Autoencoders and Restricted Boltzmann Machines

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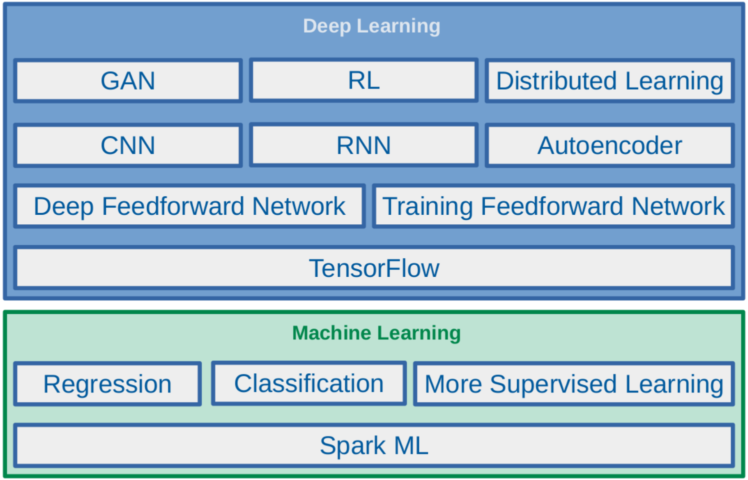
The Course Web Page

`https://id2223kth.github.io`

Where Are We?


Deep Learning
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Machine Learning
Regression
Classification
More Supervised Learning
Spark ML

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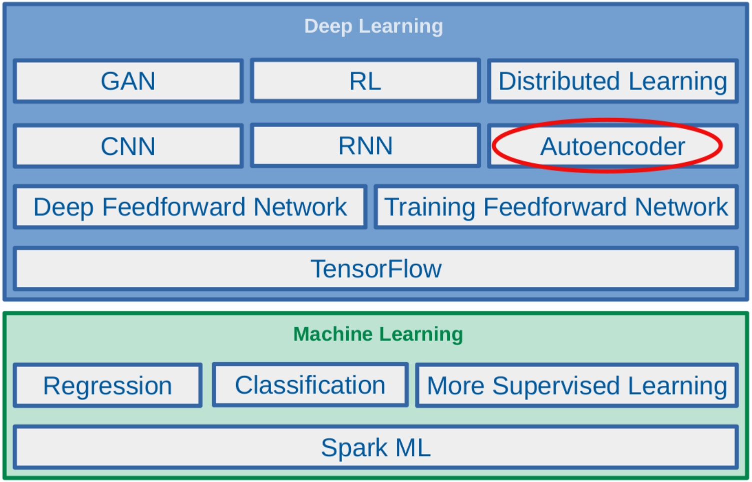
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Let's Start With An Example

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  - Even numbers are followed by their half.
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- You don't need pattern if you could quickly and easily memorize very long sequences
- But, it is hard to memorize long sequences that makes it useful to recognize patterns.

- 1970, W. Chase and H. Simon
- They observed that expert chess players were able to memorize the positions of all the pieces in a game by looking at the board for just 5 seconds.

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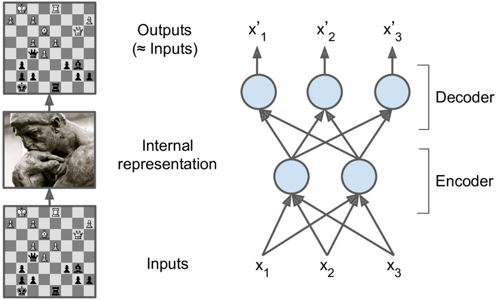
- Patterns helps them store information efficiently.

Autoencoders

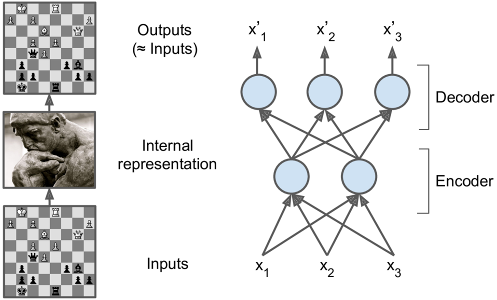
Autoencoders (1/5)

- Just like the chess players in this memory experiment.



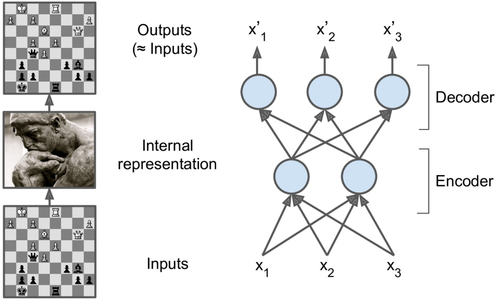
Autoencoders (1/5)

- Just like the chess players in this memory experiment.
- An autoencoder looks at the inputs, converts them to an efficient internal representation, and then spits out something that looks very close to the inputs.



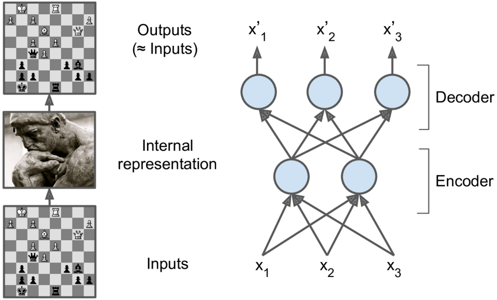
Autoencoders (2/5)

- The same architecture as a Multi-Layer Perceptron (MLP).



Autoencoders (2/5)

- The same architecture as a Multi-Layer Perceptron (MLP).
- Except that the number of neurons in the output layer must be equal to the number of inputs.



Autoencoders (3/5)

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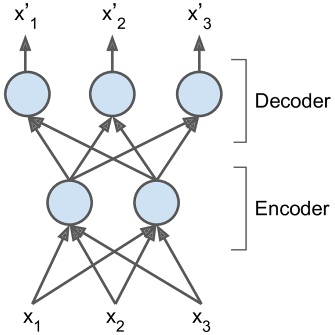

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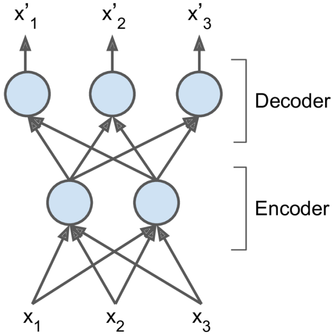
- An encoder (recognition network), $\mathbf{h} = \mathtt{f}(\mathbf{x})$
  Converts the inputs to an internal representation.



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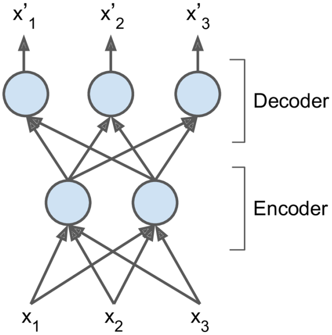

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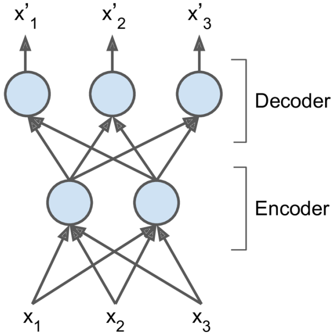
  Converts the internal representation to the outputs.
- If an autoencoder learns to set $\mathtt{g}(\mathtt{f}(\mathbf{x})) = \mathbf{x}$ everywhere, it is not especially useful, why?



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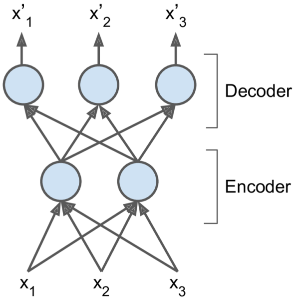
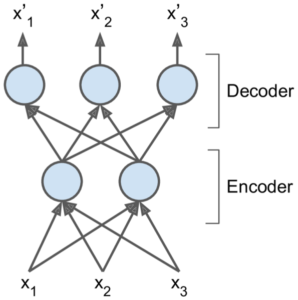


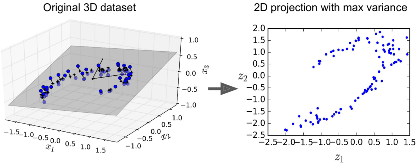
Autoencoders (4/5)

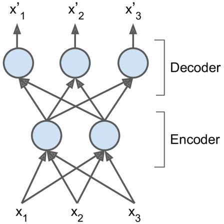
- Autoencoders are designed to be unable to learn to copy perfectly.
- The models are forced to prioritize which aspects of the input should be copied, they often learn useful properties of the data.



Autoencoders (5/5)

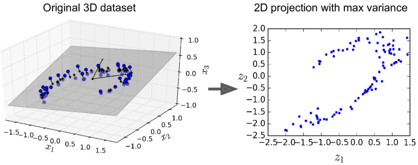
- Autoencoders are neural networks capable of learning efficient representations of the input data (called codings) without any supervision.

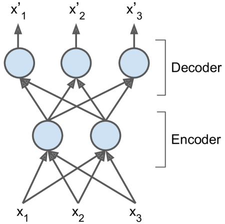




Autoencoders (5/5)

- Autoencoders are neural networks capable of learning efficient representations of the input data (called codings) without any supervision.
- Dimension reduction: these codings typically have a much lower dimensionality than the input data.





Different Types of Autoencoders

- Stacked autoencoders
- Denoising autoencoders
- Sparse autoencoders
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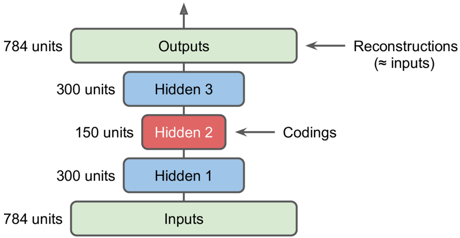

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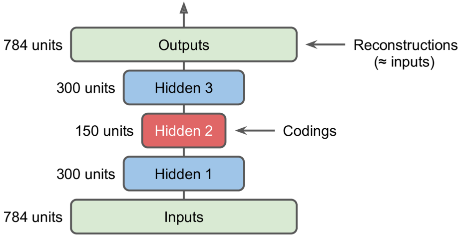
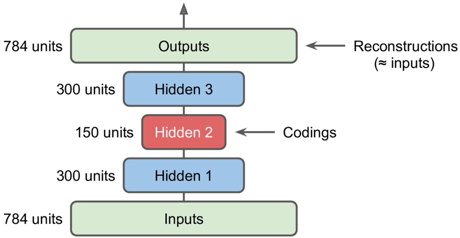
- Adding more layers helps the autoencoder learn more complex codings.



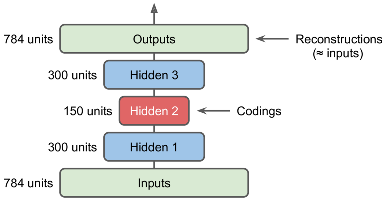
Stacked Autoencoders (1/3)

- Stacked autoencoder: autoencoders with multiple hidden layers.
- Adding more layers helps the autoencoder learn more complex codings.
- The architecture is typically symmetrical with regards to the central hidden layer.



Stacked Autoencoders (2/3)

- In a symmetric architecture, we can tie the weights of the decoder layers to the weights of the encoder layers.



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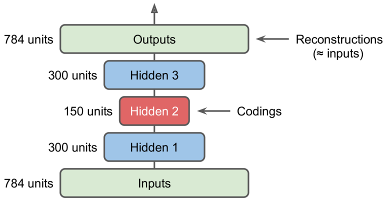
- In a network with $\mathtt{N}$ layers, the decoder layer weights can be defined as $\mathtt{w}_{\mathtt{N}-\mathtt{l}+1} = \mathtt{w}_{\mathtt{l}}^{\mathtt{T}}$, with $\mathtt{l} = 1, 2, \cdots, \frac{\mathtt{N}}{2}$.



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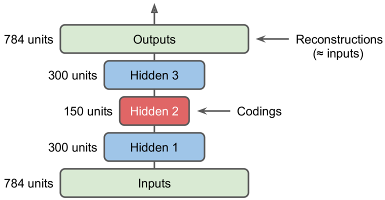
- This halves the number of weights in the model, speeding up training and limiting the risk of overfitting.



Stacked Autoencoders (3/3)

```
stacked_encoder = keras.models.Sequential([
    keras.layers.Flatten(input_shape=[28, 28]),
    keras.layers.Dense(100, activation="relu"),
    keras.layers.Dense(30, activation="relu"),
])
stacked_decoder = keras.models.Sequential([
    keras.layers.Dense(100, activation="relu", input_shape=[30]),
    keras.layers.Dense(28 * 28, activation="sigmoid"),
    keras.layers.Reshape([28, 28])
])

model = keras.models.Sequential([stacked_encoder, stacked_decoder])
```

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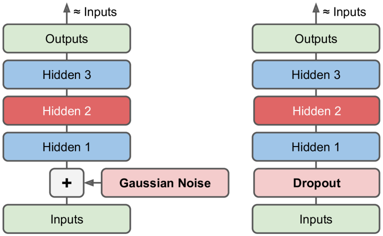
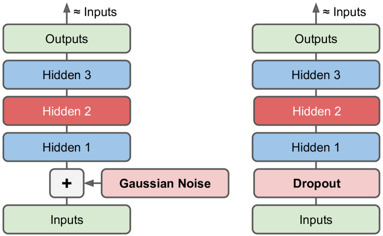
Denoising Autoencoders (1/4)

- One way to force the autoencoder to learn useful features is to add noise to its inputs, training it to recover the original noise-free inputs.



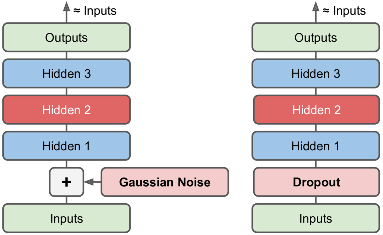
Denoising Autoencoders (1/4)

- One way to force the autoencoder to learn useful features is to add noise to its inputs, training it to recover the original noise-free inputs.
- This prevents the autoencoder from trivially copying its inputs to its outputs, so it ends up having to find patterns in the data.



Denoising Autoencoders (2/4)

- The noise can be pure Gaussian noise added to the inputs, or it can be randomly switched off inputs, just like in dropout.



Denoising Autoencoders (3/4)

```
denoising_encoder = keras.models.Sequential([
    keras.layers.Flatten(input_shape=[28, 28]),
    keras.layers.Dropout(0.5),
    keras.layers.Dense(100, activation="relu"),
    keras.layers.Dense(30, activation="relu")
])
denoising_decoder = keras.models.Sequential([
    keras.layers.Dense(100, activation="relu", input_shape=[30]),
    keras.layers.Dense(28 * 28, activation="sigmoid"),
    keras.layers.Reshape([28, 28])
])

model = keras.models.Sequential([denoising_encoder, denoising_decoder])
```

Denoising Autoencoders (4/4)

```
denoising_encoder = keras.models.Sequential([
    keras.layers.Flatten(input_shape=[28, 28]),
    keras.layers.GaussianNoise(0.2),
    keras.layers.Dense(100, activation="relu"),
    keras.layers.Dense(30, activation="relu")
])
denoising_decoder = keras.models.Sequential([
    keras.layers.Dense(100, activation="relu", input_shape=[30]),
    keras.layers.Dense(28 * 28, activation="sigmoid"),
    keras.layers.Reshape([28, 28])
])

model = keras.models.Sequential([denoising_encoder, denoising_decoder])
```

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- This forces the autoencoder to represent each input as a combination of a small number of activations.
- As a result, each neuron in the coding layer typically ends up representing a useful feature.

Sparse Autoencoders (2/2)

```
sparse_l1_encoder = keras.models.Sequential([
    keras.layers.Flatten(input_shape=[28, 28]),
    keras.layers.Dense(100, activation="selu"),
    keras.layers.Dense(300, activation="sigmoid", activity_regularizer=keras.regularizers.l1(1e-3))
])

sparse_l1_decoder = keras.models.Sequential([
    keras.layers.Dense(100, activation="selu", input_shape=[300]),
    keras.layers.Dense(28 * 28, activation="sigmoid"),
    keras.layers.Reshape([28, 28])
])

model = keras.models.Sequential([sparse_l1_encoder, sparse_l1_decoder])
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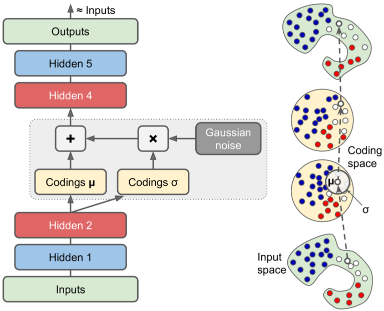
- Their outputs are partly determined by chance, even after training.
  - As opposed to denoising autoencoders, which use randomness only during training.
- They are generative autoencoders, meaning that they can generate new instances that look like they were sampled from the training set.

Variational Autoencoders (2/6)

- Instead of directly producing a coding for a given input, the encoder produces a mean coding μ and a standard deviation σ.



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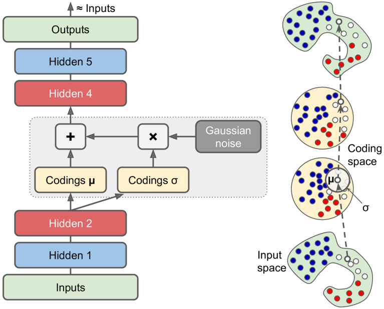
- The actual coding is then sampled randomly from a Gaussian distribution with mean μ and standard deviation σ.



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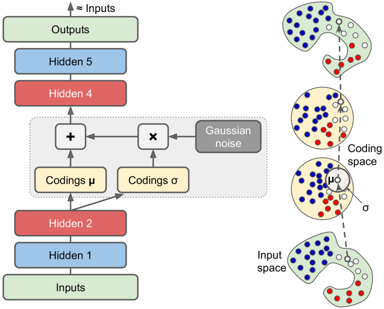
- Instead of directly producing a coding for a given input, the encoder produces a mean coding μ and a standard deviation σ.
- The actual coding is then sampled randomly from a Gaussian distribution with mean μ and standard deviation σ.
- After that the decoder just decodes the sampled coding normally.



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  - Pushes the autoencoder to have codings that look as though they were sampled from a simple Gaussian distribution.
  - Using the KL divergence between the target distribution (the Gaussian distribution) and the actual distribution of the codings.
  - $latent\_loss = -\frac{1}{2}\sum_1^K(1 + log(\sigma_i^2) - \sigma_i^2 - \mu_i^2)$

Variational Autoencoders (4/6)

- Encoder part

```
inputs = keras.layers.Input(shape=[28, 28])
z = keras.layers.Flatten()(inputs)
z = keras.layers.Dense(150, activation="relu")(z)
z = keras.layers.Dense(100, activation="relu")(z)
codings_mean = keras.layers.Dense(10)(z)
codings_log_var = keras.layers.Dense(10)(z)
codings = Sampling()([codings_mean, codings_log_var]) # normal distribution
variational_encoder = keras.models.Model(inputs=[inputs], outputs=[codings])
```

Variational Autoencoders (5/6)

- Decoder part

```
decoder_inputs = keras.layers.Input(shape=[codings_size])
x = keras.layers.Dense(100, activation="relu")(decoder_inputs)
x = keras.layers.Dense(150, activation="relu")(x)
x = keras.layers.Dense(28 * 28, activation="sigmoid")(x)
outputs = keras.layers.Reshape([28, 28])(x)
variational_decoder = keras.models.Model(inputs=[decoder_inputs], outputs=[outputs])
```

Variational Autoencoders (6/6)

```
codings = variational_encoder(inputs)
reconstructions = variational_decoder(codings)
model = keras.models.Model(inputs=[inputs], outputs=[reconstructions])

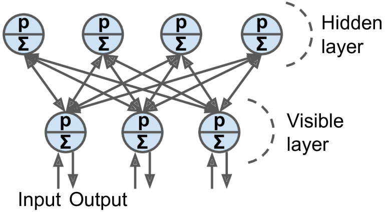
latent_loss = -0.5 * K.sum(1 + codings_log_var - K.exp(codings_log_var)
                           - K.square(codings_mean), axis=-1)
model.add_loss(K.mean(latent_loss) / 784.)
```


S = k. log W
Original illustration: Sonia Jerrems

Restricted Boltzmann Machines

Restricted Boltzmann Machines

- A Restricted Boltzmann Machine (RBM) is a stochastic neural network.



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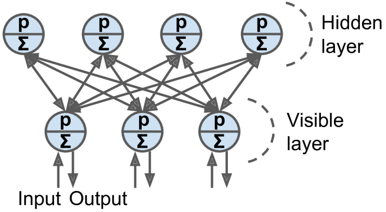
- Stochastic meaning these activations have a probabilistic element, instead of deterministic functions, e.g., logistic or ReLU.



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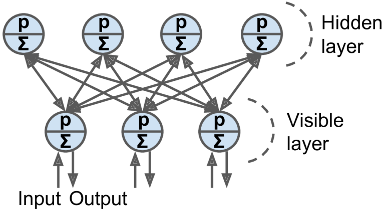
- A Restricted Boltzmann Machine (RBM) is a stochastic neural network.
- Stochastic meaning these activations have a probabilistic element, instead of deterministic functions, e.g., logistic or ReLU.
- The neurons form a bipartite graph:
  - One visible layer and one hidden layer.
  - A symmetric connection between the two layers.
  - There are no connections between neurons within a layer.



Let's Start With An Example

RBM Example (1/10)

- We have a set of six movies, and we ask users to tell us which ones they want to watch.

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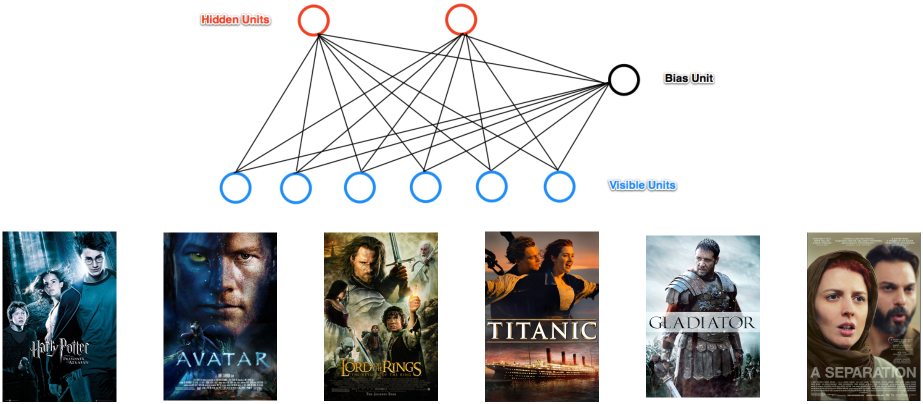
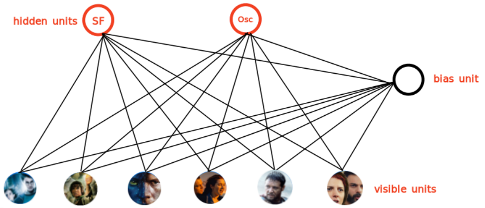
- We have a set of six movies, and we ask users to tell us which ones they want to watch.
- We want to learn two latent neurons (hidden neurons) underlying movie preferences, e.g., SF/fantasy and Oscar winners

- Our RBM would look like the following.



RBM Example (3/10)

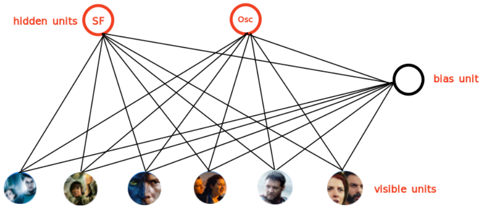
- Assume the given input x_i is the 0 or 1 for each visible neuron v_i.
  - 1: like a movie, and 0: dislike a movie



RBM Example (3/10)

- Assume the given input x_i is the 0 or 1 for each visible neuron v_i.
  - 1: like a movie, and 0: dislike a movie
- Compute the activation energy at hidden neuron h_j:

$$a(h_j) = \sum_i w_{ij} v_i$$



RBM Example (4/10)

- For each hidden neuron $\mathtt{h_j}$, we compute the probability $\mathtt{p(h_j)}$.

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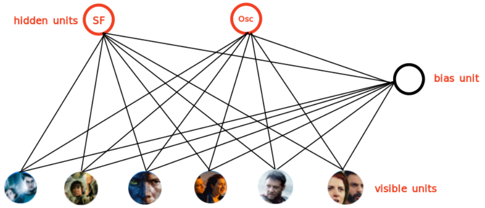

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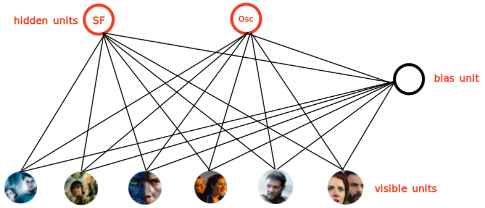
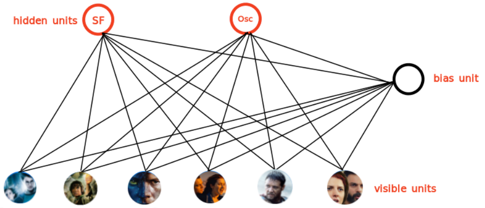
- We turn on the hidden neuron $\mathtt{h_j}$ with the probability $\mathtt{p(h_j)}$, and turn it off with probability $1 - \mathtt{p(h_j)}$.



RBM Example (5/10)

- Declaring that you like Harry Potter, Avatar, and LOTR, doesn't guarantee that the SF/fantasy hidden neuron will turn on.



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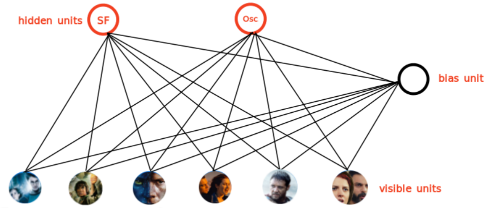
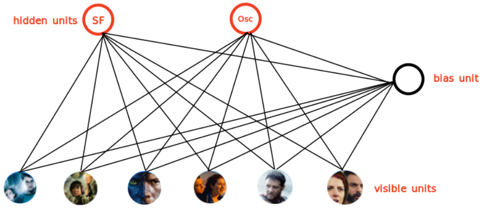


RBM Example (5/10)

- Declaring that you like Harry Potter, Avatar, and LOTR, doesn't guarantee that the SF/fantasy hidden neuron will turn on.
- But it will turn on with a high probability.
  - In reality, if you want to watch all three of those movies makes us highly suspect you like SF/fantasy in general.
  - But there's a small chance you like them for other reasons.



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- We can ask the RBM to generate a set of movie recommendations.
- The hidden neurons send messages to the visible (movie) neurons, telling them to update their states.

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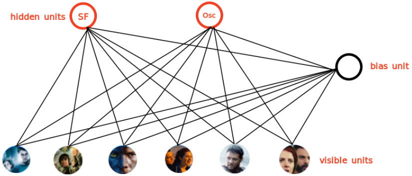
- Being on the SF/fantasy neuron doesn't guarantee that we'll always recommend all three of Harry Potter, Avatar, and LOTR.
  - For example not everyone who likes science fiction liked Avatar.

RBM Example (7/10)

- How do we learn the connection weights w_{ij} in our network?



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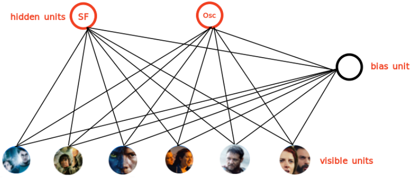

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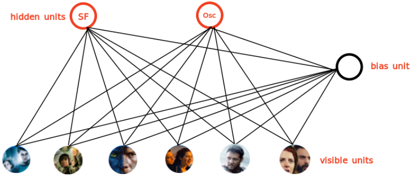

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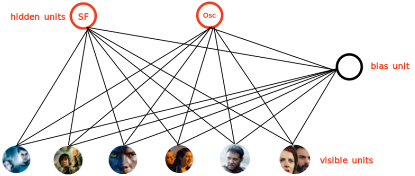
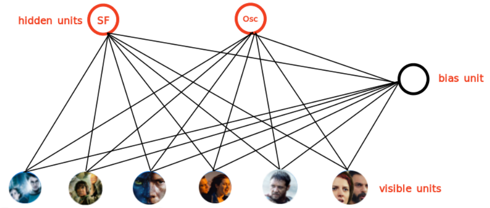
- 1. Take a training instance $\mathbf{x}$ and set the states of the visible neurons to these preferences.



RBM Example (8/10)

- 2. Update the states of the hidden neurons.



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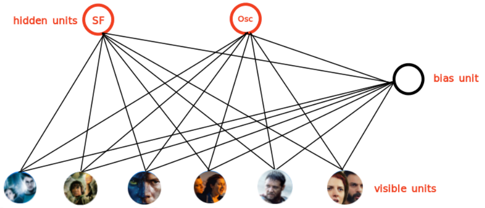

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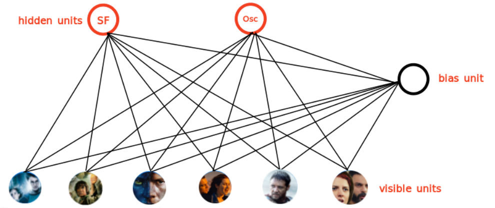

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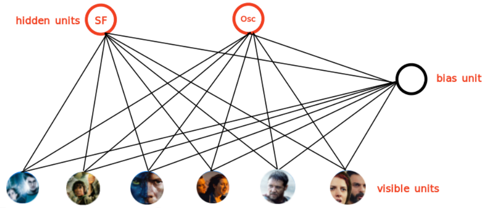
- 3. For each edge e_{ij}, compute $\text{positive}(e_{ij}) = v_i \times h_j$
  - I.e., for each pair of neurons, measure whether they are both on.



RBM Example (9/10)

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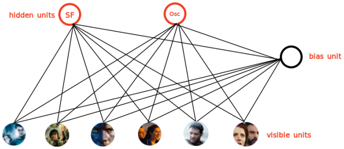

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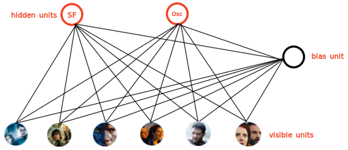

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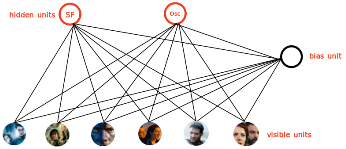

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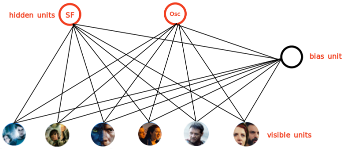

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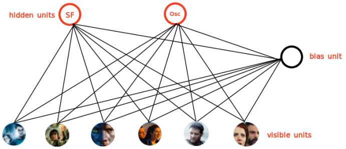
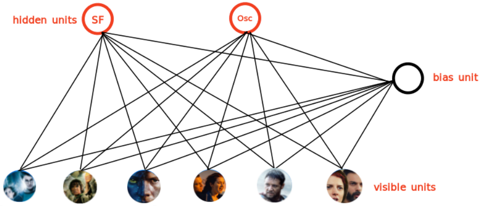
- 5. Update the hidden neurons again similar to step 2. We denote the updated hidden neurons with $\mathtt{h}'_\mathtt{j}$.
- 6. For each edge $\mathtt{e}_\mathtt{ij}$, compute $\mathtt{negative}(\mathtt{e}_\mathtt{ij}) = \mathtt{v}'_\mathtt{i} \times \mathtt{h}'_\mathtt{j}$



RBM Example (10/10)

- 7. Update the weight of each edge $\mathtt{e_{ij}}$.

$$\mathtt{w_{ij}} = \mathtt{w_{ij}} + \eta(\mathtt{positive}(\mathtt{e_{ij}}) - \mathtt{negative}(\mathtt{e_{ij}}))$$



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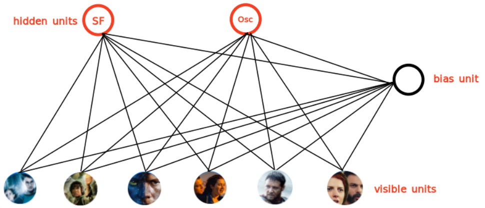

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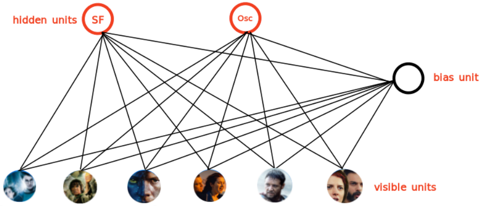
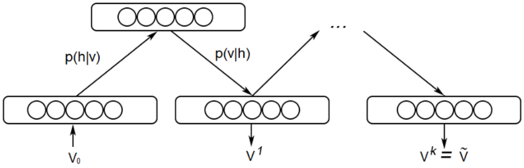
- 9. Continue until the error between the training examples and their reconstructions falls below some threshold or we reach some maximum number of epochs.



RBM Training (1/2)

- Step 1, Gibbs sampling: what we have done in steps 1-6.



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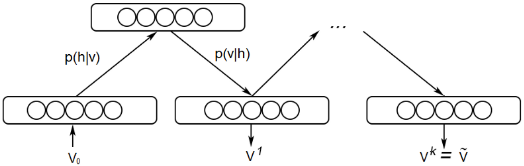

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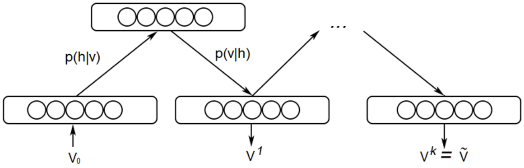
- Knowing the hidden values $\mathbf{h}$, we use $p(\mathbf{v}|\mathbf{h})$ for prediction of new input values $\mathbf{v}$.



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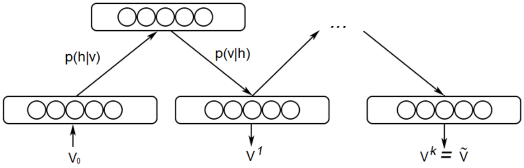
- This process is repeated k times.



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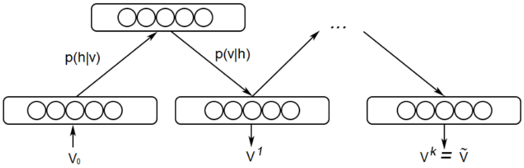

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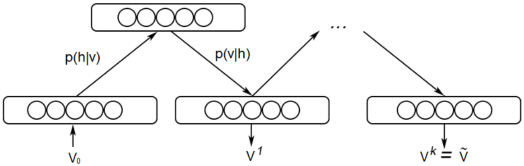
$$\begin{aligned} \mathtt{positive}(\mathbf{e}) &= \mathbf{v}_0 \times \mathrm{p}(\mathbf{h}_0|\mathbf{v}_0) \\ \mathtt{negative}(\mathbf{e}) &= \mathbf{v}_\mathtt{k} \times \mathrm{p}(\mathbf{h}_\mathtt{k}|\mathbf{v}_\mathtt{k}) \\ \mathbf{w} = \mathbf{w} + \eta(\mathtt{positive}&(\mathbf{e}) - \mathtt{negative}(\mathbf{e})) \end{aligned}$$

- $\mathbf{v}_0$ is the original input, and $\mathbf{v}_\mathtt{k}$ is the input after $\mathtt{k}$ iterations.



More Details about RBM

Energy-based Model (1/3)

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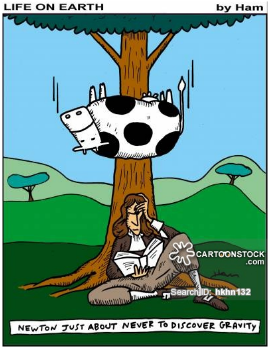
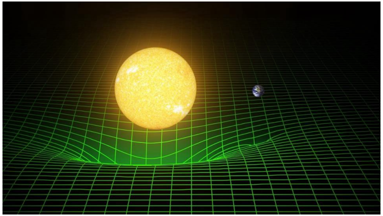
  - E.g., gravitational energy describes the potential energy a body with mass has in relation to another massive object due to gravity.



Energy-based Model (2/3)

- One purpose of deep learning models is to encode dependencies between variables.



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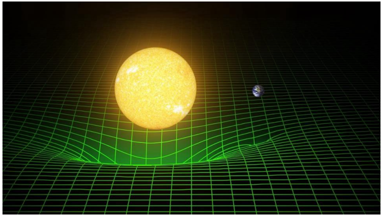

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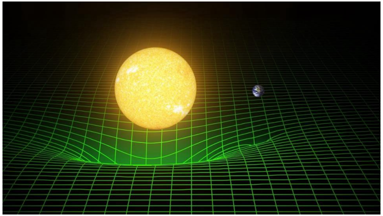

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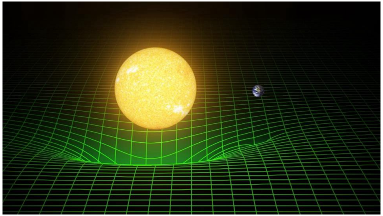
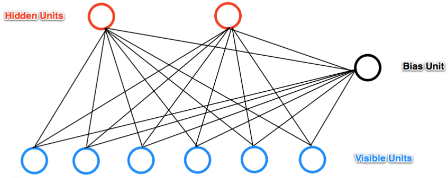
- An energy based model tries always to minimize a predefined energy function.



Energy-based Model (3/3)

- The energy function for the RBMs is defined as:

$$E(\mathbf{v}, \mathbf{h}) = -(\sum_{ij} w_{ij} v_i h_j + \sum_i b_i v_i + \sum_j c_j h_j)$$

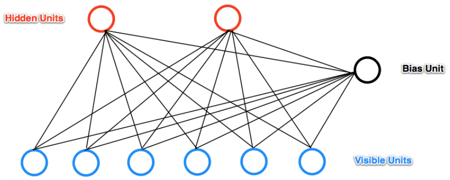


Energy-based Model (3/3)

- The energy function for the RBMs is defined as:

$$\mathtt{E}(\mathbf{v}, \mathbf{h}) = -(\sum_{\mathtt{ij}} \mathtt{w_{ij}} \mathtt{v_i} \mathtt{h_j} + \sum_{\mathtt{i}} \mathtt{b_i} \mathtt{v_i} + \sum_{\mathtt{j}} \mathtt{c_j} \mathtt{h_j})$$

- $\mathbf{v}$ and $\mathbf{h}$ represent the visible and hidden units, respectively.



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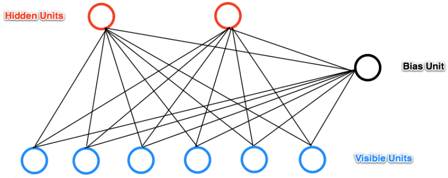

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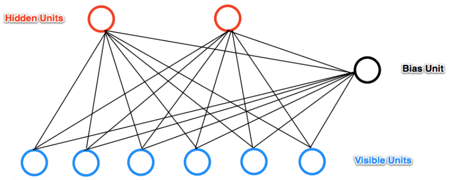
- **b** and **c** are the biases of the visible and hidden layers, respectively.



RBM Is A Probabilistic Model (1/2)

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- As in Physics we assign a probability to observe a state of $\mathbf{v}$ and $\mathbf{h}$, that depends on the overall energy of the model E.
- At each point in time the RBM is in a certain state.
  - The state refers to the values of neurons in the visible and hidden layers $\mathbf{v}$ and $\mathbf{h}$.

RBM is a Probabilistic Model (2/2)

- The probability of a certain state of $\mathbf{v}$ and $\mathbf{h}$:

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- The probability that the network assigns to a visible vector $\mathbf{v}$, is given by summing over all possible hidden vectors $\mathbf{h}$.

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Learning in Boltzmann Machines (1/2)

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- Use the maximum-likelihood estimation.
- For a model of the form $p(\mathbf{v})$ with parameters $\mathbf{w}$, the log-likelihood given a single training example $\mathbf{v}$ is:

$$\log p(\mathbf{v}|\mathbf{w}) = \log \frac{\sum_{\mathbf{h}} e^{-E(\mathbf{v},\mathbf{h})}}{\sum_{\mathbf{v},\mathbf{h}} e^{-E(\mathbf{v},\mathbf{h})}} = \log \sum_{\mathbf{h}} e^{-E(\mathbf{v},\mathbf{h})} - \log \sum_{\mathbf{v},\mathbf{h}} e^{-E(\mathbf{v},\mathbf{h})}$$

Learning in Boltzmann Machines (2/2)

- The log-likelihood gradients for an RBM with binary units:

$$\frac{\partial \log p(\mathbf{v}|\mathtt{w}_{\mathtt{ij}})}{\partial \mathtt{w}_{\mathtt{ij}}} = \mathtt{positive}(\mathtt{e}_{\mathtt{ij}}) - \mathtt{negative}(\mathtt{e}_{\mathtt{ij}})$$

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- Then, we can update the weight $\mathbf{w}$ as follows:

$$\mathtt{w}_{\mathtt{ij}}^{(\mathtt{next})} = \mathtt{w}_{\mathtt{ij}} + \eta(\mathtt{positive}(\mathtt{e}_{\mathtt{ij}}) - \mathtt{negative}(\mathtt{e}_{\mathtt{ij}}))$$


IT'S OVER
IT'S FINALLY OVER
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Summary

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- Autoencoders
  - Stacked autoencoders
  - Denoising autoencoders
  - Variational autoencoders
- Restricted Boltzmann Machine
  - Gibbs sampling
  - Contrastive divergence

Reference

- Ian Goodfellow et al., Deep Learning (Ch. 14, 20)
- Aurélien Géron, Hands-On Machine Learning (Ch. 17)

Questions?