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MGMT 670 – Business Analytics
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Homework Exercise #2

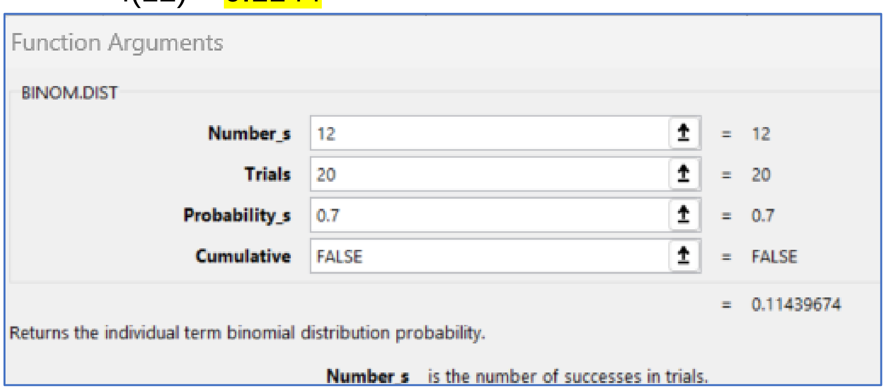
Homework exercises must be prepared individually and submitted online through Brightspace prior to the posted deadline. Up to three submissions are allowed but only the last submission will be graded. See the Syllabus for more information on Homework Exercise requirements and expectations. Any necessary modifications to this assignment will be posted to Brightspace as an announcement.

For this exercise, submit only one Microsoft Word document with all appropriate output results and graphs from Minitab and Excel into the single Word document.

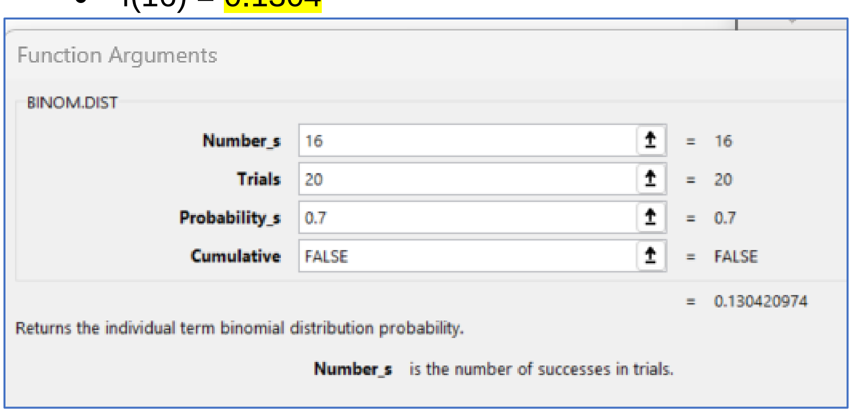
Question 1 – Binomial Formula (pg.184-189)

Consider a binomial experiment with n = 20 and p = 0.70. Be sure to show a reasonable amount of your work.

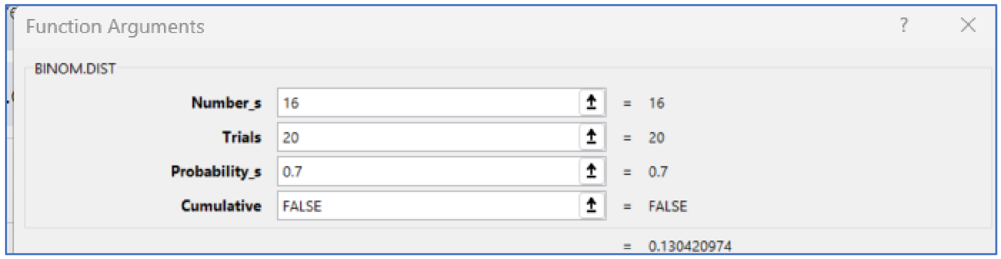
- **Compute *f*(12).**
  - f(12) = 0.1144



- **Compute *f*(16).**
  - f(16) = 0.1304



- **<u>Compute P(X ≥ 16).</u>**
  - Calculate this formula:



  - After, Calculate sum of Number_s (16, 17,18,19,20)
  - Sum all variables = 0.2374

0.130420974
0.071603672
0.027845873
0.006839337
0.000797923

- **<u>Compute P(X ≤ 15).</u>**
  - Calculate this formula:

  - After, Calculate sum of Numbers (16, 17,18,19,20)
  - Sum all variables = 0.2375

0.130420974
0.071603672
0.027845873
0.006839337
0.000797923

  - To compute P(X ≤ 15) you need to subtract (1 – Sum(16,17,18,19,20))
  - <u>P(X ≤ 15) = 0.762</u>
- **<u>Compute E[X].</u>**
  - E[X] = 20 x 0.70 = 14.0
- **<u>Compute Var[X] and σ.</u>**
  - In Excel, (=20*0.7*(1-0.7)) or 20(.70)(.30) = 4.2 Var[x]
  - STD_DEV=SQRT(=20*0.7*(1-0.7))
  - = 2.049 or 2.05
- **<u>Use the normal approximation to calculate P(X ≥ 16) even though *n* typically must be larger than 20. How does this value compare to your answer in part c?</u>**

- N = 20, P = 0.70, X = 16
- Variance = 20 x 0.70(1-0.70) = 14(0.3) = 4.2

- Mean = N x P (20 x 0.70) = 14
- Std_dev = 2.05
- **Z score = (16 – 0.5 – 14) / SQRT 4.2(2.049) = 0.732**
- **Calculate the Probability (1 - .7673) = 0.2327**

Question 2 -

A business executive, transferred from Chicago to Atlanta, needs to sell her house in Chicago quickly. The executive's employer has offered to buy the house for $210,000, but the offer expires at the end of the week. The executive does not currently have a better offer but can afford to leave the house on the market for another month. From conversations with her realtor, the executive believes the price she will get by leaving the house on the market for another month is uniformly distributed between $200,000 and $225,000.

1. If she leaves it on the market for another month, what is the probability she will get at least $215,000 for the house?
   - Min price = 200,000, Max price = 225,000, Target price = 215,000
   - (225,000 – 215,000) / 225,000 – 200,000)
   - 10,000 / 25,000
   - = 0.40 = 40%

2. If she leaves it on the market for another month, what is the probability she will get at least $210,000?
   - Min price = 200,000, Max price = 225,000, Target price = 215,000
   - (225,000 – 210,000) / 225,000 – 200,000)
   - 15,000 / 25,000
   - = 0.60 = 60%

3. Should the executive leave the house on the market for another month? Why or why not?
   - There is 60% chance of getting at least the employer's offer of $210,000 but there is a 40% of getting less than $210,000. If the executive is willing to take a risk, it could be beneficial to leave the house on the market for another month. On the other hand, if you are needing immediate funds accepting the offer would be the best choice.

Question 3 – Mean and Standard Deviation of Discrete Distributions (pg. 175-178)

A friend has asked you to analyze his stock portfolio, which consists of 10 shares of stock D and 5 shares of stock C. The probability distribution of the stock prices is shown below.

		Stock D price				
		$40	$50	$60	$70	
Stock C price	$45	0.00	0.00	0.05	0.20	0.25
	$50	0.05	0.00	0.05	0.10	0.20
	$55	0.10	0.05	0.00	0.05	0.20
	$60	0.20	0.10	0.05	0.00	0.35
		0.35	0.15	0.15	0.35	1.00

1. For each of the stocks compute the expected values and variance of one share. Also compute their covariance.
   - Expected Value (Mean) = E[Stock] = ∑ (Price x Probability)
   - Variance = Var[Stock] ∑ ((Price – E[Stock]2Squred x Probability)
   - Covariance = Cov[D,C]=∑((D−E[D])×(C−E[C])×P(D,C))

   a) Expected Value (Mean) Stock D:(40 x .35) + (50 x .15) + (60 x .15) + (70 x .35) = $55

   b) Variance Stock D: ((40 – 55)^2 x 0.35 = 78.75) + (50 – 55)^2 x .15 = 3.75)) + ((60 – 55)^2 x .15 = 3.75)) + (70 - 55)^2 x .35 = 78.75))
      - Total Variance Stock D = 165.00

   c) Expected Value (Mean) Stock C: (45 x .25) + (50 x .20) + (55 x .20) + (60 x .35) = $53.25

   d) Variance Stock C: ((45 – 53.35)^2 x 0.25 = 17.0156)) + ((50 – 53.25)^2 x 0.20 = 2.1125)) + (55 – 53.25)^2 x 0.20 = 0.6125)) + (60 – 53.25)^2 x 0.35 = 15.946))
      - Total Variance Stock C = 35.69

   e) Covariance:
      - Expected value of product: ((45·60·0.05)+(45·70·0.20)+(50·40·0.05)+(50·60·0.05)+(50·70·0.10)+(55·40·0.10)+(55·50·0.05)+(55·70·0.05)+(60·40·0.20)+(60·50·0.10)+(60·60·0.05) = 2875
      - 2875 – (53.25 X 55) (2875 – 2928.75)
      - = -53.75

2. Compute the expected value and variance of the portfolio.
   - Expected Value (Mean) = ((10 x 55.00) + (5 x 53.25)) = 816.25
   - Variance = ((10)^2 x 165.00) + (5)^2 x 35.69) + 2 x (10) x (5) x (-53.75)) = 12017.25

Question 4

During the 1960s and into the 1970s, the Mexican government pegged the value of Mexican peso to the U.S. dollar at 12 pesos per dollar. Because interest rates in Mexico were higher than those in the United States, many investors (including banks) bought bonds in Mexico to earn higher returns than were available in the United States. The benefits of the higher interest rates, however, may be limited because the government could decide to float the currency and it might lose value. Suppose the probability that the exchange rate is 12 pesos per dollar is 0.9 and the probability that the exchange rate is 24 pesos per dollar is 0.1. Assume that the investor is risk-neutral for the following calculations.

1. Assume you are a U.S. investor who is considering two options. Deposit $1,000 today in a U.S. savings account that pays 8% annual interest or deposit the converted pesos in a Mexican savings account that pays 16% annual interest. The latter option requires converting back the pesos into dollars at the end of the year. Which investment would you choose?
    - Convert $1000 to pesos at 12 pesos per dollar = 12,000 pesos
    - Deposit at 16% interest = 12,000 x 1.16 = 13,920 pesos
    - Convert back to US dollars from 12 pesos per dollar is 0.9 and 24 pesos per dollar is 0.1
        - 90% = 12 pesos (13,290 / 12) = $1,160
        - 10% = 24 pesos (13,290 / 24) = $580
    - Expected return (Price x Probability)
        - ($1,160 x 0.9) + ($580 x 0.1) = (1044) + (58) = $1,102
        - Earnings = $102
    - USD: Interest = $1,000 x 1.08 = $1,080 USD or $80
    - I would choose the Mexican Savings account comparing US ($1,080 or $80) and Mexican Savings ($1,102 or $102)

2. Now, suppose you are a Mexican investor with 12,000 pesos to invest. You can either convert the money to U.S. dollars, earn 8% interest, and convert the money back to pesos, or you can earn 16% interest in a Mexican savings account. What would you choose?
    - Convert 12,000 pesos to dollars at 12 pesos/dollar = 12,000 / 12 = $1000
    - Interest = $1,000 x 1.08 = $1,080 USD or $80
    - 12 pesos exchange at 0.9 probability: 1,080 x 12 = 12,960
    - 24 pesos exchange at 0.1 probability: 1,080 x 24 = 25,920
    - Expected return: (12,960 x 0.9) + (25,920 x 0.1) = (11,664) + (2,592) = $14,256
    - End of year amount: 12,000 x 1.16 = $13,920
    - I would choose the US dollars at 8%

3. Can you intuitively explain the strategies in the above parts?
   - Question 1: Comparing the two options for the US investor would choose the Mexican savings account because it can offer a higher expected return than the US account which I showed above.
   - Question 2: I would choose the 8% interest based on the work I have shown above.

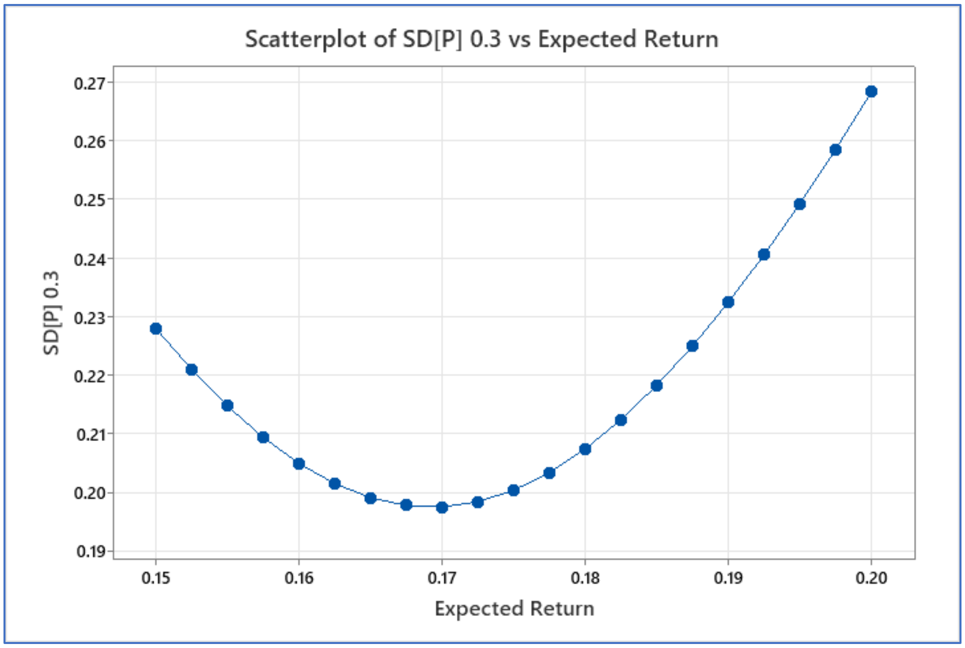
Question 5

Consider a portfolio that consists of two assets which we call A and B. Let *X* be the annual rate of return from A and *Y* denote the annual rate of return from B. Let
E[X] = 0.15*;* E[Y] = 0.20*;* Var[X] = 0.052*;* Var[Y] = 0.072 and CORR[*X, Y*] = 0.30
Use a spreadsheet to perform the following analysis.

1. Suppose the fraction of portfolio invested in asset B is *f* and the fraction invested in A is 1 - *f*. Let *f* vary from 0 to 1 in increments of 5% (*i.e.*, *f* = 0.0*;* 0.05*;* 0.10*; ... ;* 1.0). Compute the mean and standard deviation of the annual rate of return of the portfolio. Plot the standard deviation as a function of the return.

   Information:
   - E[X] = 0.15 (expected return of Asset A)
   - E[Y] = 0.20 (expected return of Asset B)
   - Var[X] = 0.053 (Variance of Asset A)
   - Var[Y] = 0.072 (Variance of Asset B)
   - Corr [X, Y] = 0.30 (correlation between A and B)
   - F in Asset B and 1 – f in Asset A:

   Equations:
   - Expected Return Equation: E[P]=f·E[Y]+(1−f)·E[X]
   - Standard Deviation Equation: SD[*P*]=(*f*2·Var[*Y*])+((1−*f*)2·Var[*X*])+(2·*f*·(1−*f*)·Corr[*X*,*Y*]·SD[*X*]·SD[*Y*])
   - Sharpe Ratio: (Expected Return – 0.5 / Standard Deviation)



f	A	B	Expected Return	SD[P] 0.3	Sharpe ratio	SD[P2] 0.1	Max SR
0	1	0	0.15	0.228035	0.43852901	0.073024598	0.626655
0.05	0.95	0.05	0.1525	0.221029	0.46373983	0.071688889	
0.1	0.9	0.1	0.155	0.214812	0.48879967	0.070563041	
0.15	0.85	0.15	0.1575	0.209454	0.51323964	0.06965872	
0.2	0.8	0.2	0.16	0.205022	0.53652747	0.068986167	
0.25	0.75	0.25	0.1625	0.201578	0.55809671	0.06855371	
0.3	0.7	0.3	0.165	0.199173	0.57738871	0.068367326	
0.35	0.65	0.35	0.1675	0.197844	0.59390268	0.068430289	
0.4	0.6	0.4	0.17	0.197614	0.60724588	0.068742962	
0.45	0.55	0.45	0.1725	0.198485	0.61717387	0.06930275	
0.5	0.5	0.5	0.175	0.200445	0.62361217	0.070104235	
0.55	0.45	0.55	0.1775	0.203461	0.62665516	0.071139471	
0.6	0.4	0.6	0.18	0.207488	0.62654345	0.072398401	
0.65	0.35	0.65	0.1825	0.212467	0.62362648	0.073869346	
0.7	0.3	0.7	0.185	0.218334	0.61831884	0.075539526	
0.75	0.25	0.75	0.1875	0.225019	0.61105875	0.07739554	
0.8	0.2	0.8	0.19	0.232452	0.60227411	0.079423801	
0.85	0.15	0.85	0.1925	0.240564	0.59235867	0.081610888	
0.9	0.1	0.9	0.195	0.249287	0.58165816	0.083943817	
0.95	0.05	0.95	0.1975	0.258561	0.57046468	0.086410235	
1	0	1	0.2	0.268328	0.55901699	0.088998545	

2. Assume that the risk-free interest rate, ***r_F* , is 0.05.** For any portfolio *P*, let E[P] and SD[*P*] denote its expected return and standard deviation of the return. The Sharpe-ratio for *P* is then given by (E[P] – r_F)/SD[*P*] . Which of the portfolios has the maximum Sharpe-ratio?

- The maximum Sharpe Ratio = (A = 0.45 and B = 0.55) and the Sharpe ratio = 0.626655161

3. Repeat the above exercise but change CORR[*X*, *Y*] to 0.1. Interpret the change in the output graph.