

# Inter-record time distribution for the Korean stock market and housing market

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#### Introduction

- Record statistics in the real world:
- Records in sports such as Olympic games
- Change of stock prices in stock exchange markets
- Change of temperature (global warming)
- In physics, record statistics plays an important role to understand the behavior of the stochastic systems:
  - A domain wall in metallic ferromagnetic materials
  - A model for the growth of networks based on record events
  - Understanding magnetization of superconductors and spin-glasses
  - An alternative indicator of quantum chaos in kicked rotor model
- Record statistics is closely related to the first passage process [1]
- Basic questions in the first-passage process in the stock market [1]
- Will I eventually break even?
- 2) How long do I have to wait until I break even?
- 3) While I am waiting to break even, how low might the stock price go?
- 4) Is it a good idea to place a limit order?

#### Motivation

- The inter-record time (IRT; record-age) distribution has been rarely investigated.
- We study the behavior of IRT distribution for Lévy walks with drift.
- We find that the IRT distribution can be useful to analyze finite single time series.
- Based on the analytical results for IRT distribution, we analyze the statistical properties of records in the Korean stock market and housing market.

#### 1-D Lévy walks with constant drift c and staying probability $p_0$

Let  $x_n$  be the position of a walker at the n-th step.

$$\begin{cases} x_n = x_{n-1} & (with \ p_0) \\ x_n = x_{n-1} + \eta_n + c & (with \ 1 - p_0) \end{cases}$$

- 1) For the Korean stock market:  $p_0 = 0$  (same with [2].)
- 2) For Seoul's housing market:  $p_0 = 0.910608$
- 3)  $p_0$  only affects the mean record number.

Jump length distribution:  $\varphi(\eta) = \int_{-\infty}^{\infty} \widehat{\varphi}(k) e^{ik\eta} dk$ 

Characteristic function of  $\varphi(\eta)$ :  $\hat{\varphi}(k) = \exp(-|lk|^{\mu})$ , specially l=1 in simulation.

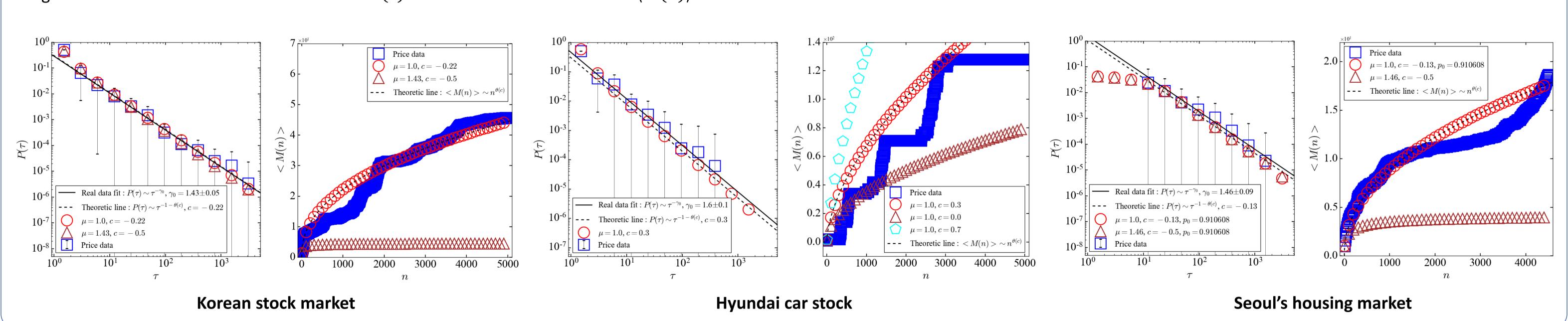
## IRT distributions on 1-D Lévy walk with constant drift

- IRT distribution  $P(\tau)$  corresponds to first-passage probability  $f_{-}(\tau)$ .
- $f_-(\tau)$  could be represented by survival probability  $q_-(\tau)$ , the relation between them is as followed.  $f_-(\tau) = -\frac{\partial}{\partial \tau}q_-(\tau)$ .

	Regime 1 $0<\mu<1$ $c\in(-\infty,\infty)$	Regime 2 $\mu=1$ $c\in(-\infty,\infty)$	Regime 3 $1<\mu<2$ $c\in(0,\infty)$	Regime 4 $\mu = 2$ $c \in (0, \infty)$	Regime 5 $1<\mu\leq 2$ $c\in (-\infty,0)$	
P( au)	$P(\tau) \sim \tau^{-1.5}$	$P(\tau) \sim \tau^{-1-\theta(c)}$	$P(\tau) \sim \tau^{-1-\mu}$	$P(\tau) \sim e^{-\frac{c^2 \tau}{2\sigma^2}} \left( \frac{c^2}{2\sigma^2} \tau^{-1.5} + \frac{3}{2} \tau^{-2.5} \right)$	$P(\tau) \sim \tau^{-\mu}$	$\theta(c) = \frac{1}{2} + \frac{1}{\pi} \arctan(c)$ $\sigma = \sqrt{2}l$
$ \begin{array}{c} 10^{0} \\ 10^{-1} \\ 10^{-2} \\ 10^{-3} \\ \hline 10^{-4} \\ 10^{-5} \\ 10^{-6} \\ 10^{-7} \\ \hline 10^{-7} \\ \hline 10^{-8} \\ \hline 10^{-8} \\ \hline 10^{0} \\ 10^{1} \\ 10^{2} \\ 10^{3} \\ 10^{4} \\ \hline  \tau \end{array} $	$ \begin{array}{c} 10^{0} \\ 10^{-1} \\ 10^{-2} \\ 10^{-3} \\ \hline 10^{-4} \\ 10^{-5} \\ 10^{-6} \\ \hline 10^{-7} \\ 10^{-8} \\ \hline 10^{-8} \\ \hline 10^{-8} \\ \hline 10^{0} \\ 10^{1} \\ 10^{2} \\ \hline 10^{1} \\ 10^{2} \\ \hline 10^{2} \\ \hline 10^{1} \\ 10^{2} \\ \hline 10^{2} \\ $	10-7	$\mu = 1.5, c = 0.5$ $P(\tau) \sim \tau^{-1-\mu}$ $\mu = 1.5, c = 1.0$ $P(\tau) \sim \tau^{-1-\mu}$ $\tau$	$ \begin{array}{c} 10^{-1} \\ 10^{-2} \\ 10^{-3} \\ \hline 10^{-4} \\ 10^{-5} \\ 10^{-6} \\ 10^{-6} \\ 10^{-7} \\ 10^{-7} \\ 10^{-8} \\ 10^{-8} \\ 10^{-8} \\ 10^{-8} \\ 10^{-1} \\ 10^{-1} \\ 10^{-2$	$ \begin{array}{c} 10^{0} \\ 10^{-1} \\ 10^{-2} \\ 10^{-3} \\ \hline 10^{-4} \\ 10^{-5} \\ 10^{-6} \\ 10^{-7} \\ \hline 10^{-8} \\ 10^{-8} \\ \hline 10^{-8} \\ \hline 10^{-1} \\ \mu = 1.5, e  P(\tau) \sim \tau^{-1} \\ 10^$	z=-1.0
Regime 1	Regime	e <b>2</b>	Regime 3	Regime 4		Regime 5

### Record statistics in empirical data

- · Korean stock market: KOSPI and KOSDAQ stock price data from the Korea exchange market (KRX). 2000.01.02~2020.04.02 (5,000 days)
- · Seoul's housing market: Daily price data per square meter for 332 administrative Dongs. 2006.01.01~2018.05.21 (4,523 days)
- Figures for each data are the IRT distribution  $P(\tau)$  and the mean record number  $\langle M(n) \rangle$ .



## Conclusion

- We analytically study the IRT distribution for Lévy walks with drift and staying probability.
- We compare the analytic results with the IRT distribution obtained from the Korean stock exchange market and housing market.
- We find that, even for the single time series, the record statistics in a stochastic process can be analyzed by the IRT distribution.