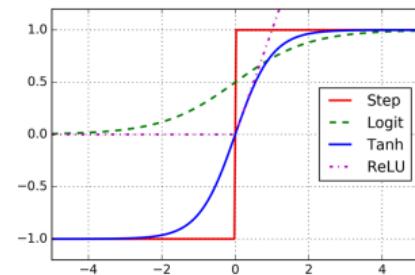
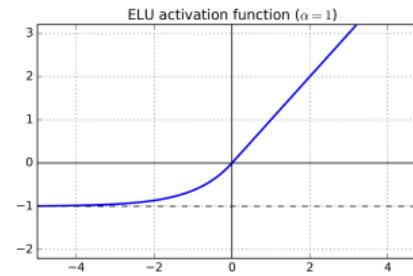
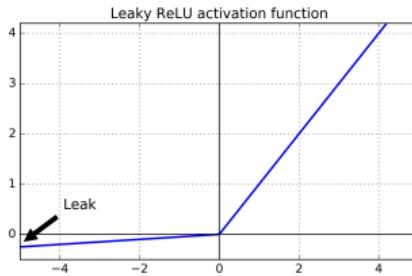
Nonsaturating Activation Functions (3/4)

- ▶ Which activation function should we use?
- ▶ In general logistic < tanh < ReLU < leaky ReLU (and its variants) < ELU



Nonsaturating Activation Functions (3/4)

- ▶ Which activation function should we use?
- ▶ In general logistic < tanh < ReLU < leaky ReLU (and its variants) < ELU
- ▶ If you care about runtime performance, then leaky ReLUs works better than ELUs.





Nonsaturating Activation Functions (4/4)

```
# leaky relu
def leaky_relu(z, name=None):
    alpha = 0.01
    return tf.maximum(alpha * z, z, name=name)

hidden = tf.layers.dense(X, n_neurons_h, activation=leaky_relu, name="hidden")
```



Nonsaturating Activation Functions (4/4)

```
# leaky relu
def leaky_relu(z, name=None):
    alpha = 0.01
    return tf.maximum(alpha * z, z, name=name)

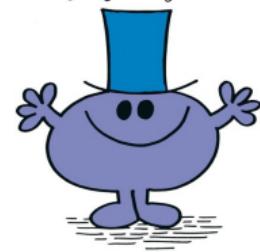
hidden = tf.layers.dense(X, n_neurons_h, activation=leaky_relu, name="hidden")
```

```
# elu
hidden = tf.layers.dense(X, n_neurons_h, activation=tf.nn.elu, name="hidden")
```

Overcoming the Vanishing Gradient

- ▶ Parameter initialization strategies
- ▶ Nonsaturating activation function
- ▶ **Batch normalization**
- ▶ Gradient clipping

by Roger Hargreaves





Batch Normalization (1/5)

- ▶ The gradient tells how to update each parameter, under the assumption that the other layers do not change.



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- ▶ The gradient tells how to update each parameter, under the assumption that the other layers do not change.
 - In practice, we update all of the layers simultaneously.
 - However, unexpected results can happen.
- ▶ Batch normalization makes the learning of layers in the network more independent of each other.
 - It is a technique to address the problem that the distribution of each layer's inputs changes during training, as the parameters of the previous layers change.



Batch Normalization (1/5)

- ▶ The gradient tells how to **update each parameter**, under the assumption that **the other layers do not change**.
 - In practice, we update all of the layers **simultaneously**.
 - However, **unexpected results can happen**.
- ▶ **Batch normalization** makes the **learning of layers** in the network more **independent of each other**.
 - It is a technique to address the problem that the **distribution of each layer's inputs** changes **during training**, as the parameters of the previous layers change.
- ▶ The technique consists of **adding an operation** in the model just **before the activation function** of each layer.



Batch Normalization (2/5)

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 - Estimates the inputs' mean and standard deviation of the current mini-batch.

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 - Estimates the inputs' mean and standard deviation of the current mini-batch.

$$\mu_B = \frac{1}{m_B} \sum_{i=1}^{m_B} x^{(i)}$$

$$\sigma_B^2 = \frac{1}{m_B} \sum_{i=1}^{m_B} (x^{(i)} - \mu_B)^2$$

- ▶ μ_B : the empirical mean, evaluated over the whole mini-batch B .
- ▶ σ_B : the empirical standard deviation, also evaluated over the whole mini-batch.
- ▶ m_B : the number of instances in the mini-batch.



Batch Normalization (3/5)

$$\hat{x}^{(i)} = \frac{x^{(i)} - \mu_B}{\sqrt{\sigma_B^2 + \epsilon}}$$
$$z^{(i)} = \gamma \hat{x}^{(i)} + \beta$$

- ▶ $\hat{x}^{(i)}$: the zero-centered and normalized input.
- ▶ γ : the scaling parameter for the layer.
- ▶ β : the shifting parameter (offset) for the layer.
- ▶ ϵ : a tiny number to avoid division by zero.
- ▶ $z^{(i)}$: the output of the BN operation, which is a scaled and shifted version of the inputs.



Batch Normalization (4/5)

- ▶ Use `tf.layers.batch_normalization`

```
# make the network
training = tf.placeholder_with_default(False, shape=(), name="training")

hidden = tf.layers.dense(X, n_neurons_h, name="hidden")
bn = tf.layers.batch_normalization(hidden, training=training)
bn_act = tf.sigmoid(bn)

logits_before_bn = tf.layers.dense(bn_act, n_outputs, name="output")
logits = tf.layers.batch_normalization(logits_before_bn, training=training)
```



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bn_act = tf.sigmoid(bn)

logits_before_bn = tf.layers.dense(bn_act, n_outputs, name="output")
logits = tf.layers.batch_normalization(logits_before_bn, training=training)

# define the cost
cross_entropy = tf.nn.sparse_softmax_cross_entropy_with_logits(labels=y, logits=logits)
cost = tf.reduce_mean(cross_entropy)

# train the model
optimizer = tf.train.GradientDescentOptimizer(learning_rate)
training_op = optimizer.minimize(cost)
```



Batch Normalization (5/5)

- We need to explicitly run the extra update operations needed by batch normalization
`sess.run([training_op, extra_update_ops], ...)`

```
extra_update_ops = tf.get_collection(tf.GraphKeys.UPDATE_OPS)
init = tf.global_variables_initializer()

with tf.Session() as sess:
    init.run()
    for epoch in range(n_epochs):
        sess.run([training_op, extra_update_ops],
                feed_dict={X: training_X, y_true: training_y, training: True})
```

Overcoming the Vanishing Gradient

- ▶ Parameter initialization strategies
- ▶ Nonsaturating activation function
- ▶ Batch normalization
- ▶ **Gradient clipping**

Roger Hargreaves





Gradient Clipping (1/2)

- ▶ **Gradient clipping**: clip the gradients during **backpropagation** so that they **never exceed some threshold**.



Gradient Clipping (1/2)

- ▶ Gradient clipping: clip the gradients during backpropagation so that they never exceed some threshold.
- ▶ In TensorFlow, the optimizer's `minimize()` function takes care of:
 1. Compute the gradients with `compute_gradients()`
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- ▶ Gradient clipping: clip the gradients during backpropagation so that they never exceed some threshold.
- ▶ In TensorFlow, the optimizer's `minimize()` function takes care of:
 1. Compute the gradients with `compute_gradients()`
 2. Apply the processed gradients with `apply_gradients()`
- ▶ To enable the gradient clipping, you must instead of calling `minimize()`, call:
 1. Compute the gradients with `compute_gradients()`
 2. Process the gradients as you wish.
 3. Apply the processed gradients with `apply_gradients()`



Gradient Clipping (2/2)

- ▶ Use `clip_by_value()`

```
# define the cost
cross_entropy = tf.nn.sparse_softmax_cross_entropy_with_logits(labels=y, logits=logits)
cost = tf.reduce_mean(cross_entropy)
```



Gradient Clipping (2/2)

- ▶ Use `clip_by_value()`

```
# define the cost
cross_entropy = tf.nn.sparse_softmax_cross_entropy_with_logits(labels=y, logits=logits)
cost = tf.reduce_mean(cross_entropy)

# train the model
threshold = 1.0

optimizer = tf.train.GradientDescentOptimizer(learning_rate)

# returns a list of (gradient, variable) pairs
grads_and_vars = optimizer.compute_gradients(cost)

capped_gvs = [(tf.clip_by_value(grad, -threshold, threshold), var)
               for grad, var in grads_and_vars]

training_op = optimizer.apply_gradients(capped_gvs)
```

Training Speed





Regular Gradient Descent Optimization (1/2)

- ▶ Gradient descent optimization algorithm
- ▶ It updates the weights $w_i^{(\text{next})} = w - \eta \frac{\partial J(w)}{\partial w_i}$
- ▶ Better optimization algorithms to improve the training speed



Regular Gradient Descent Optimization (2/2)

```
# define the cost
cross_entropy = tf.nn.sigmoid_cross_entropy_with_logits(z, y_true)
cost = tf.reduce_mean(cross_entropy)

# train the model
learning_rate = 0.1
optimizer = tf.train.GradientDescentOptimizer(learning_rate)
training_op = optimizer.minimize(cost)
```

Optimization Algorithms

- ▶ Momentum
- ▶ Nesterov momentum
- ▶ AdaGrad
- ▶ RMSProp
- ▶ Adam Optimization



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Momentum (1/4)

- ▶ **Momentum** is a concept from physics: an **object in motion** will have a **tendency to keep moving**.
- ▶ It measures the **resistance to change in motion**.
 - The **higher momentum** an object has, the harder it is to stop it.



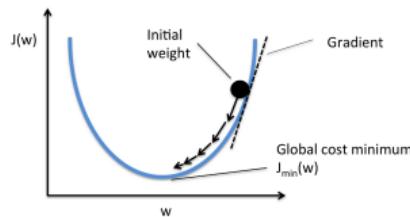


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- ▶ This is the very simple idea behind **momentum optimization**.

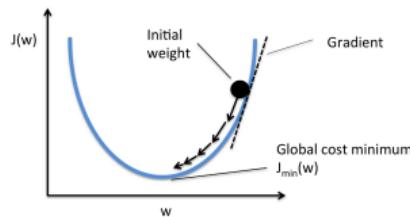
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- ▶ We can see the **change in the parameters w** as **motion**: $w_i^{(\text{next})} = w_i - \eta \frac{\partial J(w)}{\partial w_i}$



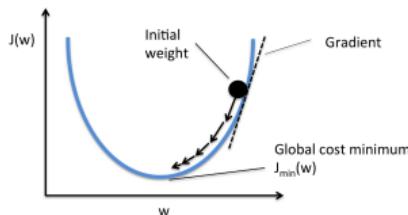
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- ▶ We can thus use the concept of momentum to give the update process a **tendency to keep moving** in the same direction.



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- ▶ We can see the **change in the parameters w** as **motion**: $w_i^{(\text{next})} = w_i - \eta \frac{\partial J(w)}{\partial w_i}$
- ▶ We can thus use the concept of momentum to give the update process a **tendency to keep moving** in the same direction.
- ▶ It can help to **escape from bad local minima pits**.





Momentum (3/4)

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- ▶ At each iteration, it adds the local gradient to the momentum vector \mathbf{m} .

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- ▶ β is called momentum, and it is between 0 and 1.
- ▶ Updates the weights by subtracting this momentum vector.

$$w_i^{(\text{next})} = w_i - \mathbf{m}_i$$



Momentum (4/4)

```
# train the model  
  
optimizer = tf.train.MomentumOptimizer(learning_rate=learning_rate, momentum=0.9)
```

Optimization Algorithms

- ▶ Momentum
- ▶ Nesterov momentum
- ▶ AdaGrad
- ▶ RMSProp
- ▶ Adam optimization

By Roger Hargreaves





Nesterov Momentum (1/3)

- ▶ Nesterov Momentum is a **small variant** to **Momentum optimization**.
- ▶ **Faster** than vanilla **Momentum optimization**.



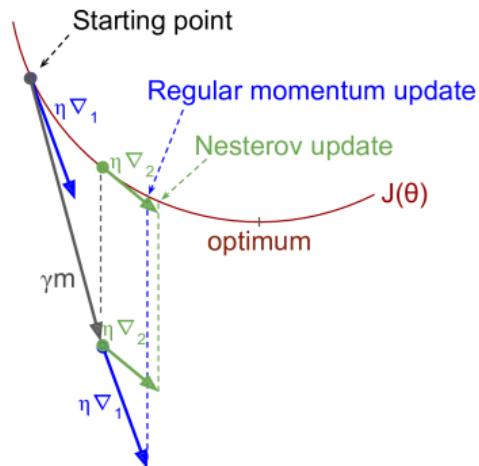
Nesterov Momentum (1/3)

- ▶ Nesterov Momentum is a small variant to Momentum optimization.
- ▶ Faster than vanilla Momentum optimization.
- ▶ Measure the gradient of the cost function slightly ahead in the direction of the momentum (not at the local position).

$$\begin{aligned} \mathbf{m}_i &= \beta \mathbf{m}_i + \eta \frac{\partial J(\mathbf{w} + \beta \mathbf{m})}{\partial \mathbf{w}_i} \\ \mathbf{w}_i^{(\text{next})} &= \mathbf{w}_i - \mathbf{m}_i \end{aligned}$$

Nesterov Momentum (2/3)

- ▶ ∇_1 represents the **gradient of the cost function** measured at the **starting point w** , and ∇_2 represents the gradient at the point located at $w + \beta m$.





Nesterov Momentum (3/3)

```
# train the model

optimizer = tf.train.MomentumOptimizer(learning_rate=learning_rate, momentum=0.9,
    use_nesterov=True)
```



Optimization Algorithms

- ▶ Momentum
- ▶ Nesterov momentum
- ▶ AdaGrad
- ▶ RMSProp
- ▶ Adam optimization





AdaGrad (1/3)

- ▶ AdaGrad keeps track of a learning rate for each parameter.
- ▶ Adapts the learning rate over time (adaptive learning rate).



AdaGrad (2/3)

- ▶ For each feature w_i , we do the following steps:

$$s_i = s_i + \left(\frac{\partial J(\mathbf{w})}{\partial w_i} \right)^2$$

$$w_i^{(\text{next})} = w_i - \frac{\eta}{\sqrt{s_i + \epsilon}} \frac{\partial J(\mathbf{w})}{\partial w_i}$$

- ▶ Parameters with **large** partial derivative of the cost have a **rapid decrease** in their **learning rate**.
- ▶ Parameters with **small** partial derivatives have a **small decrease** in their **learning rate**.



AdaGrad (3/3)

```
# train the model  
  
optimizer = tf.train.AdagradOptimizer(learning_rate=learning_rate)
```

Optimization Algorithms

- ▶ Momentum
- ▶ Nesterov momentum
- ▶ AdaGrad
- ▶ RMSProp
- ▶ Adam optimization





RMSProp (1/3)

- ▶ AdaGrad often stops too early when training neural networks.
- ▶ The learning rate gets scaled down so much that the algorithm ends up stopping entirely before reaching the global optimum.



RMSProp (2/3)

- ▶ The **RMSProp** fixed the AdaGrad problem.
- ▶ It is like the **AdaGrad problem**, but accumulates only the gradients from the **most recent iterations** (not from the beginning of training).
- ▶ For each feature w_i , we do the following steps:

$$\begin{aligned}s_i &= \beta s_i + (1 - \beta) \left(\frac{\partial J(w)}{\partial w_i} \right)^2 \\ w_i^{(\text{next})} &= w_i - \frac{\eta}{\sqrt{s_i + \epsilon}} \frac{\partial J(w)}{\partial w_i}\end{aligned}$$



RMSProp (3/3)

```
# train the model

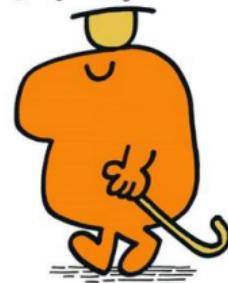
optimizer = tf.train.RMSPropOptimizer(learning_rate=learning_rate, momentum=0.9,
                                     decay=0.9, epsilon=1e-10)
```



Optimization Algorithms

- ▶ Momentum
- ▶ Nesterov momentum
- ▶ AdaGrad
- ▶ RMSProp
- ▶ Adam optimization

by Roger Hargreaves





Adam Optimization (1/3)

- ▶ Adam (Adaptive moment estimation) combines the ideas of Momentum optimization and RMSProp.



Adam Optimization (1/3)

- ▶ Adam (Adaptive moment estimation) combines the ideas of Momentum optimization and RMSProp.
- ▶ Like Momentum optimization, it keeps track of an exponentially decaying average of past gradients.



Adam Optimization (1/3)

- ▶ Adam (Adaptive moment estimation) combines the ideas of Momentum optimization and RMSProp.
- ▶ Like Momentum optimization, it keeps track of an exponentially decaying average of past gradients.
- ▶ Like RMSProp, it keeps track of an exponentially decaying average of past squared gradients.



Adam Optimization (2/3)

$$1. \quad \mathbf{m}^{(\text{next})} = \beta_1 \mathbf{m} + (1 - \beta_1) \nabla_{\mathbf{w}} J(\mathbf{w})$$

$$2. \quad \mathbf{s}^{(\text{next})} = \beta_2 \mathbf{s} + (1 - \beta_2) \nabla_{\mathbf{w}} J(\mathbf{w}) \otimes \nabla_{\mathbf{w}} J(\mathbf{w})$$

$$3. \quad \mathbf{m}^{(\text{next})} = \frac{\mathbf{m}}{1 - \beta_1^T}$$

$$4. \quad \mathbf{s}^{(\text{next})} = \frac{\mathbf{s}}{1 - \beta_2^T}$$

$$5. \quad \mathbf{w}^{(\text{next})} = \mathbf{w} - \eta \mathbf{m} \oslash \sqrt{\mathbf{s} + \epsilon}$$

- ▶ \otimes and \oslash represents the represents the element-wise multiplication and division.
- ▶ Steps 1, 2, and 5: similar to both Momentum optimization and RMSProp.
- ▶ Steps 3 and 4: since \mathbf{m} and \mathbf{s} are initialized at 0, they will be biased toward 0 at the beginning of training, so these two steps will help boost \mathbf{m} and \mathbf{s} at the beginning of training.



Adam Optimization (3/3)

```
# train the model  
  
optimizer = tf.train.AdamOptimizer(learning_rate=learning_rate)
```



Summary

Summary

- ▶ Overfitting
 - Early stopping, ℓ_1 and ℓ_2 regularization, max-norm regularization
 - Dropout, data augmentation
- ▶ Vanishing gradient
 - Parameter initialization, nonsaturating activation functions
 - Batch normalization, gradient clipping
- ▶ Training speed
 - Momentum, nesterov momentum, AdaGrad
 - RMSProp, Adam optimization





Reference

- ▶ Ian Goodfellow et al., Deep Learning (Ch. 7, 8)
- ▶ Aurélien Géron, Hands-On Machine Learning (Ch. 11)



Questions?