Record statistics and inter-record time distribution

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Record 의 정의와 Record Statistics의 주요 변수들

- Time series data : $\{x_0, x_1, \dots, x_N\}$
- Definition of record event : $x_n > \max\{x_0, x_1, \dots, x_{n-1}\}$

- Persistence (Survival probability which a walker survives **below** the origin at *n*-th step)

  : $q_-(n) = Proba[(x_1 < x_0) \wedge (x_2 < x_0) \wedge \cdots \wedge (x_n < x_0)]$

- Binary record indicator (when a new record value is the maximum value)

  : $\sigma(n) = \begin{cases} 1, & if\ the\ record\ event\ occurs\ at\ n-th\ step \\ 0, & otherwise \end{cases}$

- Record number : $M(N) = \sum_{n=1}^{N} \sigma(n)$
- Average record number : $\langle M(N) \rangle = \sum_{n=1}^{N} \langle \sigma(n) \rangle$

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Theoretical background of record statistics

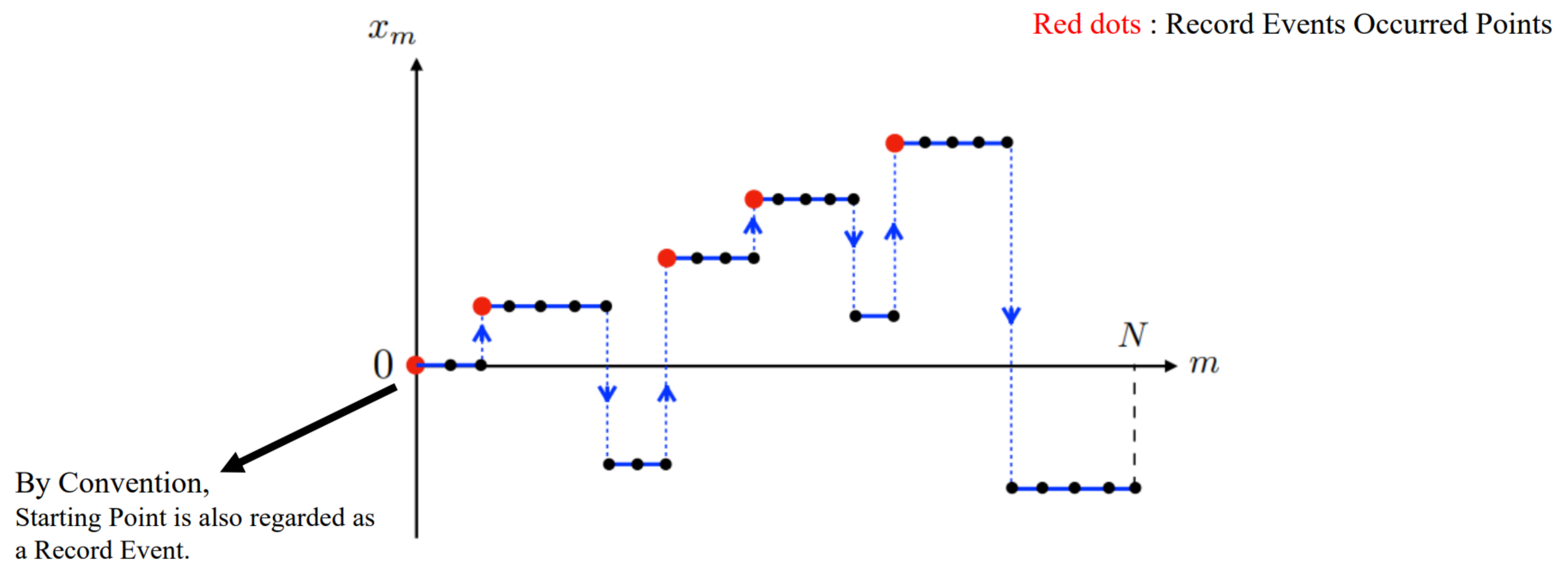

FIG 1.

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Record statistics의 선행 연구들

- Satya N. Majumdar 주도 하에 2000년도 부터 record statistics에 대한 연구 진행

- 대표적인 연구결과들
  - Universal Record Statistics of Random Walks and Lévy Flights [Phys. Rev. Lett. **101**, 050601 (2008)]
  - Record statistics for biased random walks, with an application to financial data [Phys. Rev. E **83**, 051109 (2011)]
  - Record statistics and persistence for a random walk with a drift [J. Phys. A: Math. Theor. **45**, 355002 (2012)]
  - Exact record and order statistics of random walks via first-passage ideas [World Scientific Review Volume, Chapter 1 (2013)]
  - Record statistics of a strongly correlated time series: random walks and Lévy Flights [J. Phys. A: Math. Theor. **50**, 333001 (2017)]
  - Exactly Solvable Record Model for Rainfall [Phys. Rev. Lett. **122**, 158702 (2019)]
  - Statistics of the number of records for random walks and Lévy Flights on a 1D lattice [J. Phys. A: Math. Theor. **53**, 415003 (2020)]
  - Universal record statistics for random walks and Lévy Flights with a nonzero staying probability [J. Phys. A: Math. Theor. **54**, 315002 (2021)]

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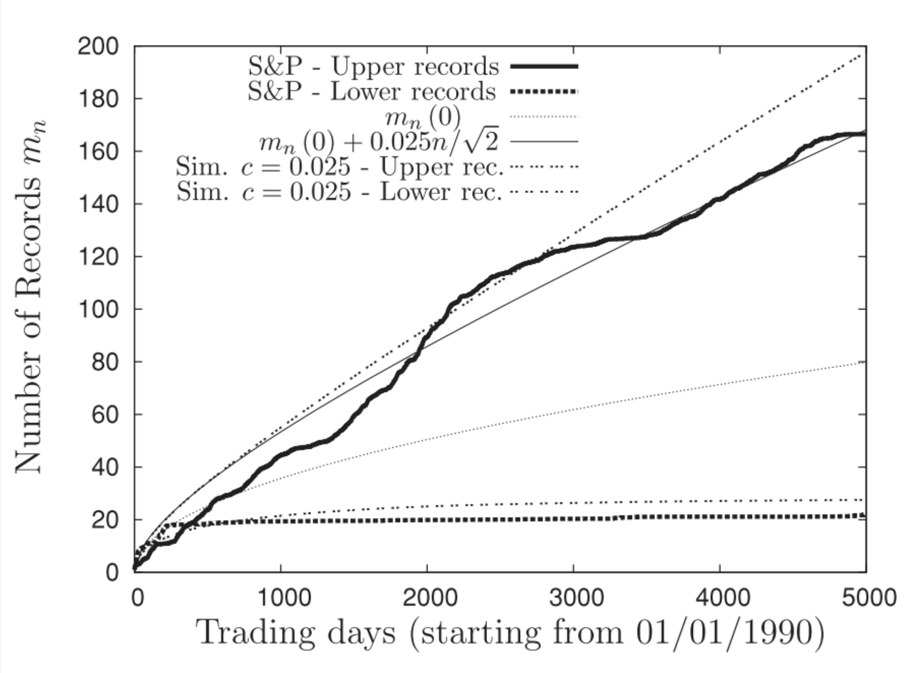
Record statistics의 주요 선행 연구 #1

- Record statistics for biased random walks, with an application to financial data [Phys. Rev. E **83**, 051109 (2011)]

- Time-series model : $x_n = x_{n-1} + \eta_n + c$,

  η (jump length) : random variables from the gaussian distribution $f(\eta)$

  $c = (constant)$



- $m_n(c)$: analytic function of record number

  $m_n(c) = \frac{2}{\sqrt{\pi}}\sqrt{n} + \frac{c}{\sigma}\frac{\sqrt{2}}{\pi}(n\arctan(\sqrt{n}) - \sqrt{n})$

- S&P500 index에 포함된 366개 주식의 가격 데이터 사용

  (1990.01.01 ~ 2009.03.31, $n = 5000$)

- $c = 0.025$ 는 개별 주식의 log price 에서 linear regression analysis 를 통해 얻어 냄.

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Record statistics의 주요 선행 연구 #2

- Record statistics and persistence for a random walk with a drift [J. Phys. A: Math. Theor. **45**, 355002 (2012)]

- Time-series model : $x_n = x_{n-1} + \eta_n + c,\quad c = (constant),\quad \eta$: random variables from the distribution $f(\eta)$
  - $f(\eta)$: symmetric Lévy α-stable distribution
  - $\hat{f}(k) = \int_{-\infty}^{\infty} f(\eta) e^{ik\eta} d\eta = \exp(-|l_\alpha k|^\alpha)$

- $\theta(c) = \frac{1}{2} + \frac{1}{\pi}\arctan(c)$
- $k_\alpha(c) = \exp\left[-\sum_{n=1}^{\infty} \frac{1}{n} \int_{cn}^{\infty} P_n(x)\, dx\right]$

	1st Regime $\alpha \in (0,1)$ $c \in \mathbb{R}$	2nd Regime $\alpha = 1$ $c \in \mathbb{R}$	3rd Regime $\alpha \in (1,2)$ $c \in \mathbb{R}^+$	4th Regime $\alpha = 2$ $c \in \mathbb{R}^+$	5th Regime $\alpha \in (1,2]$ $c \in \mathbb{R}^-$
$q_-(N)$	$\propto N^{-\frac{1}{2}}$	$\propto N^{-\theta(c)}$	$\propto N^{-\alpha}$	$\propto N^{-\frac{3}{2}} e^{-\frac{c^2 N}{2\sigma^2}}$	$\sim k_\alpha(-c)$
$\langle M(N) \rangle$	$\propto N^{\frac{1}{2}}$	$\propto N^{\theta(c)}$	$\sim k_\alpha(c) N$	$\sim k_2(c) N$	$\sim [k_\alpha(-c)]^{-1}$

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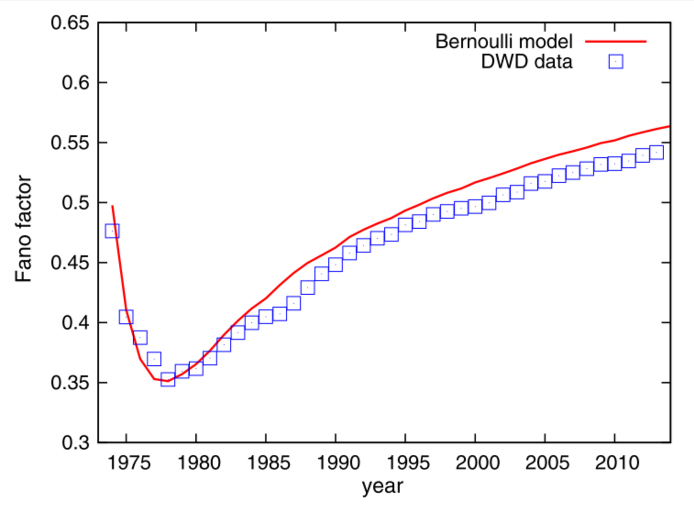
Record statistics의 주요 선행 연구들 #3

- Exactly Solvable Record Model for Rainfall [Phys. Rev. Lett. **122**, 158702 (2019)]

- Time-series model : x_n from $f(x) = p_0\delta(x) + (1-p_0)f_W(x)$. *Bernoulli model*
  - p_0 : probability with zero precipitation events occurring
  - $f_W(x)$: continuous probability density

- $\langle M(N)\rangle = \sum_{n=1}^{N}\frac{1-q^n}{n}$.
- $\langle M(N)\rangle \to \mu(t)$ as $N\to\infty$, $\quad t=(1-p_0)N$.
- $\mu(t) = \ln t + \gamma_E + \int_t^\infty \frac{e^{-z}}{z}dz$.
- $F(t) = 1 + \frac{2}{\mu(t)}\int_0^t \frac{dz}{z}e^{-z}[\mu(t)-\mu(z)-\mu(t-z)]$.
- *Bernoulli model*'s simulation : $1-p_0 = 0.5$ (the average rainfall probability)
- Daily rainfall amounts at 144 German weather stations (1961 ~ 2013)



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기존 연구와의 차별점과 보완점

- 기존 연구에서는 average record number을 중점적으로 연구 진행
- 본 연구에서는 average record number와 그 외 first-passage probability를 중점적으로 연구
- First-passage probability가 inter-record time distribution과 같다는 것으로부터 first-passage probability를 시계열 데이터로부터 직접 측정할 수 있게 됨
- Inter-record time distribution을 이용하면 적은 샘플의 시계열 데이터로부터 기존과 대비해서 더욱 정확한 record statistics를 알아낼 수 있음

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First-passage probability (theoretic part)

- First-passage probability : $f_{-}(n) = Proba[(x_1 < x_0) \wedge (x_2 < x_0) \wedge \cdots \wedge (x_n > x_0)]$

- Relation between survival probability and first-passage probability
  - $f_{-}(n) = -\frac{\partial}{\partial n} q_{-}(n) = -\left(q_{-}(n) - q_{-}(n-1)\right)$ (for discrete n)

- τ : inter-record time (age of record)
- $P(\tau)$: distribution of inter-record times

- Record가 $n = 0$ 과 $n = \tau$ 에서 발생한다고 보면, $P(\tau) = f_{-}(\tau)$.

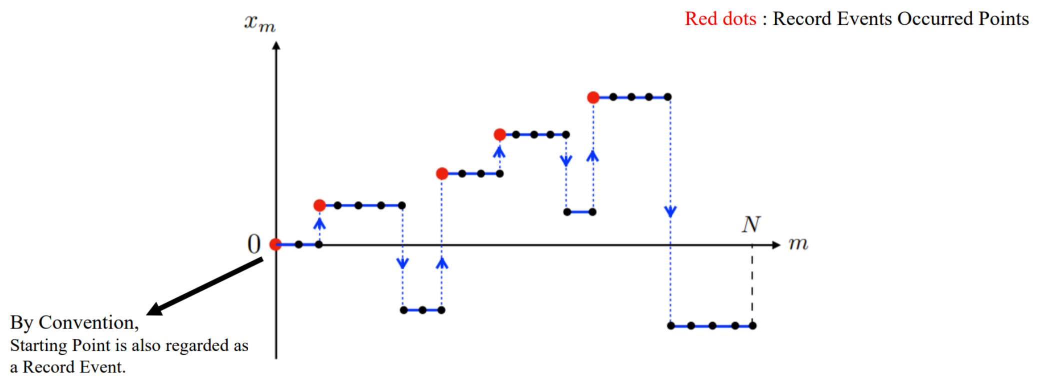

FIG 1.

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Study method (theoretic part)

- $x_n = x_{n-1} + \eta_n + c, \quad \eta$: 대칭적인 Lévy α-stable 분포 $f(\eta)$ 를 따르는 jump length
  - $\hat{f}(k) = \exp(-|l_\alpha k|^\alpha)$: characteristic function of $f(\eta)$
  - $f(\eta) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk\, \hat{f}(k) e^{-ik\eta}$
  - $P_n(x) = \int_{-\infty}^{\infty} \frac{dk}{2\pi} \left[\hat{f}(k)\right]^n e^{-ikx}$: n 번째 step에서 walker의 position distribution
  - $P_n(x) \to \frac{1}{l_\alpha n^{\frac{1}{\alpha}}} \mathcal{L}_\alpha\left(\frac{x}{l_\alpha n^{\frac{1}{\alpha}}}\right)$, for small k.
  - $\mathcal{L}_\alpha(y) \to \frac{A_\alpha}{|y|^{\alpha+1}}$ for $|y| \to \infty$. $\quad A_\alpha = \frac{1}{\pi}\sin\left(\frac{\alpha\pi}{2}\right)\Gamma(1+\alpha)$

- $\tilde{q}_-(z)$ 를 Sparre-Andersen theorem (generating function's relation)을 이용하여 구함
  - $\tilde{q}_-(z) = \sum_{n=0}^{\infty} q_-(n) z^n = \exp\left(\sum_{n=1}^{\infty} \frac{z^n}{n} \int_0^{\infty} P_n(x)dx\right)$

- $\tilde{q}_-(z)$ 로부터 다음과 같은 알려진 관계식에 의해 $\tilde{f}_-(z)$ 와 $\widetilde{M}(z)$ 구함
  - $\tilde{f}_-(z) = 1 - (1-z)\tilde{q}_-(z)$
  - $\widetilde{M}(z) = \frac{1}{(1-z)^2 \tilde{q}_-(z)}$

- Generating function 혹은 Inverse laplace transform을 이용하여 $f_-(n)$ 와 $\langle M(N)\rangle$구함

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Survival probability (symmetric Lévy walk)

- $x_n = x_{n-1} + \eta_n, \quad \eta$: symmetric Lévy α-stable 분포 $f(\eta)$ 를 따르는 jump length
  - $f(\eta) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk\, \hat{f}(k) e^{-ik\eta}, \quad \hat{f}(k) = \exp(-|l_\alpha k|^\alpha)$
  - $P_n(x)$: n 번째 step에서 walker의 position distriubtion

- Sparre-Andersen theorem
  - $\tilde{q}_-(z) = \sum_{n=0}^{\infty} q_-(n) z^n = \exp\left(\sum_{n=1}^{\infty} \frac{z^n}{n} \int_0^{\infty} P_n(x) dx\right)$
  - for symmetric Lévy walk, $\int_0^{\infty} P_n(x) dx = \frac{1}{2}$
  - $\tilde{q}_-(z) = \exp\left(\sum_{n=1}^{\infty} \frac{z^n}{2n}\right) = \exp\left(-\frac{1}{2}\ln(1-z)\right) = \frac{1}{(1-z)^{0.5}}$

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Survival probability (symmetric Lévy walk)

- Sparre-Andersen theorem
  - $\tilde{q}_-(z) = \sum_{n=0}^{\infty} q_-(n) z^n = \exp\left(\sum_{n=1}^{\infty} \frac{z^n}{n} \int_0^\infty P_n(x)dx\right)$
  - for symmetric Lévy walk, $\int_0^\infty P_n(x)dx = \frac{1}{2}$
  - $\tilde{q}_-(z) = \exp\left(\sum_{n=1}^{\infty} \frac{z^n}{2n}\right) = \exp\left(-\frac{1}{2}\ln(1-z)\right) = \frac{1}{(1-z)^{0.5}}$

- Generating function table by Robert M. Ziff
  - when $\tilde{f}(z) = \sum_{n=0}^{\infty} f(n) z^n = \frac{1}{(1-z)^s}$, $\quad f(n) = \binom{n+s-1}{n}$
  - $q_-(n) = \binom{n-\frac{1}{2}}{n} \to \frac{1}{\sqrt{\pi n}}$ for $n \to \infty$. (Stirling approximation)

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Average record number (symmetric Lévy walk)

- Joint probability of inter-record time τ and record number M in $\{x_0, x_1, \dots, x_N\}$.
  - $P(\tau, M|N) = f_-(\tau_1)f_-(\tau_2)\dots f_-(\tau_{M-1})q_-(\tau_M)\delta_{\sum_{i=1}^{M}\tau_i, N}$
  - $P(M|N) = \sum_{\tau_1=0}^{\infty}\sum_{\tau_2=0}^{\infty}\dots\sum_{\tau_M=0}^{\infty}(\delta_{\sum_{i=1}^{M}\tau_i, N} f_-(\tau_1)f_-(\tau_2)\dots f_-(\tau_{M-1})q_-(\tau_M))$
  - $\tilde{P}(Z) = \sum_{N=0}^{\infty} z^N P(M|N)$

  $= \sum_{N=0}^{\infty} z^N \left(\sum_{\tau_1=0}^{\infty}\sum_{\tau_2=0}^{\infty}\dots\sum_{\tau_M=0}^{\infty}(\delta_{\sum_{i=1}^{M}\tau_i, N} f_-(\tau_1)f_-(\tau_2)\dots f_-(\tau_{M-1})q_-(\tau_M))\right)$

  $= \sum_{\tau_1=0}^{\infty} z^{\tau_1} f_-(\tau_1)\sum_{\tau_2=0}^{\infty}\dots\sum_{\tau_M=0}^{\infty} z^{\tau_M} q_-(\tau_M)$

  $= \left[\tilde{f}_-(z)\right]^{M-1}\tilde{q}_-(z)$
- Average record number
  - $\widetilde{M}(z) = \sum_{N=0}^{\infty}\langle M(N)\rangle z^N = \sum_{N=0}^{\infty} z^N \sum_{M=N-1}^{\infty} MP(M|N)$ and $\tilde{f}_-(z) = 1 - (1-z)\tilde{q}_-(z)$
  - $\widetilde{M}(z) = \frac{1}{(1-z)^2\tilde{q}_-(z)} = \frac{1}{(1-z)^{1.5}}$
  - $\langle M(N)\rangle = \binom{n+\frac{1}{2}}{n} \to \sqrt{\pi n}$ for $n \to \infty$. (Stirling approximation)

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Lévy walk with a constant bias

- $x_n = x_{n-1} + \eta_n + c,\quad \eta$: symmetric Lévy α-stable 분포 $f(\eta)$ 를 따르는 jump length

	1st Regime $\boldsymbol{\alpha \in (0,1)}$ $\mathbf{c} \in \mathbb{R}$	2nd Regime $\boldsymbol{\alpha = 1}$ $\mathbf{c} \in \mathbb{R}$	3rd Regime $\boldsymbol{\alpha \in (1,2)}$ $\mathbf{c} \in \mathbb{R}^+$	4th Regime $\boldsymbol{\alpha = 2}$ $\mathbf{c} \in \mathbb{R}^+$	5th Regime $\boldsymbol{\alpha \in (1,2]}$ $\mathbf{c} \in \mathbb{R}^-$
$q_-(N)$	$\propto N^{-\frac{1}{2}}$	$\propto N^{-\theta(c)}$	$\propto N^{-\boldsymbol{\alpha}}$	$\propto N^{-\frac{3}{2}} e^{-\frac{c^2 N}{2\sigma^2}}$	$\sim k_{\boldsymbol{\alpha}}(-c)$
$P(\tau)$	$\propto \tau^{-1.5}$	$\propto \tau^{-\theta(c)-1}$	$\propto \tau^{-\boldsymbol{\alpha}-\mathbf{1}}$	$\propto e^{-\frac{c^2\tau}{2\sigma^2}}\left(\frac{c^2}{2\sigma^2}\tau^{-1.5} + 1.5\tau^{-2.5}\right)$	~ 0
$\langle M(N) \rangle$	$\propto N^{\frac{1}{2}}$	$\propto N^{\theta(c)}$	$\sim k_{\boldsymbol{\alpha}}(c)N$	$\sim k_2(c)N$	$\sim [k_{\boldsymbol{\alpha}}(-c)]^{-1}$

- $\theta(c) = \frac{1}{2} + \frac{1}{\pi}\arctan(c)$
- $k_\alpha(c) = \exp\left[-\sum_{n=1}^{\infty} \frac{1}{n} \int_{cn}^{\infty} P_n(x)\, dx\right]$

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Drift regimes' description

	1st Regime $\alpha \in (0,1)$ $c \in \mathbb{R}$	2nd Regime $\alpha = 1$ $c \in \mathbb{R}$	3rd Regime $\alpha \in (1,2)$ $c \in \mathbb{R}^+$	4th Regime $\alpha = 2$ $c \in \mathbb{R}^+$	5th Regime $\alpha \in (1,2]$ $c \in \mathbb{R}^-$
$q_-(N)$	$\propto N^{-\frac{1}{2}}$	$\propto N^{-\theta(c)}$	$\propto N^{-\alpha}$	$\propto N^{-\frac{3}{2}} e^{-\frac{c^2 N}{2\sigma^2}}$	$\sim k_\alpha(-c)$
$P(\tau)$	$\propto \tau^{-1.5}$	$\propto \tau^{-\theta(c)-1}$	$\propto \tau^{-\alpha-1}$	$\propto e^{-\frac{c^2\tau}{2\sigma^2}} \left(\frac{c^2}{2\sigma^2} \tau^{-1.5} + 1.5\tau^{-2.5} \right)$	~ 0
$\langle M(N) \rangle$	$\propto N^{\frac{1}{2}}$	$\propto N^{\theta(c)}$	$\sim k_\alpha(c) N$	$\sim k_2(c) N$	$\sim [k_\alpha(-c)]^{-1}$

- Regime 1 : α (jump length 분포 파라미터)와 c (constant bias)의 영향이 거의 없는 record statistics
- Regime 2 : c (constant bias) dominant record statistics
- Regime 3 : α (jump length 분포 파라미터) dominant record statistics
- Regime 4 : α (jump length 분포 파라미터)& c (constant bias) dominant record statistics
- Regime 5 : α (jump length 분포 파라미터)와 c (constant bias) 의 영향이 거의 없는 record statistics

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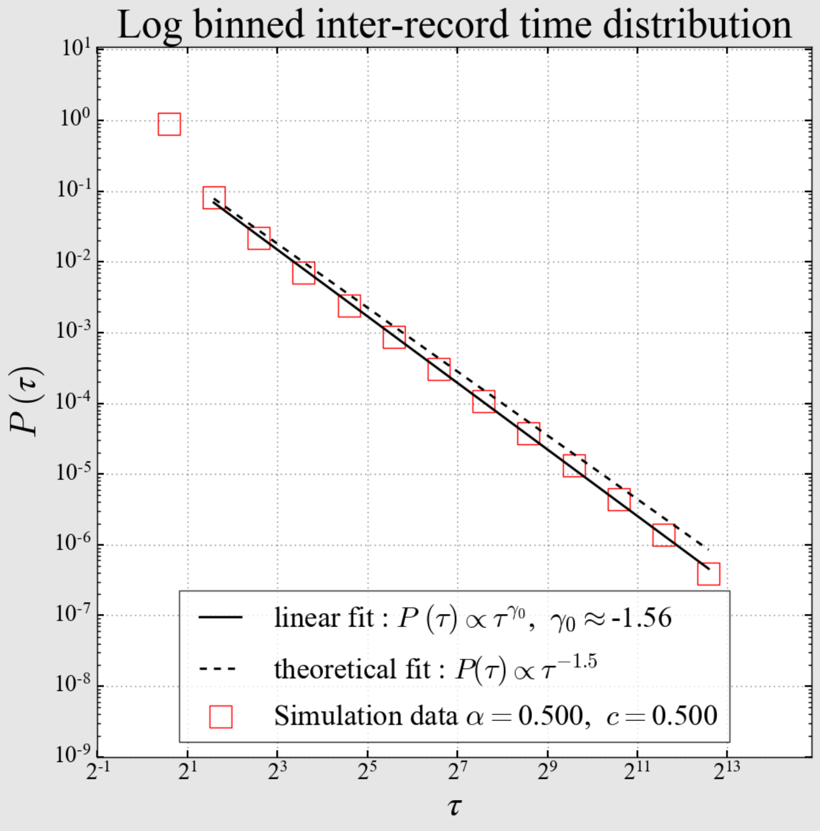
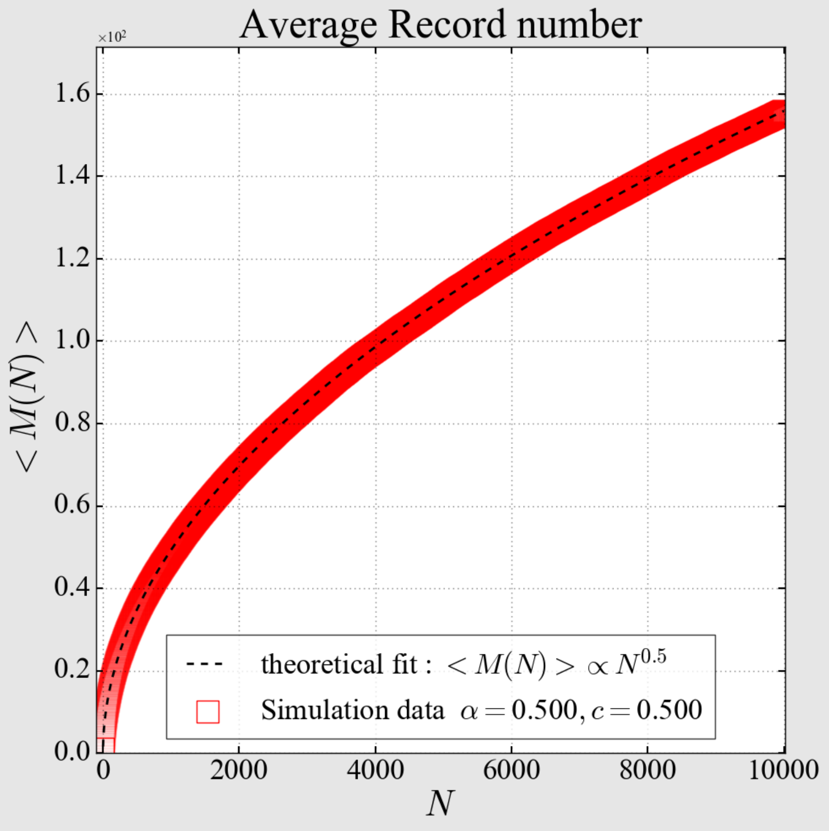
연구 결과 (Simulation : Regime 1)





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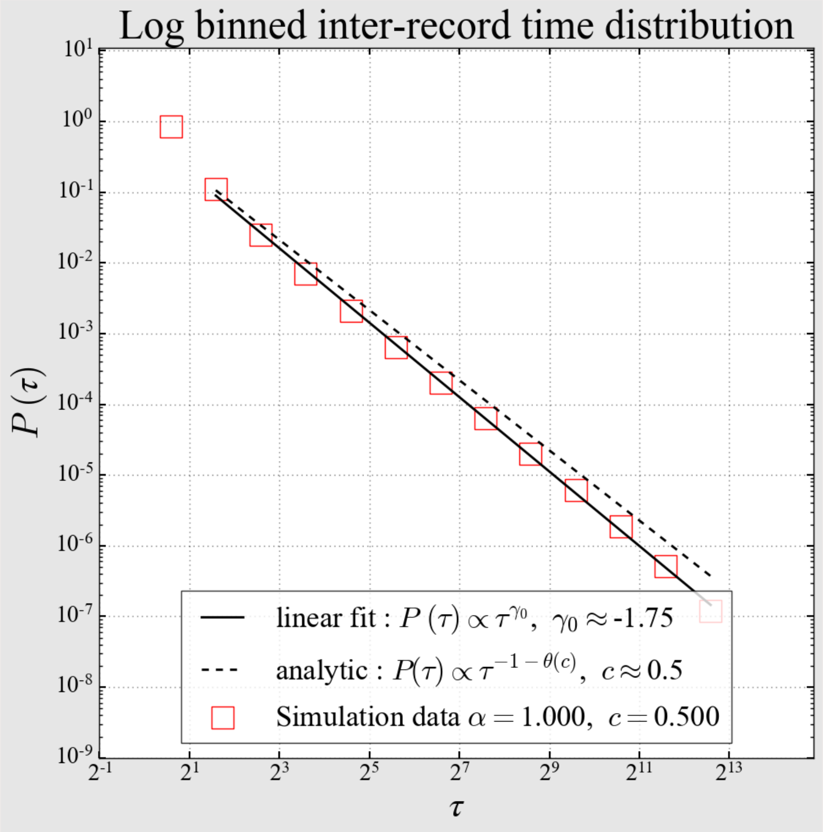
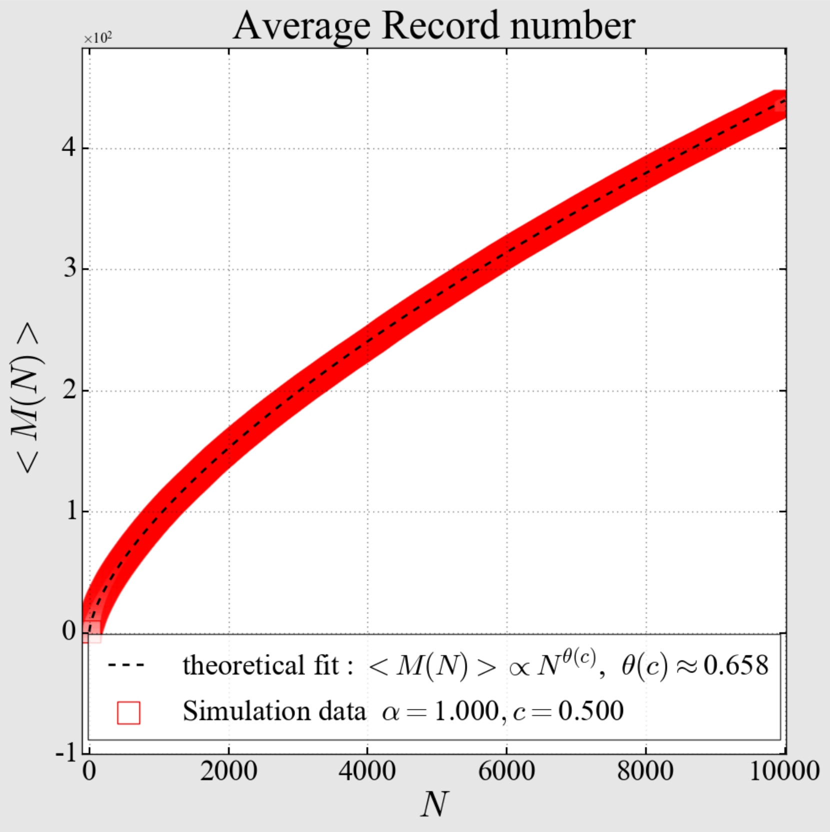
연구 결과 (Simulation : Regime 2)





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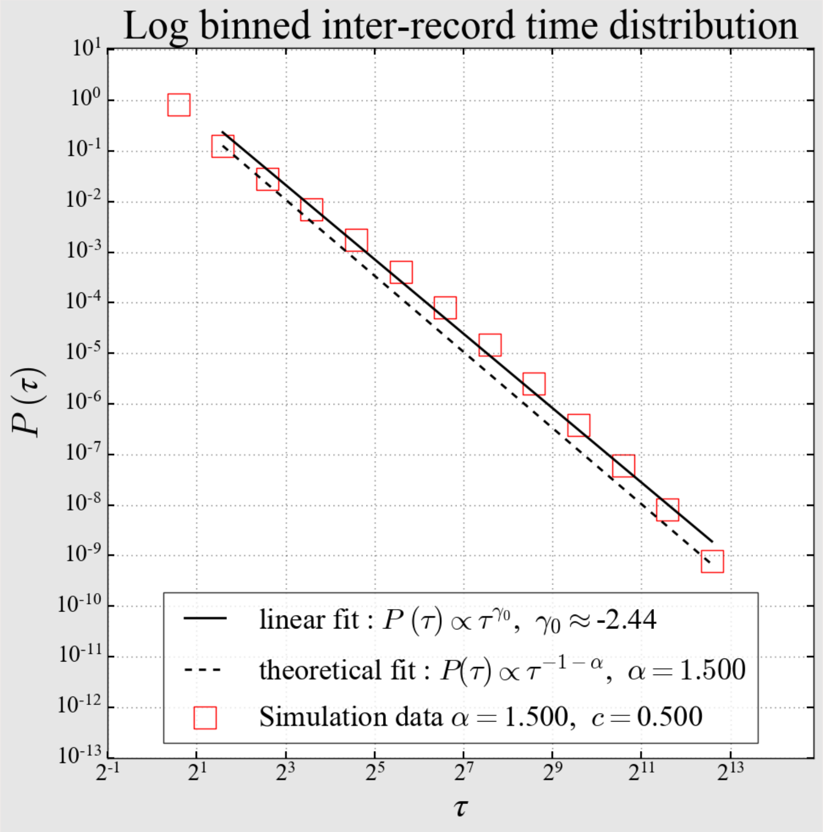
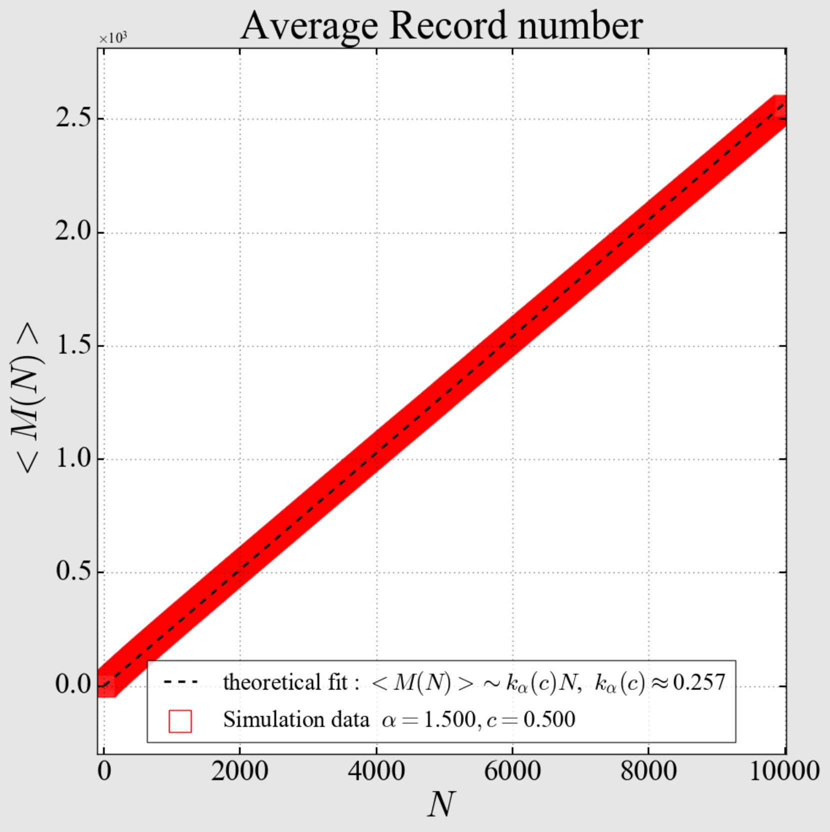
연구 결과 (Simulation : Regime 3)





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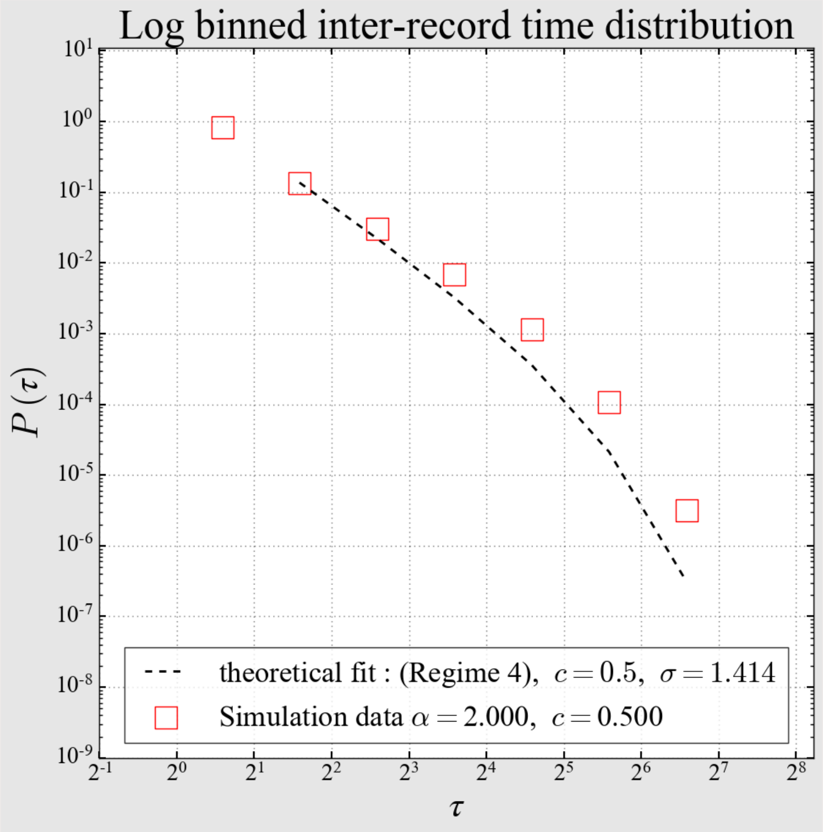
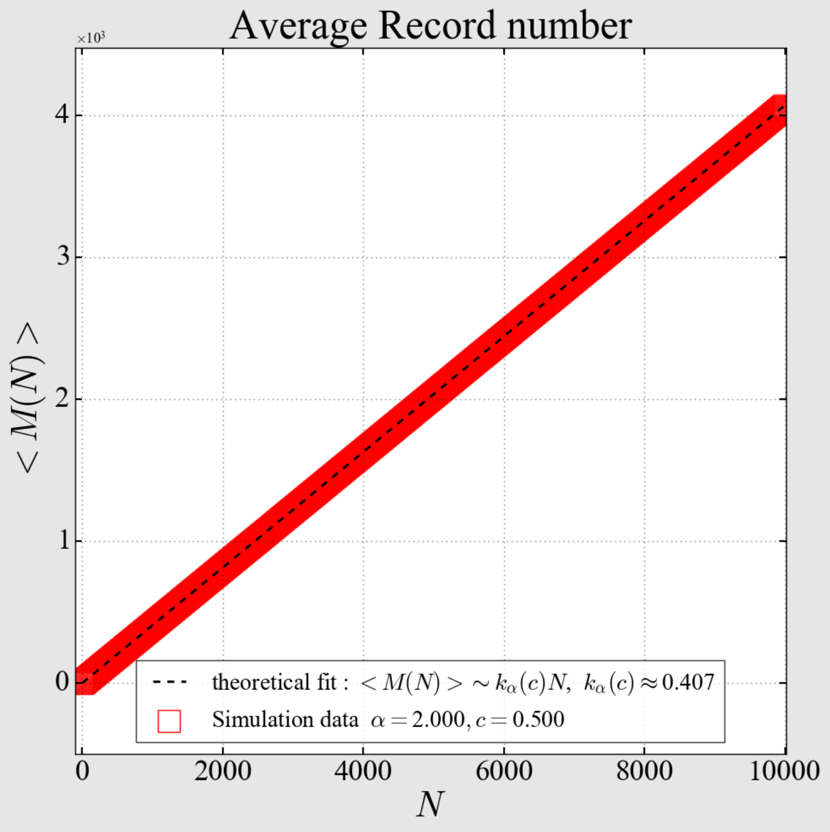
연구 결과 (Simulation : Regime 4)





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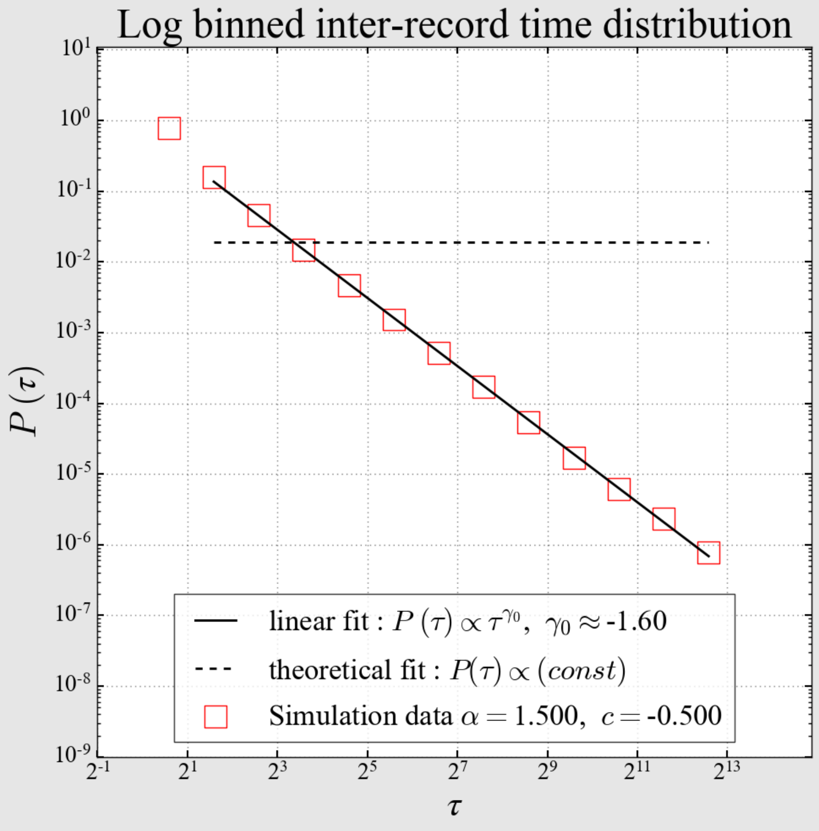
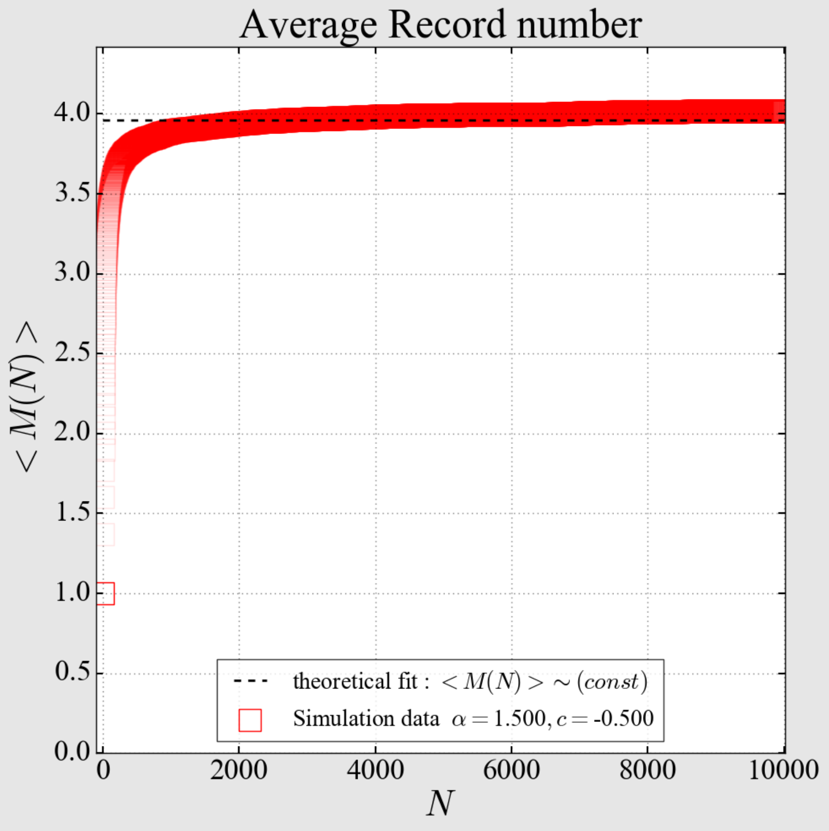
연구 결과 (Simulation : Regime 5) - 선행연구기반





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연구 결과 (Regime 5) – 이론적 보완내용

- Regime 3 (선행 연구 결과) -

- $\tilde{q}_-(z) = \sum_{n=0}^{\infty} q_-(n) z^n = \exp\left(\sum_{n=1}^{\infty} \frac{z^n}{n} \int_{cn}^{\infty} P_n(x)dx\right)$
- $\tilde{q}_-(s) = \sum_{n=0}^{\infty} q_-(n) e^{-sn} = \exp\left(\sum_{n=1}^{\infty} \frac{e^{-sn}}{n} \int_{cn}^{\infty} P_n(x)dx\right) = \exp\left(W_{c,\alpha}(s)\right) = \exp(\sum_{n=1}^{\infty} T_n)$
- $T_n \approx \frac{e^{-sn}}{n} \int_{cn^{1-\frac{1}{\alpha}}}^{\infty} \mathcal{L}_\alpha(y)dy \to \left(\frac{A_\alpha}{\alpha c^\alpha}\right) \frac{e^{-sn}}{n^\alpha}$, for $n \to \infty$. $\quad (A_\alpha = \frac{1}{\pi} \sin\left(\frac{\alpha\pi}{2}\right) \Gamma(1+\alpha))$
- $W_{c,\alpha}(s) = \sum_{n=1}^{\infty} T_n \to \boldsymbol{W_{c,\alpha}(0) - B_\alpha s^{\alpha-1}}$, for $s \to 0$. $(B_\alpha = A_\alpha \Gamma(2-\alpha)/[(\alpha(\alpha-1)c^\alpha])$
- $\boldsymbol{\tilde{q}_-(s) \to \exp(W_{c,\alpha}(0))[1 - B_\alpha s^{\alpha-1} + \cdots]}$
- $q_-(n) \to \frac{\alpha-1}{\Gamma(2-\alpha)} B_\alpha \exp(W_{c,\alpha}(0)) n^{-\alpha}$ for $n \to \infty$. using inverse laplace transform table..

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연구 결과 (Regime 5) – 이론적 보완내용

- Regime 5 (선행 연구 결과) -

- $\tilde{q}_-(z) = \sum_{n=0}^{\infty} q_-(n) z^n = \exp\left(\sum_{n=1}^{\infty} \frac{z^n}{n} \int_{cn}^{\infty} P_n(x)dx\right)$
- $\tilde{q}_-(s) = \sum_{n=0}^{\infty} q_-(n) e^{-sn} = \exp\left(\sum_{n=1}^{\infty} \frac{e^{-sn}}{n}\left(1 - \int_{|c|n}^{\infty} P_n(x)dx\right)\right) = \exp\left(W_{c,\alpha}(s)\right) = \exp(\sum_{n=1}^{\infty} T_n)$
- $T_n \approx \frac{e^{-sn}}{n}\left(1 - \int_{|c|n^{1-\frac{1}{\alpha}}}^{\infty} \mathcal{L}_\alpha(y)dy\right) \rightarrow \frac{e^{-sn}}{n} - \left(\frac{A_\alpha}{\alpha|c|^\alpha}\right)\frac{e^{-sn}}{n^\alpha},$ for $n \rightarrow \infty$. $\left(A_\alpha = \frac{1}{\pi}\sin\left(\frac{\alpha\pi}{2}\right)\Gamma(1+\alpha)\right)$
- $\boldsymbol{W_{c,\alpha}(s) = \sum_{n=1}^{\infty} T_n = -\ln(1-e^{-s}) - W_{|c|,\alpha}(s) \rightarrow -\ln(s) - W_{|c|,\alpha}(0)},$ for $s \rightarrow 0$.
- $\tilde{q}_-(s) \rightarrow \frac{\exp\left(-W_{|c|,\alpha}(0)\right)}{s},$ for $s \rightarrow 0$.
- $q_-(n) \rightarrow \exp\left(-W_{|c|,\alpha}(0)\right) = (const.)$ for $n \rightarrow \infty$. using inverse laplace transform..

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연구 결과 (Regime 5) – 이론적 보완내용

- Regime 5 (보완 결과) -

- $\tilde{q}_-(z) = \sum_{n=0}^{\infty} q_-(n) z^n = \exp\left(\sum_{n=1}^{\infty} \frac{z^n}{n} \int_{cn}^{\infty} P_n(x)dx\right)$
- $\tilde{q}_-(s) = \sum_{n=0}^{\infty} q_-(n) e^{-sn} = \exp\left(\sum_{n=1}^{\infty} \frac{e^{-sn}}{n}\left(1 - \int_{|c|n}^{\infty} P_n(x)dx\right)\right) = \exp\left(W_{c,\alpha}(s)\right) = \exp(\sum_{n=1}^{\infty} T_n)$
- $T_n \approx \frac{e^{-sn}}{n}\left(1 - \int_{|c|n^{1-\frac{1}{\alpha}}}^{\infty} \mathcal{L}_\alpha(y)dy\right) \to \frac{e^{-sn}}{n} - \left(\frac{A_\alpha}{\alpha|c|^\alpha}\right)\frac{e^{-sn}}{n^\alpha},$ for $n \to \infty.$ $\left(A_\alpha = \frac{1}{\pi}\sin\left(\frac{\alpha\pi}{2}\right)\Gamma(1+\alpha)\right)$
- $\boldsymbol{W_{c,\alpha}(s) = \sum_{n=1}^{\infty} T_n = -\ln(1 - e^{-s}) - W_{|c|,\alpha}(s) \to -\ln(s) - W_{|c|,\alpha}(0) + B'_\alpha s^{\alpha-1}},$ for $s \to 0.$
  - $B'_\alpha = A_\alpha \Gamma(2-\alpha)/[(\alpha(\alpha-1)|c|^\alpha]$
- $\tilde{q}_-(s) \to \frac{\exp\left(-W_{|c|,\alpha}(0)\right)(1+\boldsymbol{B'_\alpha s^{\alpha-1}})}{s},$ for $s \to 0.$
- $q_-(n) \to \exp\left(-W_{|c|,\alpha}(0)\right)\left(1 + \frac{B'_\alpha}{\Gamma(2-\alpha)}\right) n^{1-\alpha}$ for $n \to \infty.$ using inverse laplace transform table..

연구 결과 (Regime 5) – 이론적 보완내용

- 같은 식 상에서 Regime 3에서는 고려했던 term을 regime 5에서는 고려하지 않아서 inter-record time distribution 결과에 차이가 발생

- 기존연구결과
  - $P(\tau) = f_{-}(\tau) = -\frac{\partial}{\partial\tau} q_{-}(\tau)$
  - $q_{-}(\tau) \propto (constant)$
  - $P(\tau) \propto 0$

- 보완연구결과
  - $P(\tau) = f_{-}(\tau) = -\frac{\partial}{\partial\tau} q_{-}(\tau)$
  - $q_{-}(\tau) \propto \left(1 + \frac{B'_{\alpha}}{\Gamma(2-\alpha)}\right) \tau^{1-\alpha}$
  - $P(\tau) \propto \left(1 + \frac{B'_{\alpha}}{\Gamma(2-\alpha)}\right) \tau^{-\alpha}$

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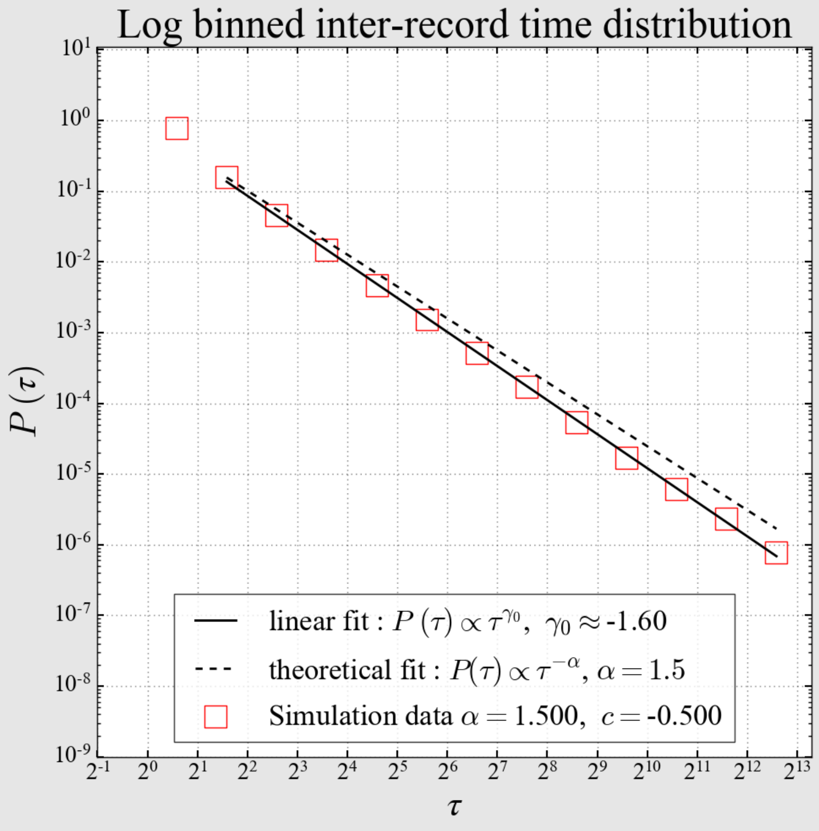
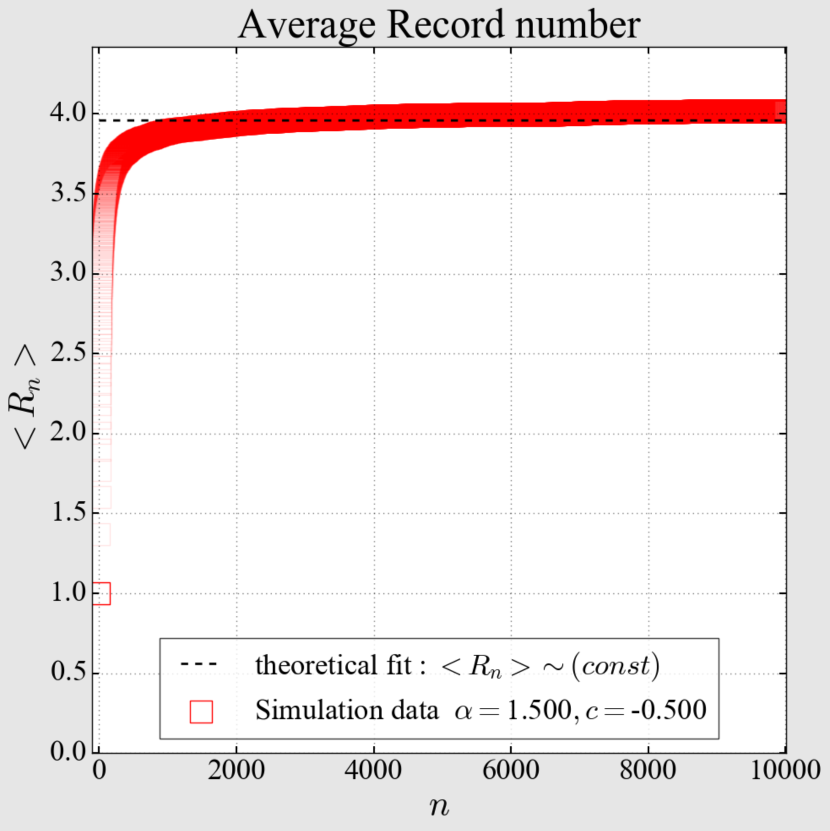
연구 결과 (Simulation : Regime 5) - 보완결과





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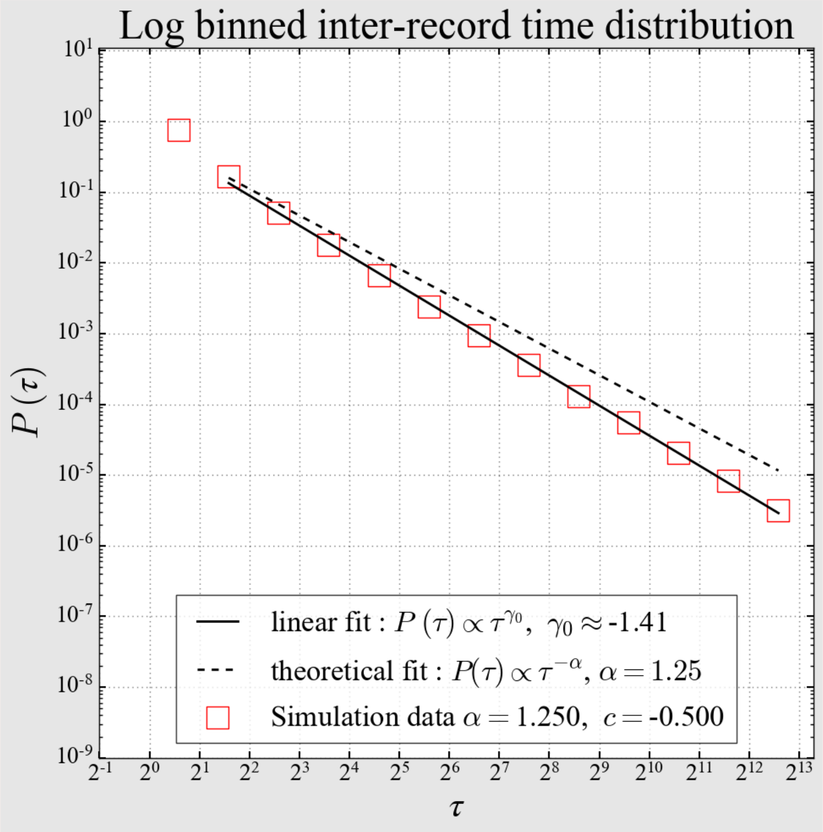
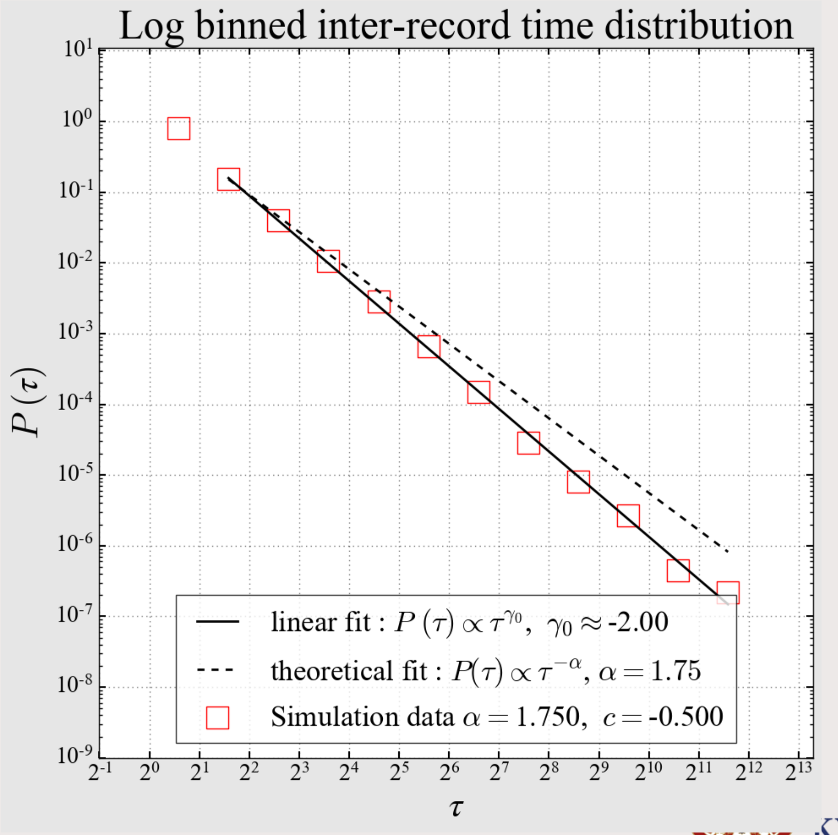
연구 결과 (Simulation : Regime 5) - 보완결과





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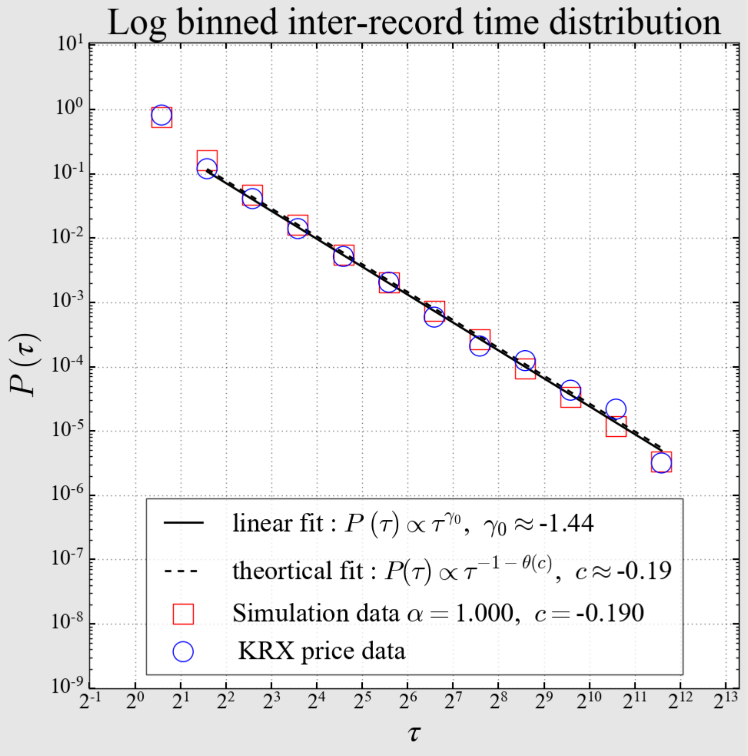
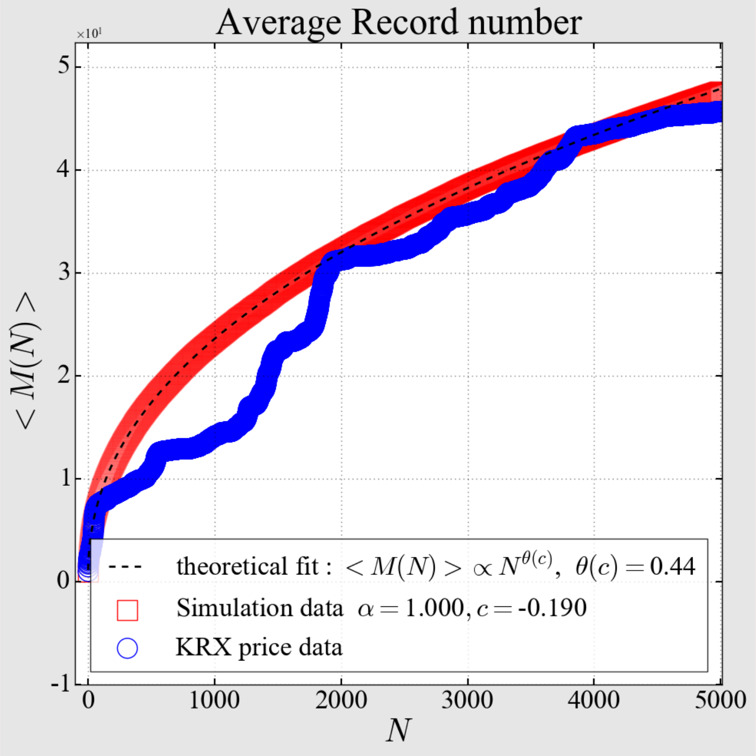
연구 결과 (한국 주식 시장 일별 가격)

2000.01.02 ~ 2020.04.02 (5000일) 에 KOSPI와 KOSDAQ에 상장된 종목들





- 2000년 초부터 2020년 초까지 한국 주식 시장은 평균적으로 regime 2 에 속해 있다는 것을 알 수 있음.
- 따라서 한국 주식시장은 constant bias가 record statistics에 영향을 주로 끼치고, 이 때 bias 값 $c \approx -0.19$ 임.

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