TORO:
A TOol to evaluate the RObustness of cause-effect chains.

Leonie Köhler, M.Sc.
Nikolas Brendes, B.Sc.
Prof. Dr.-Ing. Rolf Ernst

Version 1.0
(typeset November 19, 2019)

Abschlussbericht AP3

iTUBS
Braunschweig

Contents

1 Deliverables of AP 3

- TORO, TOol to evaluate the RObustness of cause-effect chains
- Documentation
- PowerPoint Presentation
- *Additionally*
  Joint conference paper, submission to ECRTS'20 planned

2 Introduction

Control applications in automotive software systems often contain time-critical cause-effect chains. A time-critical chain typically includes the reading of sensor data, their processing and finally the control of actuators. Satisfying specified end-to-end deadlines of cause-effect chains serves to ensure correct system behavior and can also increase driving comfort. From an implementation point of view, a cause-effect chain consists of communicating tasks that are distributed among various system components.

Designing a software application in such a way that all end-to-end deadlines of cause-effect chains are satisfied, even in the worst case, is challenging. Since software applications are regularly extended, it is another central concern to guarantee end-to-end deadlines in case of software updates without having to carry out a completely new software design.
The tool TORO (TOol to evaluate the RObustness of cause-effect chains)

- processes the model of a given software application with time-critical effect chains,
- calculates the maximum end-to-end latencies of the given effect chains,
- makes statements about the robustness of applications during software updates with regard to compliance with end-to-end deadlines.

The results can be used

- to evaluate the robustness of a given software application regarding software updates,
- to identify critical (less robust) segments of a cause-effect chain,
- to predict whether a given software update will change the timing of effect chains such that end-to-end deadlines are missed,
- to determine where and why a given software update changes the time behavior such that end-to-end deadlines are missed.

The tool TORO is based on a novel robustness analysis that was developed in 2018/19 in a collaboration between Daimler and iTUBS. The analysis is summarized in a scientific article which forms the appendix of this documentation. The submission of the article to ECRTS 2020 is planned.

Structure of the Document

Section 3 explains which inputs are expected and which outputs are provided by TORO. Section 4 explains how to install and run TORO. It also introduces three different example use cases, which are evaluated with TORO. Section **??** describes the software architecture of TORO. The appendix is a scientific article on the underlying theory of TORO.

3 Functionality

This chapter describes the terms and methods that are used by the tool TORO.

3.1 Inputs: System Specification

This section explains what type of systems are accepted by TORO as input. In section 3.1.1, a general system model is introduced first. All systems that fulfill this system model can be analyzed by TORO. Section 3.1.2 lists some practical examples to which the system model applies.

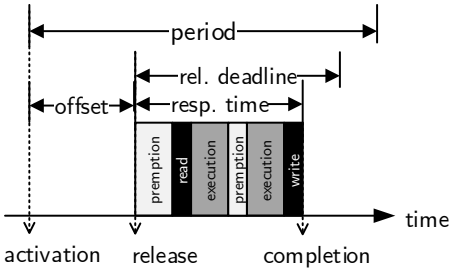
3.1.1 System Model

To analyze the timing of one or more cause-effect chains, a model of the hardware-software system is required. The system model represents the hardware platform and the application software neglecting all aspects that do not determine the timing of the system. In the following, the system model for TORO is specified.

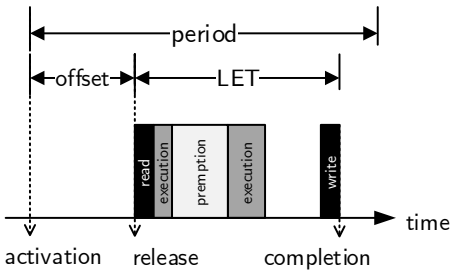
A *hardware platform* $\mathcal{P} = (\mathcal{R}, \mathcal{E})$ is a directed graph. A vertex of the graph is an element from the set of resources $\mathcal{R}$. The edges $\mathcal{E}$ correspond to the directed physical connections between resources. A *resource* $R \in \mathcal{R}$ is characterized by the property that it provides processing or communication service to tasks according to a given scheduling policy. A resource can thus be a processor core, a data bus, etc. We assume that the clocks of all resources are perfectly synchronized.
A *task* $\tau_i \in \mathcal{T}$ is a service-consuming entity. This definition is very general and corresponds to the concept of a task in the real-time community. Since consumed service can be processing service or communication service; we do not need to distinguish between notions like 'software tasks' and 'messages'. The jth instance of a task τ_i is called job $\tau_i(j)$. Two variants of a task exist:

Definition 1 (Bounded execution time task)**.** *The bounded execution time (BET) task is characterized by the following attributes*

- Periodic activation model. *A task τ_i is periodically activated with period P_{τ_i}.*
- Release offset. *A constant release offset Φ_{τ_i} applies after each activation of task τ_i.*
- Worst-case response time. *The response time of a job $\tau_i(j)$ is the duration between its release and completion. It is bounded from above by the worst-case response time $WCRT_{\tau_i}$.*
- Relative deadline. *The relative deadline d_{τ_i} is added to the release instant of a job $\tau_i(j)$ to determine the latest acceptable moment of completion of job $\tau_i(j)$.*
- Read-execute-write semantics. *A task with read-process-write semantics reads all required inputs before it starts to process. All outputs are written to registers directly after the data processing is finished.*
- Allocation. *A task is associated with the resource from which it receives service.*



(a) Periodic BET task with release offset.



(b) LET task with release offset.

Figure 1: Task model.

Definition 2 (Logical execution time task)**.** *The logical execution time (LET) task is characterized by*

- Periodic activation model. *A task τ_i is periodically activated with period P_{τ_i}.*
- Release offset. *A constant release offset Φ_{τ_i} may applies after each activation of task τ_i.*
- Logical execution time. *A task with logical execution time (LET) semantics reads all required inputs at its release time. All outputs are written to registers with the elapse of the LET_{τ_i}.*
- Allocation. *A task is associated with the resource from which it receives service.*

An *application* is a directed graph $\mathcal{A} = (\mathcal{T}, \mathcal{D})$, where the set of vertices $\mathcal{T}$ represents the tasks in the application and the set of directed edges $\mathcal{D}$ represents the data-flow dependencies among the tasks.

Definition 3 (Cause-effect chain)**.** *A cause-effect chain $C = (\tau_1^c, \tau_2^c, \ldots, \tau_n^c)$ is a finite directed walk in an application $\mathcal{A}$ which is executed on a hardware platform $\mathcal{P}$.*

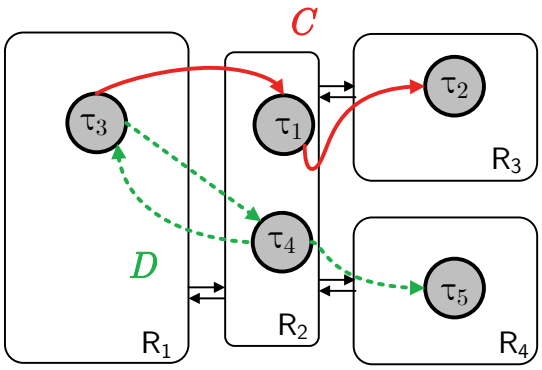
Definition 4 (Instance of a cause-effect chain)**.** *An instance of a cause-effect chain $C(j_1, j_2, \ldots, j_n)$ is a sequence of jobs $(\tau_1^c(j_1), \tau_2^c(j_2), \ldots, \tau_n^c(j_n))$, where each job $\tau_{k+1}^c(j_{k+1})$ reads data from the predecessor job $\tau_k^c(j_k)$.*

The concepts of a cause-effect chain and an instance of a cause-effect chain are illustrated in Figure 2. The possible instances of a cause-effect chain can be determined by a data flow analysis, which checks whether a data flow through the sequence of jobs $\tau_1^c(j_1)\tau_2^c(j_2), \ldots, \tau_n^c(j_n)$ is at all possible.

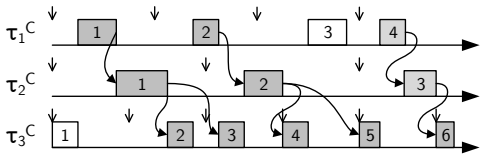
The tool TORO supports time-triggered and LET-triggered cause-effect chains:

Definition 5 (Time-triggered cause-effect chain)**.** *A time-triggered cause-effect chain C^{tt} is a cause-effect chain which has the following properties:*

- *Each task τ_i^c in C^{tt} is a periodically activated BET task with period $P_{\tau_i^c}$ and release offset $\Phi_{\tau_i^c}$.*



(a) Exemplary cause-effect chains $C = (\tau_1^c, \tau_2^c, \tau_3^c) = (\tau_3, \tau_1, \tau_2)$ and $D = (\tau_1^D, \tau_2^D, \tau_3^D, \tau_4^D, \tau_5^D) = (\tau_3, \tau_4, \tau_3, \tau_4, \tau_5)$ in an application.



(b) Exemplary instances of a cause-effect chain $C = (\tau_{C_1}^c, \tau_2^c, \tau_3^c)$:
$C(1,1,2) = (\tau_1^c(1), \tau_2^c(1), \tau_3^c(2))$
$C(1,1,3) = (\tau_1^c(1), \tau_2^c(1), \tau_3^c(3))$
$C(2,2,4) = (\tau_1^c(2), \tau_2^c(2), \tau_3^c(4))$
$C(2,2,5) = (\tau_1^c(2), \tau_2^c(2), \tau_3^c(5))$
$C(4,3,6) = (\tau_1^c(4), \tau_2^c(3), \tau_3^c(6))$

Figure 2: Modeling cause-effect chains.

- *The deadline of each task τ_i^c in C^{tt} is arbitrary.*
- *At the system start t_0, all tasks in C^{let} are activated with release offsets $\Phi_{\tau_i^c}$.*

Definition 6 (LET-triggered cause-effect chain)**.** *A LET-triggered cause-effect chain C^{let} is a cause-effect chain which has the following properties:*

- *Each task τ_i^c in C^{let} is a LET task.*
- *The deadline of each task τ_i^c in C^{let} is arbitrary.*
- *At the system start t_0, all tasks in C^{let} are activated with release offsets $\Phi_{\tau_i^c}$.*

TORO assumes that each task can satisfy its deadline or LET.

3.1.2 Exemplary Analyzable Systems

Application on a single-core ECU under the following assumptions

- Only time-triggered and LET-triggered cause-effect chains.
- Each task meets its deadline or LET.

Application on a multi-core ECU under the following assumptions

- Only time-triggered and LET-triggered cause-effect chains.
- Each task meets its deadline or LET.
- All cores have a common clock, all schedules start simulateneously.

Distributed application on a systems with multiple ECUS connected by data buses and networks under the following assumptions

- Only time-triggered and LET-triggered cause-effect chains. *Note that messages can also be modeled as tasks.*
- Each task meets its deadline or LET.
- All clocks are synchronized, all schedules start simulateneously.

3.2 Computations

Please refer to the paper in the appendix for details.

3.3 Outputs: Latencies and Robustness Margins

This section describes the expected outputs of the analysis, which are both numerical and graphical.

3.3.1 Maximum End-to-End Latencies

There are different definitions for end-to-end latencies of cause-effect chains. Each definition is relevant in specific contexts, but the most important one is the following:

Definition 7 (End-to-end latency, also: data age)**.** *The end-to-end latency of an instance of a cause-effect chain, denoted as $Lat(C(j_1, j_2, \ldots, j_n))$, is the maximum amount of time that may elapse from the release of the first job $\tau_1^c(j_1)$ to the completion of the last job $\tau_n^c(j_n)$ in $C(j_1, j_2, \ldots, j_n)$.*

Definition 8 (Maximum end-to-end latency)**.** *The maximum end-to-end latency $\overline{Lat}(C)$ of a cause-effect chain C is an upper bound on the end-to-end latency of any of its instances*

$$\overline{Lat}(C) = \max_{C(j_1,j_2,\ldots,j_n)\in\mathcal{C}} \{Lat(C(j_1, j_2, \ldots, j_n))\}.$$

The maximum end-to-end latency describes how long it can take until fresh input data at the beginning of the cause-effect chain (e.g. a sensor sample) impacts the output of a cause-effect chain (e.g. actuator control value). This property is important for the reactivity of the system and practical timing requirements often do not relate to the deadline of tasks but in particular to end-to-end deadlines. Tasks deadlines are auxiliary values which emerge in the course of design and implementation.

Definition 9 (End-to-end deadline)**.** *An end-to-end deadline d_C^{e2e} is the maximum tolerable end-to-end latency of a cause-effect chain C.*

3.3.2 Robustness Margins For a Single Cause-Effect Chain

While the maximum end-to-end latency of a cause-effect chain is an important property of the present design, robustness margins indicate to what degree the tasks in a cause-effect chain can be extended in terms of their worst-case response time, resp. LET, without violating the end-to-end deadline.

Theorem: Robustness test for a single time-triggered cause-effect chain

Let $C = (\tau_1^c, \tau_2^c, \ldots, \tau_n^c)$ be a time-triggered cause-effect chain with the property that each task τ_k^c in C has a worst-case response time $WCRT_{\tau_k^c}$ satisfying the task deadline $d_{\tau_k^c}$. Moreover, the cause-effect chain C also satisfies its end-to-end deadline d_C^{e2e}. Let $\Delta WCRT_{\tau_k^c} \geq 0$ be the increase in the worst-case response time of each task τ_k^c in C that is caused by a software update.

The time-triggered cause-effect chain C still satisfies its end-to-end deadline d_C^{e2e} under the increase of the worst-case response times of its tasks τ_k^c by $\Delta WCRT_{\tau_k^c}$, if

$$\forall \tau_k^c \in C : \Delta WCRT_{\tau_k^c} < RM^C(\tau_k^c).$$

Theorem: Robustness test for a single LET-triggered cause-effect chain

Let $C = (\tau_1^c, \tau_2^c, \ldots, \tau_n^c)$ be a LET-triggered cause-effect chain with the property that each task τ_k^c in C satisfies its LET $LET_{\tau_k^c}$. Moreover, the cause-effect chain C also satisfies its end-to-end deadline d_C^{e2e}. Let $\Delta LET_{\tau_k^c} \geq 0$ be a planned increase of the $LET_{\tau_k^c}$ of a task τ_k^c.

The LET-triggered cause-effect chain C still satisfies its end-to-end deadline d_C^{e2e} under the increase of the LETs of its tasks τ_k^c by $\Delta LET_{\tau_k^c}$, if

$$\forall \tau_k^c \in C : \Delta LET_{\tau_k^c} < RM^C(\tau_k^c).$$

3.3.3 Robustness Margins For Multiple Cause-Effect Chains

A task of an application may be part of several cause-effect chains. Consequently, increasing the worst-case response time (or LET) of this particular task may have an impact on all cause-effect chains that share this task. We can find new robustness margins $RM(\tau_k^c)$ (instead of: $RM^C(\tau_k^c)$) which guarantee that *all* cause-effect chains in the application still satisfy their deadlines under a disturbance $\Delta WCRT_{\tau_k^c}$ resp. $\Delta LET_{\tau_k^c}$.

Theorem (Robustness test for time-triggered cause-effect chains)

Let $\mathcal{S}$ be a set of time-triggered cause-effect chains. Each cause-effect chain $C = (\tau_1^c, \tau_2^c, \ldots, \tau_n^c)$ in $\mathcal{S}$ has the property that the worst-case response time $WCRT_{\tau_k^c}$ of every task τ_k^c in C satisfies the task deadline $d_{\tau_k^c}$. Moreover, each cause-effect chain C in $\mathcal{S}$ also satisfies its end-to-end deadline d_C^{e2e}. Let $\Delta WCRT_{\tau_k^c} \geq 0$ be the increase in the worst-case response time of a task τ_k^c in $\mathcal{T}$ that is caused by a software update.

All time-triggered cause-effect chain C in $\mathcal{S}$ still satisfy their end-to-end deadlines d_C^{e2e} under the increase of the worst-case response times of its tasks τ_k^c by $\Delta WCRT_{\tau_k^c}$, if

$$\forall C : \forall \tau_k^c \in C : \Delta WCRT_{\tau_k^c} < RM(\tau_k^c).$$

Theorem (Robustness test for LET-triggered cause-effect chains)

Let $\mathcal{S}$ be a set of LET-triggered cause-effect chains. Each cause-effect chain $C = (\tau_1^c, \tau_2^c, \ldots, \tau_n^c)$ in $\mathcal{S}$ has the property that every task τ_k^c in C satisfies its LET

$LET_{\tau_k^c}$. Moreover, each cause-effect chain C in $\mathcal{S}$ also satisfies its end-to-end deadline d_C^{e2e}. Let $\Delta LET_{\tau_k^c} \geq 0$ be a planned increase of the $LET_{\tau_k^c}$ of a task τ_k^c. All LET-triggered cause-effect chain C in $\mathcal{S}$ still satisfy their end-to-end deadlines d_C^{e2e} under the increase of the LETs of its tasks τ_k^c by $\Delta LET_{\tau_k^c}$, if

$$\forall C : \forall \tau_k^c \in C : \ \Delta LET_{\tau_k^c} < RM(\tau_k^c).$$

4 Usability

This chapter explains how to install and use the tool TORO.

4.1 Installation

This section explains how to install the tool TORO.

- Install a Python Interpreter (version 2.7 or 3). You may want to install a distribution (e.g. Python(x,y)) which contains already many useful packages.
- The tool TORO will be delivered in a zipped folder `Toro.zip`. This folder has the following content:
  - **Main script `toro_main.py`**,
  - **Libraries (pycpa, toro) in the folder `libs`**. The libraries under `./libs` are dynamically included. They do not need to be installed unless you want to change their location.
  - **Input folder `data`.** Systems to be analyzed can be stored here.
- Unzip the zip folder to your preferred destination.

4.2 Use

This section gives an overview how to use the tool TORO.

- Open a terminal and change to the `/Toro` folder.
- The tool TORO is called via the script `toro_main.py`. A path is passed to the script as an argument to specify the location of the system to be analyzed. It is recommended to store the system to be analyzed in the provided `data` folder. If several systems are to be analyzed, a subfolder can be created in `data` for each system to be analyzed. The folder structure is illustrated in Figure 3.
- An exemplary call of the tool would then look as follows:

```
user@computer:~/Toro$ python toro_main.py ./data
```

- The tool TORO then executes, searching for specified systems. From the set of found systems, one ore more can interactively be selected for analysis. Then the same procedure is repeated for each system to be analyzed:
  - User query to identify the type of system and verify assumptions about the system.
  - Parsing the csv-files which specify the system (see Section 4.3.
  - Calculation of maximum end-to-end latencies and robustness margins.
  - Generation of diagrams. The files are written to the folder in which the analyzed system is specified.

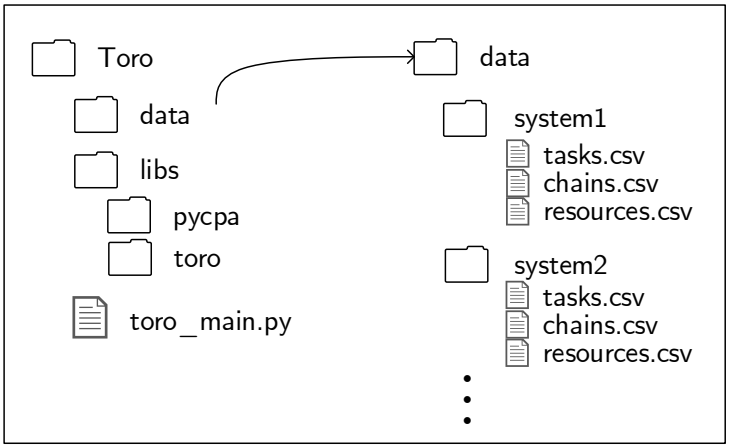


Figure 3: Folder structure

– Creation of a file containing numerical results. The file is written to the folder in which the analyzed system is specified.

4.3 Input Files

This section explains how to specify systems that the tool TORO should analyze.

Firstly, TORO calls the method `get_system_dirs(dir)` and searches the specified folder for systems to be analyzed. Each individual system should be encapsulated in a dedicated folder as illustrated in Figure 3. This system folder should contain three semicolon-separated csv-files, namely

- `chains.csv`,
- `resources.csv`,
- `tasks.csv`.

These files can be created in Microsoft Excel, for example, and can then be exported as CSV files. If TORO finds more than one system in the specified top-level folder (e.g. `.` `data`), the user can choose from a list which system(s) should be examined. An example output would look like this:

```
The following systems have been found:
       1:  system1
       2:  system2
       2:  system3
       0:  all
To select systems enter their ID or a comma-separated list of
IDs.  For instance, enter 1 to select the first of the listed
systems, enter 1,3 to select the first and the third of the
listed systems, or enter '0' to select all systems:
```

The following sections 4.3.1-4.3.3 explain the structure of the CSV files. Then sections 4.3.4-4.3.6 demonstrate the three possible types of use cases and which information needs to be filled in.

4.3.1 Input File 'resources.csv'

The `resources.csv` file describes the execution platform of the system by listing the different resources (CPUs, CAN bus etc.) and the scheduling policy that is applied on each resource. The `resources.csv` file should have the following column headers:

name Type: String
Each computing core and each data bus, on which at least one chain task is executed, is represented by a so-called *resource*. This field specifies the name of the resource.

scheduler Type: String
e.g., `SPPScheduler` or `SPNPScheduler`

4.3.2 Input File 'tasks.csv'

The `tasks.csv` file specifies tasks that are executed on the platform, it should have the following column headers:

task_name Type: String
Unique task ID

period Type: Integer
Activation period

offset Type: Integer
Release offset

priority Type: Integer
Note that 0 is the highest priority.

wcet Type: Integer
Worst Case Execution Time

resource Type: String
Resource that services the task. The name should match a resource from file `pycpa_resources.csv`.

bcrt Type: Integer
Best Case Response Time

wcrt Type: Integer
Worst Case Response Time

let Typ: Integer
Logical Execution Time

4.3.3 Input File 'chains.csv'

The `chains.csv` file specifies which tasks are part of each cause-effect chain; it should have the following column headers:

chain_name Type: String
Unique cause-effect chain ID

e2e_deadline Type: Integer
End-to-end deadline

members Type: String
The member tasks must be listed in correct order and the task names must match those in the task definitions. The list of member tasks comprises as many cells in a row as needed.

4.3.4 Use Case 1:Cause-effect chains with BET tasks, WCRT for each chain task known

- applicable to many systems
- the name, period, offset, WCRT of every task in a cause-effect chain must be known

For *Use Case 1* given that

- all clocks in the system are synchronized,
- all tasks in the cause-effect chains are BET tasks,
- task deadlines are implicit,

(which is checked in an interactive user query), it is sufficient to specify the following details of the entire systems. An example system of type *'Use Case 1'* is depicted in Figure 4.
For the example system, the file `resources.csv` would contain:

name	scheduler
unknown	unknown

With regard to the tasks that are part of the cause-effect chains to be analyzed, we have in `tasks.csv`

task_name	period	offset	priority	wcet	resource	bcrt	wcrt	let

Note that a number of fields in `resources.csv` and `tasks.csv` can be left empty or declared as 'unknown' because the WCRTs are already available and do not need to be computed from these parameters.

The cause-effect chains in `chains.csv` are specified by

chain_name	e2e_deadline	members

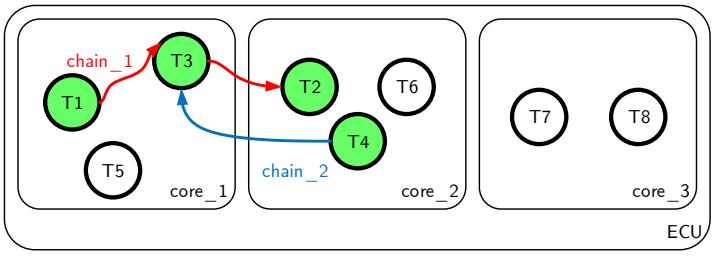


Figure 4: Use case 1; relevant parts of the system for the analysis are marked in color.

4.3.5 Use Case 2: Cause-effect chains with BET tasks, WCRT of each chain task computable

- only applicable to systems with static priority preemptive (SPP) und static priority non-preemptive (SPNP) scheduling
- not only tasks that are part of cause-effect chains must be specified but also the entire background load with all parameters

For *Use Case 2* given that

- all clocks in the system are synchronized,
- ALL tasks are BET tasks (not only those in the listed cause-effect chains),
- task deadlines are implicit,
- ALL tasks in the system must be known with their parameters,
- ALL resources must be known with their scheduling algorithms (SPP or SPNP) and the task-to-resource mapping,

(which is checked in an interactive user query), it is sufficient to specify the following details of the entire systems. An example system of type *'Use Case 2'* is depicted in Figure 5. For the example system, the file `resources.csv` would contain:

Name	Scheduler

With regard to the tasks that are part of the cause-effect chains to be analyzed, we have not only the chain tasks but also the tasks of the background load with all parameters required to compute the WCRTs.

task_name	period	offset	priority	wcet	resource	bcrt	wcrt	let

The cause-effect chains are specified again by

chain_name	e2e_deadline	members

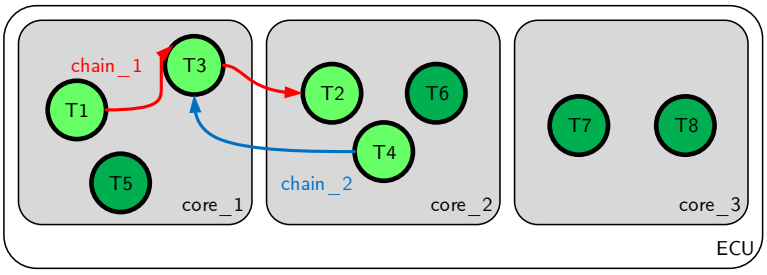


Figure 5: Use case 2; relevant parts of the system for the analysis are marked in color

4.3.6 Use Case 3: Cause-effect chains with LET tasks, LET for each chain task known

- applicable to LET systems

For *Use Case 3* given that

- all clocks in the system are synchronized,
- all tasks in the cause-effect chains are LET tasks,
- all LET tasks have implicit deadlines,

(which is checked in an interactive user query), it is sufficient to specify the following details of the entire systems. An example system of type *'Use Case 3'* is depicted in Figure 6.
For the example system, the file `resources.csv` would contain:

name	scheduler
unknown	unknown

With regard to the tasks that are part of the cause-effect chains to be analyzed, we have in `tasks.csv`

task_name	period	offset	priority	wcet	resource	bcrt	wcrt	let

The cause-effect chains are specified by

chain_name	e2e_deadline	members

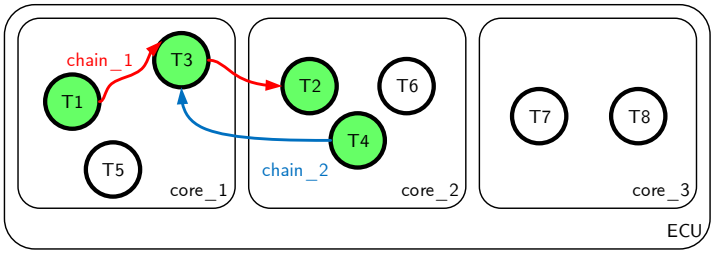


Figure 6: Use case 3; relevant parts of the system for the analysis are marked in color.

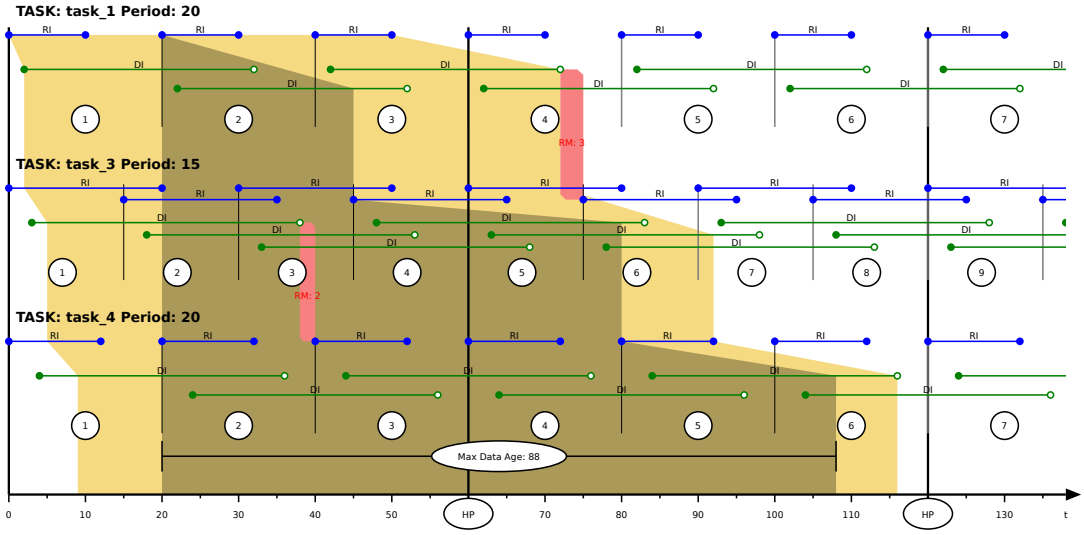


Figure 7: Interval diagram.

4.4 Output Files

Outputs are written into the folder of the specified system.

4.4.1 Log File

The numberical results are written to the text file `RESULTS_LOG.txt`.

4.4.2 Interval Diagram

In the interval diagram, the individual jobs are represented as numbered circles. The blue-marked read intervals represent the earliest possible to latest possible time, when a job can read data. The green-marked data intervals bound the period of time in which the output data of a job is available to other jobs. The short vertical lines stand for the period of a task. The long vertical lines show the hyper period (HP).

The yellow area covers all instances of the cause-effect chain which are relevant for the computation of the maximum end-to-end latency and the robustness margins.
The dark area highlights one instance of a cause-effect chains which actually has the maximum end-to-end latency.
The red areas illustrate some selected robustness margins, which are valid for the isolated cause-effect chain (not for the set of specified cause-effect chains in `chains.csv`.

4.4.3 Reachability Graph

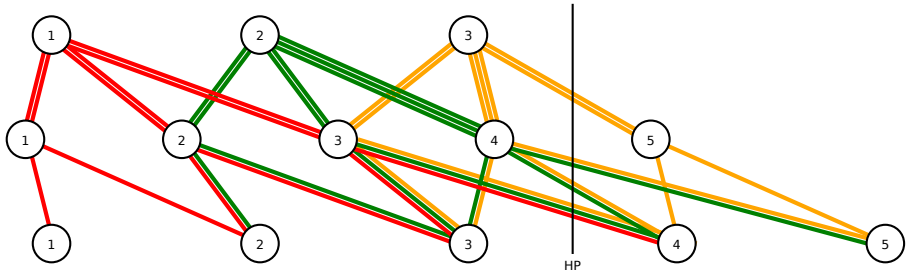


Figure 8: Reachability graph

The reachability graph results from the overlap of read intervals and data intervals of jobs (data flow analysis). In the reachability graph, all possible paths of starting in from the first hyperperiod are represented. These paths are required for the computation of the maximum end-to-end latency and the robustness margins. The colored marking of the individual paths makes it easy to recognize where a path begins and ends. The numbers stand for the respective job number.

4.4.4 Overview Graph

The overview graph represents the specified cause-effect chains as a tasks with data flow dependencies. Moreover, it lists the computed maximum end-to-end latencies for each cause-effect chain. Finally, it indicates the robustness margins for the isolated chains and in the presence of all chains.

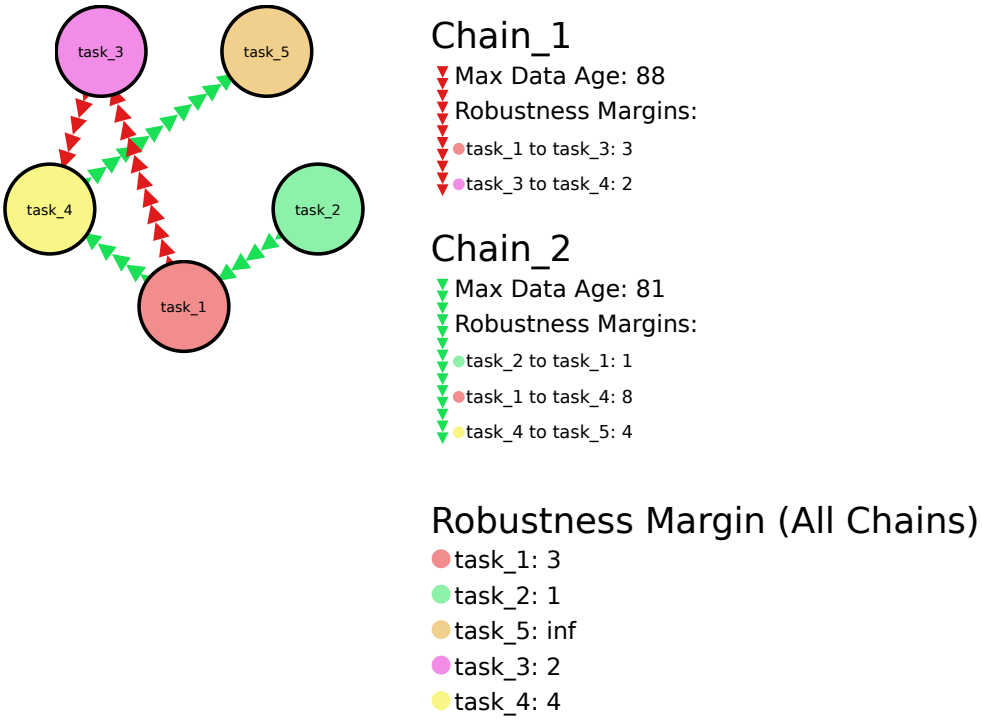


Figure 9: Summarizing diagram.

5 Internals

The tool TORO relies on the Python packages

- `toro`
- `pycpa`

and the main script

- `toro_main.py`

Details about the `pycpa` package can be found in the online documentation at `https://pycpa.readthedocs.io`. The other two components are discussed in more detail in the following sections 5.1 and 5.2.

5.1 User Script

The main script `toro_main.py` consists of two methods (cf. Figure 10):

get_system_dirs(dir)

The script `toro_main.py` starts with the main method and passes the call parameter (folder path) to the `get_system_dirs(dir)` method.
The method `get_system_dirs(dir)` searches for existing systems under the specified path. If more than one system is found, the user can choose from a list which systems should to be analyzed.

perform_analysis(dir)

The method `perform_analysis(dir)`

- starts a user query to identify the type of system and verify assumptions about the system,
- parses the csv-files which specify the system,
- calculates the maximum end-to-end latencies and robustness margins,
- generates of diagrams,
- logs the numerical results (`RESULTS_LOG.txt`),

for each of the selected systems.

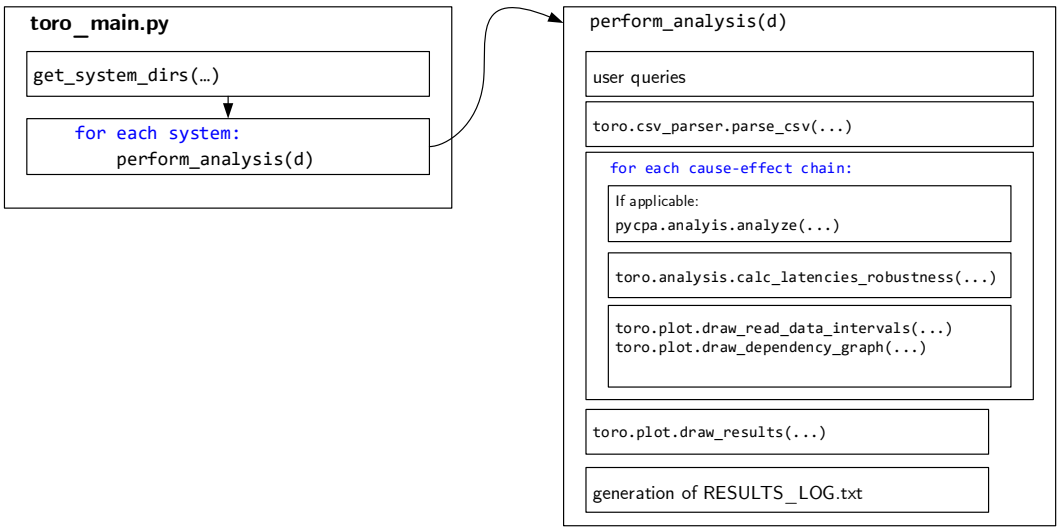


Figure 10: Internals of the main script `toro_main.py`

5.2 Toro Package

This section describes the modules in the package `toro`.

toro.model

This module defines the data structures required within the software. It is an extension of the module of the same name from the `pycpa` package.

toro.analysis

This module contains the `calc_latencies_robustness` class. When an object of this class is initialized and a chain is passed as an argument, then the maximum latency and the robustness margins for each task are computed.

toro.csv_parser

This module reads the CSV-files from the specified folder and transfers them into the software's internal data structure. If files are not available or data is not properly defined, an error message is issued. The parser is called via the `prase_csv` class, which requires the folder path of the system to be analyzed as argument.

toro.plot

The module plots creates graphical representations of the analysis results; the three different types of graphs have been discussed in the section 4.4. The module consists of several classes, each is designed for one of the graphical output formats.

`image` The `image` class provides the basic functions to draw circles, lines, text and other elements in an SVG image.

`draw_read_data_intervals` This class draws the read and data intervals of jobs in a representative time window on the basis of the analysis results. In addition, the computed properties – maxmimum end-to-end latency and the robustness margins for the isolated chain – can be drawn using the options "all", "none", "first" and "last". A further option is to include the "Dependency Polygon":

- max_data_age = all | first | last | none
- robustness_margin = all | first | last | none
- dependency_polygon = True | False.

`draw_dependency_graph` The reachability graph illustrates possible instances of a cause-effect chain. As the call parameter are the analysis results required.

`draw_results` The relevant results of all tasks and cause-effect chains of a system are displayed in an overview graph. Chains and tasks are used as call parameters.

A Appendix

Bounds on Latencies and Robustness of Cause-Effect Chains in Automotive Real-Time Systems

Leonie Köhler
Institute for Computer and Network Engineering, TU Braunschweig, Germany

Max-Jonas Friese
Daimler AG, Stuttgart, Germany

Simon Bagschik
Institute for Computer and Network Engineering, TU Braunschweig, Germany

Rolf Ernst
Institute for Computer and Network Engineering, TU Braunschweig, Germany

Abstract

In automotive real-time systems, the primary timing constraints relate to cause-effect chains in the application software. A cause-effect chain describes the entire process of reading sensor data, computing a control algorithm, driving actuators, and must satisfy a given end-to-end deadline. Therefore, it is not sufficient to compute response times of isolated tasks to verify correct timing, but also the end-to-end latency of the cause-effect chains must be derived. This paper extend existing approaches for the computation of end-to-end latencies to cover realistic systems models with distributed, mixed-triggered cause-effect chains. This paper also addresses a often neglected but very important aspect in industrial practice, namely the robustness of timing properties in the presence of software updates. Specifically, we derive robustness margins which guarantee that if software extensions stay within this bounds, then the end-to-end deadlines of cause-effect chains can still be satisfied.

2012 ACM Subject Classification Computer systems organization → Embedded software; Computer systems organization → Real-time systems; Software and its engineering → Real-time schedulability

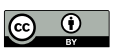
Keywords and phrases Embedded real-time systems, embedded software, cause-effect chains, bounded execution time, logical execution time

Digital Object Identifier 10.4230/LIPIcs.CVIT.2016.23

1 Introduction

A timing-critical cause-effect chain typically includes the following steps: reading and processing of sensor signals, computing a control law, and control of actuators. Given the distributed nature of sensors, ECUs, and actuators in a car, a cause-effect chain in an automotive real-time system is typically distributed over several system components and requires off-chip communication via data busses and/or networks. There are two major problems related to the topic of timing-critical cause-effect chain, which are also addressed in this paper:

1. Computing an upper bound on the end-to-end latency of a distributed cause-effect chain. The performance analysis of timing-critical cause-effect chains (hereafter: CEC-analysis) has been topic of a number of publications. It builds on top of the classical scheduling analysis, which computes worst-case response times for individual tasks. The focus of the CEC-analysis lies on the computation of the maximum end-to-end latency on the basis of results from the classical scheduling theory. To the best knowledge of the authors, existing CEC-analysis is either limited to the scope of cause-effect chains on a single core, a



data bus, a network etc., and/or a specific type of cause-effect chain, which is composed of exclusively time-triggered, logical execution time (LET)-triggered, or event-triggered tasks as will be detailed in the section on related work. The first goal of this paper is therefore to bring together the separate results in the CEC-analysis together to compute end-to-end latency for complex distributed cause-effect chains.

2. Bounding the impact of software updates on the maximum end-to-end latency of cause-effect chains. The second goal is to explore the robustness of time-triggered cause-effect chains. In industrial practice, application software is not constantly re-written but evolves from an approved and tested prior version which is known to satisfy the timing requirements. A very important problem, equally important to the problem of computing maximum end-to-end latencies of cause-effect chains, is to evaluate how robust a given software design with a set of timing-critical cause-effect chains towards updates. Ideally the software should be extensible without violating given end-to-end deadlines of cause-effect chains. The problem of quantifying the robustness of distributed cause-effect chains towards updates has not been treated so far to the best knowledge of the authors. This paper proposes a robustness metric and specifies a rule how to compute it for a given software design with time-triggered cause-effect chains.

This paper is structured as follows. To begin with, we discuss related work and introduce our system model. Then we address the first issue and develop a CEC-analysis providing an upper bound on the end-to-end latency of a complex distributed cause-effect chain. Thereafter the robustness of cause-effect chains is explored. Finally, we apply the presented theory to an industrial case study.

2 Related Work

The related work on computing a maximum end-to-end latency for a given cause-effect chain can be classified according to (1) the types of cause-effect chains covered, and (2) the (non-)distribution of the cause-effect chain over several elements of the execution platform, as shown in Table 1.

The existing work distinguishes four different types of cause-effect chains

- *time-triggered cause-effect chain* where each task is periodically activated with(out) activation/release offset,
- *LET-triggered cause-effect chains* where each task is periodically activated with(out) activation/release offset and has a time-deterministic read and write behavior,
- *sporadically-triggered cause-effect chain* where the activation pattern of each task is specified by a minimum and maximum inter-arrival time,
- *event-triggered cause-effect chain* where the header task is activated by an arbitrary activation pattern while all other tasks are exclusively activated by the completion event of the predecessor event in the chain.

but does not provide methods how to compute end-to-end latencies of a cause-effect chain which is built from different types. Our paper fill this gap.

While design space exploration for the improvement of a given (set of) cause-effect chains has been recently treated, e.g., by [18] [3], the extensibility or robustness of a already designed cause-effect chain has not been treated so far to the best knowledge of the authors. However, robustness is a very important aspect in industrial practice where changes to a design should be incremental and a complete re-design with each software update should be avoided.

	Non-distributed	Distributed
Time-triggered	Forget et al. 2010 [9][8] Becker et al. 2016 [1] Becker et al. 2017 [2]	Pagetti et al. 2011 [15] Puffitsch et al. [16]
LET-triggered	Becker et al. 2017 [2] Martinez et al. 2018 [14]	
Sporadically-triggered		Dürr et al. 2019 [5]
Event-triggered	Schlatow et al. 2016 [17] Girault et al. 2018 [10]	CPA [11] Network calculus [13] and others
Mixed-triggered	Becker et al. 2017 [2]	

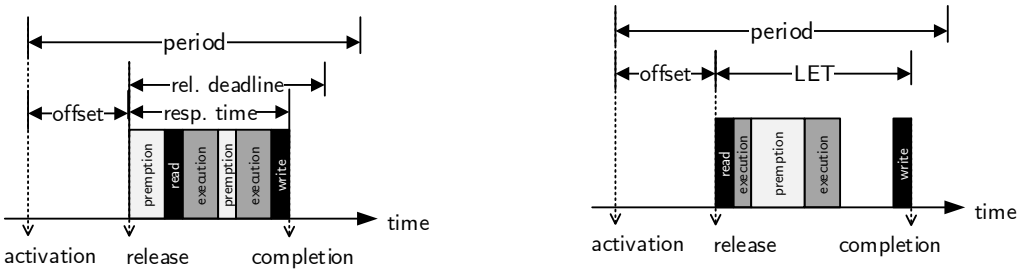
Table 1 Related work on computing the maximum end-to-end latency of cause-effect chains.

3 System Model

A *hardware platform* $\mathcal{P} = (\mathcal{R}, \mathcal{E})$ is a directed graph. A vertex of the graph is an element from the set of resources $\mathcal{R}$. The edges $\mathcal{E}$ correspond to the directed physical connections between resources. A *resource* $R \in \mathcal{R}$ is characterized by the property that it provides processing or communication service to tasks according to a given scheduling policy. A resource can thus be a processor core, a data bus, etc. In this paper, we assume that the clocks of all resources are perfectly synchronized.

A *task* $\tau_i \in \mathcal{T}$ is a service-consuming entity. This definition is very general and corresponds to the concept of a task in the real-time community. Since consumed service can be processing service or communication service; we do not need to distinguish between notions like "software tasks" and "messages". The jth instance of a task τ_i is called job $\tau_i(j)$. In the context of this paper, two variants of a task exist:

- the ***bounded execution time (BET)* task** characterized by the following attributes
  - *Arbitrary activation model.* Let $\underline{\delta}_i(n)$, resp. $\overline{\delta}_i(n)$, denote the minimum, resp. maximum, distance of n consecutive activation events of task τ_i.
  - *Release offset.* A constant release offset Φ_{τ_i} applies after each activation of task τ_i.
  - *Execution time.* The amount of execution time that a job $\tau_i(j)$ may require is bounded from above by the worst-case execution time $WCET_{\tau_i}$.
  - *Response time.* The response time of a job $\tau_i(j)$ is the duration between its release and completion. It is bounded by the best-case response time $BCRT_{\tau_i}$ and the worst-case response time $WCRT_{\tau_i}$.
  - *Relative deadline.* The relative deadline d_{τ_i} is added to the release instant of a job $\tau_i(j)$ to determine the latest acceptable moment of completion of job $\tau_i(j)$.
  - *Read-execute-write semantics.* A task with read-process-write semantics reads all required inputs before it starts to process. All outputs are written to registers directly after the data processing is finished.
  - *Allocation.* A task is associated with the resource from which it receives service.
  - **Note:** All existing methods for end-to-end latency analysis build on top of conventional worst-case response time analysis for tasks. Also in this paper, we assume that the worst-case response times (WCRTs) as well as activation models of tasks have already been computed before hand.
- the ***logical execution time (LET)* task** characterized by



(a) Periodic BET task with release offset.

(b) LET task with release offset.

Figure 1 Task model.

- *Periodic activation model.* A task τ_i is periodically activated with period P_{τ_i}.
- *Release offset.* A constant release offset Φ_{τ_i} applies after each activation of task τ_i.
- *Logical execution time.* A task with logical execution time (LET) semantics reads all required inputs at its release time. All outputs are written to registers with the elapse of the LET_{τ_i}.
- *Allocation.* A task is associated with the resource from which it receives service.

An *application* is a directed graph $\mathcal{A} = (\mathcal{T}, \mathcal{D})$, where the set of vertices $\mathcal{T}$ represents the tasks in the application and the set of directed edges $\mathcal{D}$ represents the data-flow dependencies among the tasks.

▶ **Definition 1** (Cause-effect chain). *A cause-effect chain* $C = (\tau_1^c, \tau_2^c, \ldots, \tau_n^c)$ *is a finite directed walk in an application* $\mathcal{A}$ *which is executed on a hardware platform* $\mathcal{P}$.

▶ **Definition 2** (Instance of a cause-effect chain). *An instance of a cause-effect chain* $C(j_1, j_2, \ldots, j_n)$ *is a sequence of jobs* $(\tau_1^c(j_1), \tau_2^c(j_2), \ldots, \tau_n^c(j_n))$, *where each job* $\tau_{k+1}^c(j_{k+1})$ *reads data from the predecessor job* $\tau_k^c(j_k)$.

The concepts of a cause-effect chain and an instance of a cause-effect chain are illustrated in Figure 2. The possible instances of a cause-effect chain can be determined by a data flow analysis, which checks whether a data flow through the sequence of jobs $\tau_1^c(j_1)\tau_2^c(j_2), \ldots, \tau_n^c(j_n)$ is at all possible.

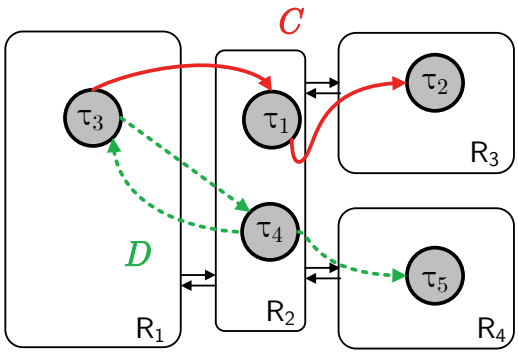
4 Data Flow Analysis: Computing Instances of a Cause-Effect Chain

The objective of this section is to present a data flow analysis, which delivers all possible instances of a given cause-effect chain. Becker et al. [2] introduced a data flow analysis for a single-resource system with a fully periodic task set. The fundamental principle of this data flow analysis, however, is valid for our more general system model with multiple resources and tasks with an arbitrary activation model as shown in Theorem 5.

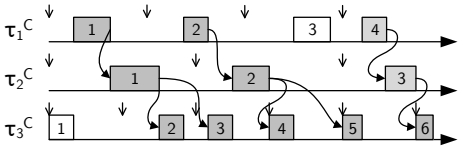
▶ **Definition 3** (Read interval). *The read interval of a job* $\tau_i^c(j_i)$, *denoted as*

$$RI\left(\tau_i^c(j_i)\right) = [\underline{R}\left(\tau_i^c(j_i)\right), \overline{R}\left(\tau_i^c(j_i)\right)],$$

is the time interval from the earliest possible instant to the latest possible instant at which job $\tau_i^c(j_i)$ *can read its input data without violating its deadline or LET.*



(a) Exemplary cause-effect chains $C = (\tau_1^c, \tau_2^c, \tau_3^c) = (\tau_3, \tau_1, \tau_2)$ and $D = (\tau_1^D, \tau_2^D, \tau_3^D, \tau_4^D, \tau_5^D) = (\tau_3, \tau_4, \tau_3, \tau_4, \tau_5)$ in an application.



(b) Exemplary instances of a cause-effect chain $C = (\tau_{C_1}^c, \tau_2^c, \tau_3^c)$:

$C(1,1,2) = (\tau_1^c(1), \tau_2^c(1), \tau_3^c(2))$
$C(1,1,3) = (\tau_1^c(1), \tau_2^c(1), \tau_3^c(3))$
$C(2,2,4) = (\tau_1^c(2), \tau_2^c(2), \tau_3^c(4))$
$C(2,2,5) = (\tau_1^c(2), \tau_2^c(2), \tau_3^c(5))$
$C(4,3,6) = (\tau_1^c(4), \tau_2^c(3), \tau_3^c(6))$

Figure 2 Modeling cause-effect chains.

▶ **Definition 4** (Data interval). *The data interval of a job $\tau_{C_i}^c(j_i)$, denoted as*

$$DI\left(\tau_i^c(j_i)\right) = \left[\underline{D}\left(\tau_i^c(j_i)\right), \overline{D}\left(\tau_i^c(j_i)\right)\right],$$

is the time interval from the earliest possible instant to the latest possible instant at which output data of job $\tau_i^c(j_i)$ is available without violating its deadline or LET.

▶ **Theorem 5** (Data flow analysis for a general cause-effect chain). *Assume an application $\mathcal{A}$ which is executed on a hardware platform as described in Section 3. Let the application $\mathcal{A}$ consists only of schedulable tasks, i.e., each task satisfies its deadline or LET. A finite directed walk in application $\mathcal{A}$ is the cause-effect chain C.*
Then all possible instances of a cause-effect chain C are contained in the (super)set

$$\mathcal{C} = \{C(j_1, j_2, \ldots, j_n) \mid j_1, j_2, \ldots, j_n \in \mathbb{N} \wedge \forall k \in \mathbb{N} : 1 \leq k \leq n-1 : RI\left(\tau_{k+1}^c(j_{k+1})\right) \cap DI\left(\tau_k^c(j_k)\right) \neq \emptyset\}$$

where

$$RI\left(\tau_i^c(j_i)\right) = \left[\underline{R}\left(\tau_i^c(j_i)\right), \overline{R}\left(\tau_i^c(j_i)\right)\right]$$

$$\underline{R}\left(\tau_i^c(j_i)\right) = \begin{cases} \underline{\delta}_{\tau_i^c}(j_i) + \Phi_{\tau_i^c} & \text{if } \tau_i^c \text{ is a BET task} \\ (j_i - 1) \cdot P_{\tau_i^c} + \Phi_{\tau_i^c} & \text{if } \tau_i^c \text{ is a LET task} \end{cases}$$

$$\overline{R}\left(\tau_i^c(j_i)\right) = \begin{cases} \overline{\delta}_{\tau_i^c}(j_i) + \Phi_{\tau_i^c} + d_{\tau_i^c} - BCRT_{\tau_i^c} & \text{if } \tau_i^c \text{ is a BET task} \\ (j_i - 1) \cdot P_{\tau_i^c} + \Phi_{\tau_i^c} & \text{if } \tau_i^c \text{ is a LET task} \end{cases}$$

and

$$DI\left(\tau_i^c(j_i)\right) = \left[\underline{D}\left(\tau_i^c(j_i)\right), \overline{D}\left(\tau_i^c(j_i)\right)\right]$$

$$\underline{D}\left(\tau_i^c(j_i)\right) = \begin{cases} \underline{\delta}_{\tau_i^c}(j_i) + \Phi_{\tau_i^c} + BCRT_{\tau_i^c} & \text{if } \tau_i^c \text{ is a BET task} \\ (j_i - 1) \cdot P_{\tau_i^c} + \Phi_{\tau_i^c} + LET_{\tau_i^c} & \text{if } \tau_i^c \text{ is a LET task} \end{cases}$$

$$\overline{D}\left(\tau_i^c(j_i)\right) = \begin{cases} \overline{\delta}_{\tau_i^c}(j_i + 1) + \Phi_{\tau_i^c} + WCRT_{\tau_i^c} & \text{if } \tau_i^c \text{ is a BET task} \\ j_i \cdot P_{\tau_i^c} + \Phi_{\tau_i^c} + LET_{\tau_i^c} & \text{if } \tau_i^c \text{ is a LET task.} \end{cases}$$

Proof. An instance of a cause-effect chain $C(j_1, j_2, \ldots, j_n)$ requires by definition that each job in the chain except the header job reads data from the predecessor job. For a job $\tau_{k+1}^c(j_{k+1})$ in the cause-effect chain C to be able to read from the predecessor $\tau_k^c(j_k)$, it is necessary that the read interval of job $\tau_{k+1}^c(j_{k+1})$ overlaps with the data interval of job $\tau_k^c(j_k)$ as was already observed in [2]:

$$RI\left(\tau_{k+1}^c(j_{k+1})\right) \cap DI\left(\tau_k^c(j_k)\right) \neq \emptyset \Rightarrow \tau_{k+1}^c(j_{k+1}) \textit{ can read from } \tau_k^c(j_k)$$
$$\text{where } k \in \mathbb{N} : 1 \leq k \leq n-1. \tag{1}$$

Therefore, any selection of jobs $(\tau_1^c(j_1), \tau_2^c(j_2), \ldots, \tau_n^c(j_n))$ with $j_1, j_2, \ldots, j_n \in \mathbb{N}$ which satisfies the data flow constraint in Eq. 1 for all $k \in \mathbb{N} : 1 \leq k \leq n-1$ is potentially an instance of the cause-effect chain C.

It remains to show that the stated interval boundaries are correct.

- The earliest point in time, when a job $\tau_i^c(j_i)$ may read its input data is at its release. By definition, the earliest possible instant of activation is $\underline{\delta}_{\tau_i^c}(j_i)$ (BET task), resp. $(j_i - 1) \cdot P_{\tau_i^c}$ (LET task), to which a constant release offset $\Phi_{\tau_i^c}$ is added.
- If a job $\tau_i^c(j_i)$ of a BET task τ_i^c is timely as required, then the latest possible instant for the job to start execution is by $BCRT_{\tau_i^c}$ time units before the deadline. Thus, this is also the latest possible moment to read the inputs.

  A job $\tau_i^c(j_i)$ of a LET task τ_i^c will always read at the release time.
- Outputs of job $\tau_i^c(j_i)$ can be produced soonest at $BCRT_{\tau_i^c}$ after the earliest point of release given by $\underline{\delta}_{\tau_i^c}(j_i) + \Phi_{\tau_i^c}$ (BET task), resp. at $LET_{\tau_i^c}$ after the release at $(j_i - 1) \cdot P_{\tau_i^c} + \Phi_{\tau_i^c}$ (LET task).
- Outputs of job $\tau_i^c(j_i)$ are available until they are overwritten by the next job $\tau_i^c(j_i + 1)$, which happens latest at $\overline{\delta}_{\tau_i^c}(j_i{+}1){+}\Phi_{\tau_i^c}{+}WCRT_{\tau_i^c}$ (BET task), resp. $j_i{\cdot}P_{\tau_i^c}{+}\Phi_{\tau_i^c}{+}LET_{\tau_i^c}$.

◀

It is clear that the proposed data flow analysis for a general cause-effect chain C is a template which needs to be enriched which information on the actual cause-effect chain under analysis with regard to the activation model, the relative deadline and the response time bounds. A practical difficulty is that an infinite amount of job combinations must be tested to construct the complete set of possible chain instances $\mathcal{C}$.

Fortunately, the calculation can be simplified for specific settings. Firstly, the special case of a time-triggered cause-effect chain C^{tt} allows to limit the numerical effort to construct due $\mathcal{C}$ to the existence of a hyper period. A related second case is the LET-triggered cause-effect chain C^{let}. Results for C^{tt} and C^{let} are given in Theorem 9. Note that Becker et al. proposed in [1] [2] a similar theorem but here we reduced the set of assumptions with respect to the analyzable system. Becker et al. only allowed single-resource systems with fully periodic task sets with implicit deadlines. Finally, the special case of an event-triggered cause-effect chain C^{et} is discussed, where the termination event of one job activates the successor job in the chain, leads to a restricted set of possible instances of C^{et}.

▶ **Definition 6** (Time-triggered cause-effect chain). *A time-triggered cause-effect chain C^{tt} is a cause-effect chain which has the following properties:*

- *Each task τ_i^c in C^{tt} is a periodically activated BET task with period $P_{\tau_i^c}$ and release offset $\Phi_{\tau_i^c}$.*
- *The deadline of each task τ_i^c in C^{tt} is arbitrary.*
- *At the system start t_0, all tasks in C^{let} are activated.*

▶ **Definition 7** (LET-triggered cause-effect chain). *A LET-triggered cause-effect chain C^{let} is a cause-effect chain which has the following properties:*

- *Each task τ_i^c in C^{let} is a LET task.*
- *The deadline of each task τ_i^c in C^{let} is arbitrary.*
- *At the system start t_0, all tasks in C^{let} are activated.*

With the proposed definition of a LET-triggered cause-effect chain C^{let}, we cover advanced aspects of LET-based system designs

- LET systems with offsets [12]
- LET systems with arbitrary deadlines [4] [12]
- System-level LET systems with multiple resources [6].

▶ **Lemma 8** (Read/data intervals for time/LET-triggered cause-effect chains). *Assume an application $\mathcal{A}$ which is executed on a hardware platform as described in Section 3. Let the application $\mathcal{A}$ consists only of schedulable tasks, i.e., each task satisfies its deadline or LET. Let a path in application $\mathcal{A}$ be a time-triggered cause-effect chain C^{tt}, resp. a LET-triggered cause-effect chain C^{let}, then the boundaries for the read intervals of chain tasks are*

$$\underline{R}\left(\tau_i^c(j)\right) = (j-1)\cdot P_i + \Phi_i$$

$$\overline{R}\left(\tau_i^c(j)\right) = \begin{cases} (j-1)\cdot P_i + \Phi_i + d_{\tau_i^c} - BCRT_{\tau_i^c} & \text{for } C^{tt} \\ (j-1)\cdot P_i + \Phi_i & \text{for } C^{let} \end{cases}$$

and the boundaries for the data intervals of chain tasks are

$$\underline{D}\left(\tau_i^c(j)\right) = \begin{cases} (j-1)\cdot P_i + \Phi_i + BCRT_{\tau_i^c} & \text{if } \tau_i^c \text{ in } C^{tt} \\ (j-1)\cdot P_i + \Phi_i + LET_{\tau_i^c} & \text{if } \tau_i^c \text{ in } C^{let} \end{cases}$$

$$\overline{D}\left(\tau_i^c(j)\right) = \begin{cases} j\cdot P_i + \Phi_i + WCRT_{\tau_i^c} & \text{if } \tau_i^c \text{ in } C^{tt} \\ j\cdot P_i + \Phi_i + LET_{\tau_i^c} & \text{if } \tau_i^c \text{ in } C^{let}. \end{cases}$$

Proof. The results of Theorem 5 for LET tasks can be reused because a LET-triggered chain C^{let} consists exclusively of LET tasks. The interval boundaries for BET tasks in a time-triggered chain C^{tt} can also be derived from Theorem 5, if the constraint of periodic activation instants with release offset is considered. ◀

▶ **Theorem 9** (Data flow analysis for time-triggered and LET-triggered cause-effect chains). *Assume an application $\mathcal{A}$ which is executed on a hardware platform as described in Section 3. Let the application $\mathcal{A}$ consists only of schedulable tasks, i.e., each task satisfies its deadline or LET. Let a path in application $\mathcal{A}$ be the time-triggered cause-effect chain C^{tt}, resp. the LET-triggered cause-effect chain C^{let}.*

The set of potential instances of the time-triggered cause-effect chain C^{tt}, resp. the LET-triggered cause-effect chain C^{let}, with the property that the header job $\tau_1^c(j_1)$ falls into the first hyper period $[0, lcm(P_1, P_2, \ldots, P_n))$ is

$$\begin{aligned} \mathcal{C}_{HP} = \{C(j_1, j_2, \ldots, j_n) \mid\ & j_1, j_2, \ldots, j_n \in \mathbb{N} \\ & \wedge\ (j_1 - 1)\cdot P_1 + \Phi_1 \leq lcm(P_1, P_2, \ldots, P_n)\} \\ & \wedge\ \forall k \in \mathbb{N} : 1 \leq k \leq n-1 : RI\left(\tau_{k+1}^c(j_{k+1})\right) \cap DI\left(\tau_k^c(j_k)\right) \neq \emptyset\} \end{aligned}$$

Then all instances of a time-triggered cause-effect chain C^{tt}, resp. of a LET-triggered cause-effect chain C^{let}, are contained in the set

$$\begin{aligned} \mathcal{C} = \{C(j_1 + m\cdot J_1, j_2 + m\cdot J_2, \ldots, j_n + m\cdot J_n) \mid\ & C(j_1, j_2, \ldots, j_n) \in \mathcal{C}_{HP} \\ \wedge\ m \in \mathbb{N} \wedge\ \forall 1 \leq x \leq n : J_x = \lceil lcm(P_1, P_2, \ldots, P_n)/P_x \rceil\}. & \end{aligned}$$

Proof. The claim of this proof is that all instances of the time-triggered cause-effect chain C^{tt}, resp. LET-triggered cause-effect chain C^{let} can be constructed from the set $\mathcal{C}_{HP}$. The set $\mathcal{C}_{HP}$ contains all potential instances of C^{tt}, resp. C^{let}, with the property that the header job $\tau_1^c(j_1)$ falls into the first hyper period of the chain tasks $[0, lcm(P_1, P_2, \ldots, P_n))$.
The justification is as follows. It is well known fact that the activation pattern of a periodic task set with/without release offsets repeats after each hyper period. Since the read intervals and data intervals for time/LET-triggered cause-effect chain only depend on activation instants and constants, also the pattern of read intervals and data intervals repeats after each hyper period. This leads to the fact that the pattern of chain instances also repeat after each hyper period. ◀

▶ **Definition 10** (Event-triggered cause-effect chain). *An event-triggered cause-effect chain C^{et} is cause-effect chain which has the following property:*

- *All tasks in C^{et} are BET tasks without release offset and an arbitrary deadline.*
- *The first task τ_1^c in the chain is activated according to an arbitrary activation model, while any τ_i^c with $1 < i \leq n$ is exclusively activated by the completion event of the predecessor task τ_{i-1}^c.*

▶ **Theorem 11** (Data flow analysis for event-triggered cause-effect chains). *Assume a system as described in Section 3 with a schedulable task set, i.e., each task satisfies its deadline. Then all instances of an event-triggered cause-effect chain C^{et} are contained in the set*

$$\mathcal{C}^{et} = \{C(j_1, j_2, \ldots, j_n) \mid j_1, j_2, \ldots, j_n \in \mathbb{N} \wedge j_1 = j_2 = \ldots = j_n\}.$$

Proof. The condition $j_1 = j_2 = \ldots = j_n$ is a direct consequence of the definition of an event-triggered cause-effect chain. ◀

5 End-to-End Latency Analysis

Feiertag et al. [7] list different definitions for end-to-end latencies of cause-effect chains. Each definition is relevant in specific contexts, but the most important one is the following:

▶ **Definition 12** (End-to-end latency, also: data age). *The end-to-end latency of an instance of a cause-effect chain, denoted as $Lat(C(j_1, j_2, \ldots, j_n))$, is the maximum amount of time that may elapse from the release of the first job $\tau_1^c(j_1)$ to the completion of the last job $\tau_n^c(j_n)$ in $C(j_1, j_2, \ldots, j_n)$.*

▶ **Definition 13** (Maximum end-to-end latency). *The maximum end-to-end latency $\overline{Lat}(C)$ of a cause-effect chain C is an upper bound on the end-to-end latency of any of its instances*

$$\overline{Lat}(C) = \max_{C(j_1, j_2, \ldots, j_n) \in \mathcal{C}} \{Lat(C(j_1, j_2, \ldots, j_n))\}.$$

The maximum end-to-end latency describes how long it can take until fresh input data at the beginning of the cause-effect chain (e.g. a sensor sample) impacts the output of a cause-effect chain (e.g. actuator control value). This property is important for the reactivity of the system and practical timing requirements often do not relate to the deadline of tasks but in particular to end-to-end deadlines. Tasks deadlines are auxiliary values which emerge in the course of design and implementation.

▶ **Definition 14** (End-to-end deadline). *An end-to-end deadline d_C^{e2e} is the maximum tolerable end-to-end latency of a cause-effect chain C.*

We suppose hereafter that a cause-effect chain C can be decomposed into segments where each segment is either a time-triggered, an LET-triggered or an event-triggered cause-effect chain. This assumption on decomposability is not very restrictive and allows indeed to cover many real-world scenarios.

▶ **Definition 15.** *A cause-effect chain C is called decomposable, if it can be represented by a concatenation of time-triggered, LET-triggered, and event-triggered cause-effect chains C_i, such that $C = (C_1, C_2, \ldots, C_N)$.*

For each type of chain segment, we discuss briefly already known methods from related work how to derive the maximum end-to-end latency.

▶ **Theorem 16** (Maximum end-to-end latency of a time-triggered or LET-triggered chain, [2])**.** *An upper bound on the end-to-end latency of a time-triggered, resp. LET-triggered, cause-effect chain $C = (\tau_1^c, \tau_2^c, \ldots, \tau_n^c)$ is*

$$\overline{Lat}(C) = \begin{cases} \max\limits_{C(j_1,\ldots,j_n)\in\mathcal{C}_{HP}} \left\{\left((j_n-1)\cdot P_{\tau_n^c} + \Phi_{\tau_n^c} + WCRT_{\tau_n^c}\right) - \left((j_1-1)\cdot P_{\tau_1^c} + \Phi_{\tau_1^c}\right)\right\} \\ \qquad \textit{if } C \textit{ is time-triggered} \\ \max\limits_{C(j_1,\ldots,j_n)\in\mathcal{C}_{HP}} \left\{\left((j_n-1)\cdot P_{\tau_n^c} + \Phi_{\tau_n^c} + LET_{\tau_n^c}\right) - \left((j_1-1)\cdot P_{\tau_1^c} + \Phi_{\tau_1^c}\right)\right\} \\ \qquad \textit{if } C \textit{ is LET-triggered.} \end{cases}$$

Proof. The end-to-end latency of some instance of a time-triggered cause-effect chain, resp. LET-triggered cause-effect chain, is

$$Lat(C(j_1,\ldots,j_n)) = \begin{cases} \left((j_n-1)\cdot P_{\tau_n^c} + \Phi_{\tau_n^c} + WCRT_{\tau_n^c}\right) - \left((j_1-1)\cdot P_{\tau_1^c} + \Phi_{\tau_1^c}\right). \\ \qquad \text{if } C \text{ is time-triggered} \\ \left((j_n-1)\cdot P_{\tau_n^c} + \Phi_{\tau_n^c} + LET_{\tau_n^c}\right) - \left((j_1-1)\cdot P_{\tau_1^c} + \Phi_{\tau_1^c}\right) \\ \qquad \text{if } C \text{ is LET-triggered} \end{cases}$$

which follows directly from Definitions 18, 6, 7. The maximum end-to-end latency is then

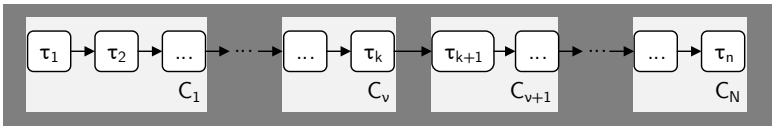
$$\overline{Lat}(C) = \max_{C(j_1,\ldots,j_n)\in\mathcal{C}_{HP}} \{Lat(C(j_1,\ldots,j_n))\}.$$

which corresponds to Definition 19 except that the set $\mathcal{C}$ can be reduced to $\mathcal{C}_{HP}$ because we know from Theorem 9 that chain instances repeat after each hyper period. ◀

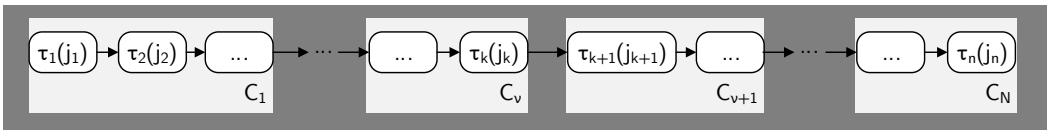
▶ **Theorem 17** (Maximum end-to-end latency of an event-triggered chain, [11])**.** *An upper bound on the end-to-end latency of an event-triggered cause-effect chain $C = (\tau_1^c, \tau_2^c, \ldots, \tau_n^c)$ is*

$$\overline{Lat}(C) = \sum_{k=1}^{n} WCRT_{\tau_k^c}.$$

Proof. By the definition of an event-triggered cause-effect chain, we know that (1) the completion event of one task τ_k^c in the chain C is the activation event of the successor task τ_{k+1}^c, (2) activation and release events coincide. Since the delay between the release of a task until its completion cannot be longer than its worst-case response time, the maximum end-to-end latency of the cause-effect chain C cannot be longer than the sum of the worst-case response times of all tasks in C. ◀



(a) Data flow graph at task level



(b) Data flow graph at job level

Figure 3 Data flow graph of a decomposable cause-effect chain

A tighter upper bound on the end-to-end latency of an event-triggered chain can be computed by applying methods from, e.g., [17] [10].

To finally derive the maximum end-to-end latency of an entire cause-effect chain, it is a viable approach to compute the maximum end-to-end latencies of the known sub-chains and add the maximum transition latencies between two successive sub-chains.

▶ **Definition 18** (Transition latencies between successive sub-chains). *Let the cause-effect chain C contain two successive sub-chains C_ν and $C_{\nu+1}$. Furthermore, let job $\tau_k^c(j_k)$ be the last job of C_ν and job $\tau_{k+1}^c(j_{k+1})$ be the first job of $C_{\nu+1}$ in an instance of C such that $RI\left(\tau_{k+1}^c(j_{k+1})\right) \cap DI\left(\tau_k^c(j_k)\right) \neq \emptyset$. Then the transition latency between successive sub-chains $TLat(C_\nu(\ldots, j_k), C_{\nu+1}(j_{k+1}, \ldots))$ is the maximum amount of time that may elapse between the completion of job $\tau_k^c(j_k)$ and the release of job $\tau_{k+1}^c(j_{k+1})$.*

▶ **Definition 19** (Maximum transition latency between successive sub-chains). *The maximum transition latency between sub-chains $\overline{TLat}(C_\nu, C_{\nu+1})$ is defined as*

$$\overline{TLat}(C_\nu, C_{\nu+1}) = \max\{TLat(C_\nu(\ldots, j_k), C_{\nu+1}(j_{k+1}, \ldots)) \mid j_k, j_{k+1} \in \mathbb{N} \wedge RI\left(\tau_{k+1}^c(j_{k+1})\right) \cap DI\left(\tau_k^c(j_k)\right) \neq \emptyset\}.$$

▶ **Theorem 20** (Maximum end-to-end latency of a cause-effect chain). *Let C be decomposable cause-effect chain, which can can be represented by a concatenation of time-triggered, LET-triggered, and event-triggered cause-effect chains C_i, such that $C = (C_1, C_2, \ldots C_N)$. Then the maximum end-to-end latency of the cause-effect chain C can be bounded from above by*

$$\overline{Lat}(C) \leq \sum_{\nu=1}^{N} \overline{Lat}(C_\nu) + \sum_{\nu=1}^{N-1} \overline{TLat}(C_\nu, C_{\nu+1}).$$

Proof. The end-to-end latency of an instance of a cause-effect chain C can be divided into the end-to-end latencies of its segments $Lat(C_\nu)$ and the transition latency $TLat(C_\nu, C_{\nu+1})$ between the segments. If each of these latency is maximized, we obtain a safe upper of the maximum end-to-end latency of the entire cause-effect chain C. ◀

From Theorems 16 and 17, we already know how to compute the maximum end-to-end latencies of the sub-chains of the decomposable cause-effect chain. In the following theorem, we discuss how to derive the maximum transition latency between successive sub-chains.

	$\mathbf{C_2}$ **is...**		
$\mathbf{C_1}$ **is...**	C^{tt}	C^{let}	C^{et}
C^{tt}	$TLat(C^{tt}, C^{tt})$	$TLat(C^{tt}, C^{let})$	$TLat(C^{tt}, C^{et})$
C^{let}	$TLat(C^{let}, C^{tt})$	$TLat(C^{let}, C^{let})$	$TLat(C^{let}, C^{et})$
C^{et}	$TLat(C^{et}, C^{tt})$	$TLat(C^{et}, C^{let})$	$TLat(C^{et}, C^{et})$

Table 2 Types of transition latencies between successive sub-chains

▶ **Theorem 21** (Constructing the maximum transition latency between successive sub-chains)**.** *The maximum transition latency between two successive sub-chains $C_\nu, C_{\nu+1}$ in a cause-effect chain C is bounded by*

$$\overline{TLat}(C_\nu, C_{\nu+1}) \leq \begin{cases} \overline{\delta}_{\tau_k^c}(2) + WCRT_{\tau_k^c} - BCRT_{\tau_k^c} & \text{if } \tau_k^c \text{ is a BET task} \\ P_{\tau_k^c} & \text{if } \tau_k^c \text{ is a LET task.} \end{cases}$$

where τ_k^c is the last task in C_ν and τ_{k+1}^c is the first task in $C_{\nu+1}$.

Proof. The transition latency between job $\tau_k^c(j_k)$ of sub-chain C_ν and job $\tau_{k+1}^c(j_{k+1})$ of sub-chain $C_{\nu+1}$ starts with the completion of job $\tau_k^c(j_k)$ and ends with the release of job $\tau_{k+1}^c(j_{k+1})$. Since jobs $\tau_k^c(j_k)$, $\tau_{k+1}^c(j_{k+1})$ are successive jobs in an instance of the decomposable cause-effect chain C, we know that $RI\left(\tau_{k+1}^c(j_{k+1})\right) \cap DI\left(\tau_k^c(j_k)\right) \neq \emptyset$. Consequently, the time from the completion of job $\tau_k^c(j_k)$ and the release of job $\tau_{k+1}^c(j_{k+1})$ cannot not be longer than the data interval of job $\tau_k^c(j_k)$, which implies that

$$\forall j_k, j_{k+1} \in \mathbb{N} : RI\left(\tau_{k+1}^c(j_{k+1})\right) \cap DI\left(\tau_k^c(j_k)\right) \neq \emptyset :$$

$$\begin{aligned} TLat(C_\nu, C_{\nu+1}) &\leq \overline{D}\left(\tau_i^c(j)\right) - \underline{D}\left(\tau_i^c(j)\right) \\ &\leq \begin{cases} \overline{\delta}_{\tau_k^c}(2) + WCRT_{\tau_k^c} - BCRT_{\tau_k^c} & \text{if } \tau_k^c \text{ is a BET task} \\ P_{\tau_k^c} & \text{if } \tau_k^c \text{ is a LET task.} \end{cases} \end{aligned}$$

◀

Since a decomposable cause-effect chain C consists only of time-triggered, LET-triggered or event-triggered cause-effect chains, there are nine different types of transition latencies as listed in Table 2. We can profit from this knowledge and refine the results from Theorem 21.

▶ **Corollary 22.**

$$TLat(C^{tt}, C^{tt}) = P_{\tau_k^c} + WCRT_{\tau_k^c} - BCRT_{\tau_k^c}$$

▶ **Corollary 23.**

$$TLat(C^{tt}, C^{let}) = P_{\tau_k^c} + WCRT_{\tau_k^c} - BCRT_{\tau_k^c}$$

▶ **Corollary 24.**

$$TLat(C^{tt}, C^{et}) = \begin{cases} 0 & \text{if Job } \tau_{k+1}^c(j_{k+1}) \text{ is activated by} \\ & \text{the completion event of job } \tau_k^c(j_k) \\ P_{\tau_k^c} + WCRT_{\tau_k^c} - BCRT_{\tau_k^c} & \text{else} \end{cases}$$

▶ **Corollary 25.**

$$TLat(C^{let}, C^{tt}) = P_{\tau_k^c}$$

▶ **Corollary 26.**

$$TLat(C^{let}, C^{let}) = P_{\tau_k^c}$$

▶ **Corollary 27.**

$$TLat(C^{let}, C^{et}) = \begin{cases} 0 & \text{if job } \tau_{k+1}^c(j_{k+1}) \text{ is activated by the completion event of job } \tau_k^c(j_k) \\ P_{\tau_k^c} & \text{else} \end{cases}$$

▶ **Corollary 28.**

$$TLat(C^{et}, C^{tt}) = \overline{\delta}_{\tau_k^c}(2) + WCRT_{\tau_k^c} - BCRT_{\tau_k^c}$$

▶ **Corollary 29.**

$$TLat(C^{et}, C^{let}) = \overline{\delta}_{\tau_k^c}(2) + WCRT_{\tau_k^c} - BCRT_{\tau_k^c}$$

▶ **Corollary 30.**

$$TLat(C^{et}, C^{et}) = \begin{cases} 0 & \text{if job } \tau_{k+1}^c(j_{k+1}) \text{ is activated by} \\ & \text{the completion event of job } \tau_k^c(j_k) \\ \overline{\delta}_{\tau_k^c}(2) + WCRT_{\tau_k^c} - BCRT_{\tau_k^c} & \text{else} \end{cases}$$

6 Robustness of Cause-Effect Chains

A challenge related to software, which includes cause-effect chains, is to design it in a robust manner, so that the required upper bounds on end-to-end latencies are respected even in the presence of software updates. Such software extensions may introduce entirely new tasks or increase the worst-case execution time (WCET) of existing tasks. Thus the amount of workload to be serviced by the system grows and slows down the execution of cause-effect chains which existed prior to the extension. The timing impact of functional extensions can be seen as a disturbance with respect to a cause-effect chain of interest C. A robust software design should be able to tolerate a limited disturbance: meaning that if the disturbance occurs, the cause-effect chain C can still meet its end-to-end deadline.

6.1 Robustness of Time-Triggered Cause-Effect Chains

Consider an application with a time-triggered cause-effect chain $C^{tt} = (\tau_1^c, \tau_2^c, \ldots, \tau_n^c)$. In the initial software design, each task τ_k^c in C^{tt} has a worst-case response time $WCRT_{\tau_k^c}$ satisfying the task deadline. Moreover, the end-to-end deadline is satisfied in the initial software design. We propose to quantify the impact of a software update as an increase in the worst-case response time of each task τ_k^c in C^{tt} and call it disturbance $\Delta WCRT_{\tau_k^c}$:

$$WCRT'_{\tau_k^c} = WCRT_{\tau_k^c} + \Delta WCRT_{\tau_k^c} \qquad \text{where } \Delta WCRT_{\tau_k^c} \geq 0. \tag{2}$$

To quantify the robustness of the cause-effect chain C^{tt} with regard to disturbances, we introduce the following definitions.

▶ **Definition 31** (Robustness margin for a job in a time-triggered cause-effect chain)**.** *The robustness margin of a job $\tau_k^c(j_k)$ with $k \in \mathbb{N} : 1 \leq k \leq n-1$ in an instance of a time-triggered cause-effect chain $C(j_1, j_2, \ldots, j_n) = (\tau_1^c(j_1), \tau_2^c(j_2), \ldots, \tau_n^c(j_n))$ is*

$$RM^C(\tau_k^c(j_k)) = \underline{R}\left(\tau_{k+1}^c(q_{\tau_k^c(j_k)} + 1)\right) - \overline{D}\left(\tau_k^c(j_k)\right)$$

where

$$q_{\tau_k^c(j_k)} = \max\left\{x \in \mathbb{N} \mid \underline{R}\left(\tau_{k+1}^c(x)\right) \cap \overline{D}\left(\tau_k^c(j_k)\right)\right\}.$$

▶ **Definition 32** (Robustness margin for a task in a time-triggered cause-effect chain)**.** *The robustness margin of a task τ_k^c in a time-triggered cause-effect chain C is*

$$RM^C(\tau_k^c) = \begin{cases} \min\left\{RM^C(\tau_k^c(j_k)) \mid j_k \in \mathcal{J}_{HP,k}\right\} & \text{if } 1 \leq k \leq n-1 \\ d_C^{e2e} - \overline{Lat}(C) & \text{if } k = n \end{cases}$$

where

$$\mathcal{J}_{HP,k} = \{\tau_k^c(j_k) : j_k \in \mathbb{N} \wedge (\exists C(j_1, j_2, \ldots, j_n) \in \mathcal{C}_{HP} : \tau_k^c(j_k) \in C(j_1, j_2, \ldots, j_n))\}.$$

▶ **Theorem 33** (Robustness test)**.** *Let $C = (\tau_1^c, \tau_2^c, \ldots, \tau_n^c)$ be a time-triggered cause-effect chain with the property that each task τ_k^c in C has a worst-case response time $WCRT_{\tau_k^c}$ satisfying the task deadline $d_{\tau_k^c}$. Moreover, the cause-effect chain C also satisfies its end-to-end deadline d_C^{e2e}. Let $\Delta WCRT_{\tau_k^c} \geq 0$ be the increase in the worst-case response time of each task τ_k^c in C that is caused by a software update.*

The time-triggered cause-effect chain C still satisfies its end-to-end deadline d_C^{e2e} under the increase of the worst-case response times of its tasks τ_k^c by $\Delta WCRT_{\tau_k^c}$, if

$$\forall \tau_k^c \in C : \Delta WCRT_{\tau_k^c} < RM^C(\tau_k^c)$$

The time-triggered cause-effect chain C still satisfies its end-to-end deadline d_C^{e2e} under the increase of the worst-case response times of its tasks τ_k^c by $\Delta WCRT_{\tau_k^c}$ and its task deadlines $d_{\tau_k^c}$, if

$$\forall \tau_k^c \in C : \Delta WCRT_{\tau_k^c} < \min\{RM^C(\tau_k^c),\ d_{\tau_k^c} - WCRT_{\tau_k^c}\}.$$

Proof. Consider a time-triggered cause-effect chain $C = (\tau_1^c, \tau_2^c, \ldots, \tau_n^c)$ with the property that each task τ_k^c in C has a worst-case response time $WCRT_{\tau_k^c}$ satisfying the task deadline $d_{\tau_k^c}$. Moreover, the cause-effect chain C also satisfies its end-to-end deadline d_C^{e2e}. The data flow analysis proposed in Theorem 9 allows to construct the set $\mathcal{C}_{HP}$, which contains all possible instances of a cause-effect chain C within a hyper period. The pattern of C-instances is repeated after each hyper period, so that it is sufficient to consider $\mathcal{C}_{HP}$ to derive all possible end-to-end latencies and find the maximum. To compute the maximum end-to-end latency of C^{tt}, we have to evaluate all chain instances in the set $\mathcal{C}_{HP}$ according to Theorem 16.

Disturbances $\Delta WCRT_{\tau_k^c}$ can have an impact on the maximum end-to-latency of C according to Theorem 16, if (1) $\Delta WCRT_{\tau_n^c} > 0$ and / or (2) the set $\mathcal{C}_{HP}$ changes. Let us briefly discuss how the set $\mathcal{C}_{HP}$ is modified in the presence of disturbances $\Delta WCRT_{\tau_k^c}$. Increasing the worst-case response of tasks τ_k^c in C by $\Delta WCRT_{\tau_k^c}$

- does not impact the read intervals of tasks τ_k^c,

- does not impact the lower boundary of the data interval of tasks τ_k^c,
- increases the upper boundary of the data interval of tasks τ_k^c.

Consequently, the set of chain instances in one hyper period may comprise more elements under disturbances ($\mathcal{C}'_{HP}$) than in the absence of disturbances ($\mathcal{C}_{HP}$), such that $\mathcal{C}_{HP} \subseteq \mathcal{C}'_{HP}$. Our intention is now to find out how large the disturbances $\Delta WCRT_{\tau_k^c}$ can be such that C^{tt} still satisfies its end-to-end deadline.

From Theorem 16, we can see that

- if $\Delta WCRT_{\tau_n^c} < d_C^{e2e} - \overline{Lat}(C)$, i.e., the disturbance $\Delta WCRT_{\tau_n^c}$ does not become larger than the slack between the end-to-end deadline and the maximum end-to-end latency $\overline{Lat}(C^{tt})$ before the software update, *and*
- if $\Delta WCRT_{\tau_k^c}$ are such that $\mathcal{C}_{HP} = \mathcal{C}'_{HP}$, i.e., no additional chain instances are created,

then C will still satisfies its end-to-end deadline even in the presence of disturbances $\Delta WCRT_{\tau_k^c}$.

It would be useful to have some upper bound $RM^C(\tau_k^c)$ with the property that if $\forall \tau_k^c : 1 \leq k \leq n : \Delta WCRT_{\tau_k^c} < RM^C(\tau_k^c)$, then the two above conditions are fulfilled. We derive the robustness margins $RM^C(\tau_k^c)$ in the following.

With regard to the first condition, we have $\Delta WCRT_{\tau_n^c} < d_C^{e2e} - \overline{Lat}(C)$ which leads to

$$\Delta WCRT_{\tau_n^c} < d_C^{e2e} - \overline{Lat}(C) = RM^C(\tau_n^c).$$

With regard to the second condition, we need to identify under which conditions no new chain instances are created even if the upper boundary of the data interval $\overline{D}\left(\tau_k^c(j_k)\right)$ is increased by $\Delta WCRT_{\tau_k^c}$ for $1 \leq k \leq n-1$. Note already here that the data interval $\overline{D}\left(\tau_n^c(j_n)\right)$ can no have impact because $\tau_n^c(j_n)$ is the last job in the chain.

To begin with, we explore the situation without disturbances.

Let the set of jobs

$$\mathcal{J}_{HP,k} = \{\tau_k^c(j_k) : j_k \in \mathbb{N} \wedge \exists C(j_1, j_2, \ldots, j_n) \in \mathcal{C}_{HP} : \tau_k^c(j_k) \in C(j_1, j_2, \ldots, j_n)\}$$

corresponds to all jobs of task τ_k^c that are part of the chain instances in $\mathcal{C}_{HP}$.

Let job $\tau_{k+1}^c(q_{\tau_k^c(j_k)})$ be the last job of task τ_{k+1}^c that reads data from job $\tau_k^c(j_k) \in \mathcal{J}_{HP,k}$. As job $\tau_{k+1}^c(q_{\tau_k^c(j_k)} + 1)$ (or any later job of task τ_{k+1}^c) does not read from $\tau_k^c(j_k)$, we have

$$\forall j_k \in \mathcal{J}_{HP,k} : \overline{D}\left(\tau_k^c(j_k)\right) < \underline{R}\left(\tau_{k+1}^c(q_{\tau_k^c(j_k)} + 1)\right)$$

which is equivalent to

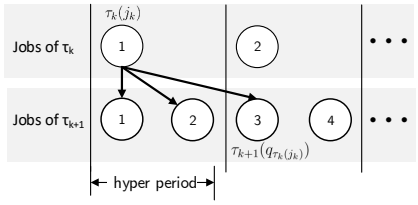
$$\forall j_k \in \mathcal{J}_{HP,k} : j_k \cdot P_{\tau_k^c} + \Phi_{\tau_k^c} + WCRT_{\tau_k^c} < ((q_{\tau_k^c(j_k)} + 1) - 1) \cdot P_{\tau_{k+1}^c} + \Phi_{\tau_{k+1}^c}.$$

In the presence of disturbances, the non-overlap of the data interval and read interval is still required if no new chain instances are to be created, such that

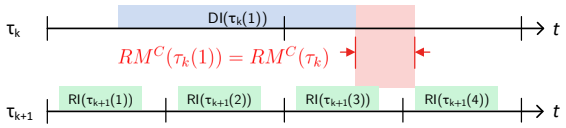
$$\forall j_k \in \mathcal{J}_{HP,k} : j_k \cdot P_{\tau_k^c} + \Phi_{\tau_k^c} + (WCRT_{\tau_k^c} + \Delta WCRT_{\tau_k^c}) < ((q_{\tau_k^c(j_k)} + 1) - 1) \cdot P_{\tau_{k+1}^c} + \Phi_{\tau_{k+1}^c}$$

which can be transformed to

$$\begin{aligned}
\Delta WCRT_{\tau_k^c} &< \min_{j_k \in \mathcal{J}_{HP,k}} \left\{ q_{\tau_k^c(j_k)} \cdot P_{\tau_{k+1}^c} + \Phi_{\tau_{k+1}^c} - j_k \cdot P_{\tau_k^c} - \Phi_{\tau_k^c} - WCRT_{\tau_k^c} \right\} \\
&= \min_{j_k \in \mathcal{J}_{HP,k}} \left\{ \underline{R}\left(\tau_{k+1}^c(q_{\tau_k^c(j_k)} + 1)\right) - \overline{D}\left(\tau_k^c(j_k)\right) \right\} \\
&= \min_{j_k \in \mathcal{J}_{HP,k}} \left\{ RM^C(\tau_k^c(j_k)) \right\} = RM^C(\tau_k^c)
\end{aligned}$$



(a) The set $\mathcal{C}_{HP}$ represented by a reachability graph. Here, we have $\mathcal{J}_{HP,k} = \{\tau_k^c(1)\}$ and $q_{\tau_k^c(j_k)} = 3$.



(b) Data intervals and read intervals for the jobs in the set $\mathcal{C}_{HP}$. Illustration of the robustness margin $RM^C(\tau_k^c(1))$, which is equal to $RM^C(\tau_k^c)$ in the example.

Figure 4 Example how to compute the robustness margin for a task in a time-triggered cause-effect chain C.

We require therefore that

$$\forall \tau_k^c \in C : \Delta WCRT_{\tau_k^c} < RM^C(\tau_k^c).$$

if C is supposed to satisfy its end-to-end deadline under disturbances $\Delta WCRT_{\tau_k^c}$. ◀

Note that even if the disturbances $\Delta WCRT_{\tau_k^c}$ that will be induced by a planned software update are not known exactly beforehand, the robustness margins allow to predict which tasks are critical in preserving the timeliness of the cause-effect chain.

6.2 Dependent Time-Triggered Cause-Effect Chains

Assume there are several cause-effect chains in an application which have dependencies due to common tasks (cf. Figure 2a where cause-effect chains C, D contain both task τ_3), then a robustness margin for a common task τ_i can be defined as follows.

▶ **Definition 34.** *The robustness margin of a task τ_i in an application with only time-triggered cause-effect chains is*

$$RM(\tau_i) = \min\{RM^C(\tau_k^c) : \tau_k^c = \tau_i \wedge \tau_k^c \in C \in \mathcal{A}\}.$$

Clearly, the end-to-end latencies of all time-triggered cause-effect chains in an application are respected under disturbances $\Delta WCRT_{\tau_k^c}$ if

$$\forall C : \forall \tau_k^c : 1 \leq k \leq n : \ \Delta WCRT_{\tau_k^c} < RM(\tau_k^c).$$

6.3 Robustness of LET-Triggered Cause-Effect Chains

LET-triggered cause-effect chains have an inherent robustness because any change in the actual response time of a task τ_k^c is hidden by the logical execution time $LET_{\tau_k^c}$ under the condition that the response time does not exceed the logical execution time. If, however, this condition is violated, then it is an option to increase the logical execution time. Thus the problem is whether it is possible to increase the logical execution times $LET_{\tau_k^c}$ of a given software design by $\Delta LET_{\tau_k^c}$

$$LET'_{\tau_k^c} = LET_{\tau_k^c} + \Delta LET_{\tau_k^c} \qquad \text{where } \Delta LET_{\tau_k^c} \geq 0. \tag{3}$$

while still satisfying the end-to-end deadline. It possible to re-use the above presented robustness analysis, but a small modification must be made: In contrast to the case of time-triggered cause-effect chain, adding $\Delta LET_{\tau_k^c}$ to $LET_{\tau_k^c}$ increases only the upper boundary

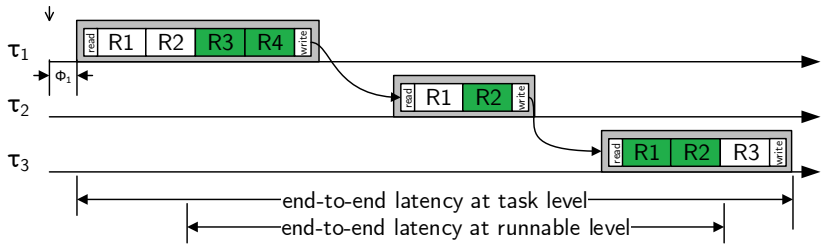


Figure 5 Cause-effect chain at runnable level

but also the lower boundary of the data interval of task τ_k^c. A possible solution is to set $\underline{D}\left(\tau_k^c(j_k)\right) := (j_k - 1) \cdot P_{\tau_k^c} + \Phi_i$ which is also a safe lower but less tight boundary of the data interval of job $\tau_i^c(j)$. This will lead to more paths in the data flow analysis (larger super set of possible paths), but this does not impact the accuracy of the computed maximum end-to-end latency according to Lemma 3 in [2].

6.4 More Detailed Abstraction Level

In automotive software architecture, a hierarchical task model is common where a task is composed of runnables. If the abstraction level of runnables is applied in the system model, an instance of a cause-effect chain is not a sequence of jobs but a sequence of runnable-instances as shown in Figure 5. A possible definition of an end-to-end latency at the runnable level is as follows.

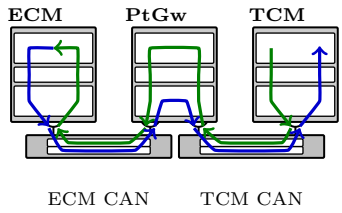
▶ **Definition 35** (End-to-end latency at runnable level)**.** *The end-to-end latency of an instance of a cause-effect chain modeled at runnable level is the maximum amount of time that may elapse from the release of the first runnable to the completion of the last runnable in the chain instance.*

With regard to the end-to-end latency, the precision gained at the runnable level compared to the task level is not considerable and decreases with the number of tasks in the cause-effect chain. The difference in precision is even smaller, if another definition of an end-to-end latency at the runnable level is chosen which includes, for instance, the reading phase of the first task and/or the writing phase last task in the chain.

7 Experiments

7.1 Case Study

Collaboration with Max-Jonas Friese. To be continued.



To test our approach we used a case-study with a set-up commonly found in automotive powertrains. It features three ECUs connected via two CAN-FD networks as depicted in Figure **??**. Furthermore, it contains realistic data for one cause-effect chain spanning over three ECUs. The CPC is the central powertrain controller which acts as a proxy for the powertrain domain but also manages the inner-domain communication between the transmission control module (TCM) and the engine control module (ECM). The case-study comprises data for communication artifacts on the CANs, as well as for the software deployed on the ECUs. In this, it contains a challenge commonly found in industry: incomplete data. During the different stages of system engineering implementation details are often not available, e.g. because a supplier is responsible for parts of an ECUs software. For the analysis of end-to-end latencies, the temporal behavior of these parts must be approximated with the help of suitable models.

The cause-effect chain we analyze is part of the torque request and deliver functionality of the powertrain. More precisely, it the chain which is set off when the transmission controller needs to reduce torque, e.g. at a gear change. For this, a signal is generated in the TCM and send in a frame on the TCM CAN to the CPC. The CPC checks the requests before forwarding it the ECM. The ECM then tries to reduce torque to the extend possible and answers to the CPC with the torque reduction actually provided. The answer of the ECM is directly routed from the ECM CAN to the TCM CAN for fast reception at the TCM. The cuase-effect chain ends with the occurrence of the updated value for the provided torque in the memory of the TCM.

8 Conclusion

Acknowledgment

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