Journal of the
Experimental Analysis of Behavior

JOURNAL OF THE EXPERIMENTAL ANALYSIS OF BEHAVIOR 2016, **106,** 145–155 NUMBER 2 (SEPTEMBER)

AN ALTERNATIVE APPROACH TO CALCULATING AREA-UNDER-THE-CURVE (AUC) IN DELAY DISCOUNTING RESEARCH

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Applied to delay discounting data, Area-Under-the-Curve (AUC) provides an atheoretical index of the rate of delay discounting. The conventional method of calculating AUC, by summing the areas of the trapezoids formed by successive delay-indifference point pairings, does not account for the fact that most delay discounting tasks scale delay pseudoexponentially, that is, time intervals between delays typically get larger as delays get longer. This results in a disproportionate contribution of indifference points at long delays to the total AUC, with minimal contribution from indifference points at short delays. We propose two modifications that correct for this imbalance via a base-10 logarithmic transformation and an ordinal scaling transformation of delays. These newly proposed indices of discounting, $AUC_{\log d}$ and $AUC_{\mathrm{or}d}$, address the limitation of AUC while preserving a primary strength (remaining atheoretical). Re-examination of previously published data provides empirical support for both $AUC_{\log d}$ and $AUC_{\mathrm{or}d}$. Thus, we believe theoretical and empirical arguments favor these methods as the preferred atheoretical indices of delay discounting.

Key words: delay discounting, Area-Under-the-Curve, AUC

Delay Discounting Background

The value of a reward can be dependent on when it becomes available for use or consumption, such that delayed rewards are frequently discounted (Chung & Herrnstein, 1967; Green & Myerson, 1996). This decrease in the subjective value of a reward as a function of the delay until receipt of the reward (Mazur, 1987; Rachlin, Raineri, & Cross, 1991) is called delay discounting. Delay discounting likely impacts many decisions in which an outcome is experienced at a later point in time, making delay discounting a useful construct for understanding intertemporal decision-making as well as behaviors associated with impulsive choice (and its inverse).

High rates of delay discounting are associated with substance use (MacKillop et al., 2011; Yi, Mitchell, & Bickel, 2010), disordered eating (Weller, Cook, Avsar, & Cox, 2008), gambling (Alessi & Petry, 2003; Petry & Madden, 2010; Reynolds, 2006), and predict clinical outcomes such as cigarette and marijuana relapse (Krishnan-Sarin et al., 2007; Sheffer et al., 2012; Sheffer et al., 2014; Stanger et al., 2012), as well as unhealthy behaviors such as risky sexual behavior, fewer preventive health screenings, and infrequent exercise (Daugherty & Brase, 2010; MacKillop et al., 2011; Story, Vlaev, Seymour, Darzi, & Dolan, 2014; Yi et al., 2010). Indeed, its role as a predictor and correlate of health behaviors highlights the theoretical and clinical utility of the delay discounting construct.

Delay discounting in humans is commonly assessed using tasks designed to determine the present subjective value of a delayed outcome. For example, a binary choice task may require an individual to indicate preference between a smaller immediate monetary reward and a larger delayed monetary reward (Johnson & Bickel, 2002; Kirby & Marakovic, 1995; Kirby, Petry, & Bickel, 1999). Using either fixed-choice or titrating procedures where the immediate reward is systematically varied, patterns of preference for the smaller-immediate and larger-delayed rewards allow the experimenter to estimate the present subjective value of the delayed reward. This amount is called the *indifference point* to indicate theoretical indifference on the part of the individual between the delayed reward and the estimated present subjective value. Determination of indifference points for the delayed reward at multiple delays allows for the calculation of a

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doi: 10.1002/jeab.219

delay discount rate as well as comparison of different models of delay discounting.

Original models of delay discounting hypothesized that the decrease in the present subjective value of a delayed outcome occurred in an exponential fashion, indicating that the subjective value of an outcome decreased at a constant rate (Samuelson 1937; Streich & Levy, 2007). For theoretical and empirical reasons (Frederick, Loewenstein, & O'Donoghue, 2002), the exponential model has largely been discarded as a descriptor of human discounting behavior (for exceptions, see Green & Myerson, 1993; Noor, 2011), with the hyperbolic model (Green & Myerson, 1996; Kirby & Marakovic, 1995; Mazur, 1987; Rachlin et al., 1991) and variations thereof (e.g., the hyperbola-like model of Green, Fry, & Myerson, 1994) gaining traction. Nonetheless, there remains a lack of consensus regarding the exact mathematical function underlying delay discounting. To avoid assumptions regarding the "correct" model when it is secondary to the assessment of delay discounting (e.g., in repeated-measures studies where changes in rate of delay discounting are examined), Myerson and colleagues (Myerson, Green, & Warusawitharana, 2001) introduced an alternative method for estimating delay discount rates called Area-Under-the-Curve (AUC).

Area-Under-the-Curve is a model-free estimate of delay discounting, calculated as the area of multiple polygons formed by plotting each indifference point as a function of delay. To calculate AUC, straight lines are drawn from each indifference point to the successive (by delay) indifference point and to the abscissa. The area for each of the resulting series of polygons is calculated as a typical trapezoid, $(x_2-x_1)\left[\frac{y_1+y_2}{2}\right]$, where x_1 and x_2 are the delays associated with successive indifference points, and y_1 and y_2 are the indifference point values associated with those delays. The areas of these trapezoids are then summed for a total AUC value that, when delays are scored as a proportion of the maximum delay of the specific delay discounting assessment (i.e., standardized), can range from 0 (steepest discounting) to 1 (no discounting). AUC values are highly correlated with the index of delay discounting derived from the hyperbolic discounting model (i.e., hyperbolic k; Myerson et al., 2001).

Demonstration of the Problematic Nature of AUC for Calculating Delay Discount Rate

To estimate delay discounting, measures of intertemporal choice inherently consist of multiple delay points that are used to assess an individual's preference between immediate and delayed rewards. Yet common measures of delay discounting lack a systematic selection of delay points. Conventional measures of intertemporal choice contain pseudoexponential delay scales, with unequal spacing between delay points (Rachlin, et al., 1991). Although no explicit rationale for the pseudoexponential scaling of delays is stated, the vast majority of researchers incorporate this scaling of time into the selection of delays in human studies of delay discounting, with intervals between delays increasing as delays get longer. For example, the delay discounting task for a number of studies (e.g., Yi & Landes, 2012) determined indifference points at the following series of delays: 1 day, 1 week, 1 month, 6 months, 1 year, 5 years, and 25 years. Procedures that use different delays still scale delay similarly, with few exceptions[1].

Although the pseudoexponential scaling of delays may reflect the psychophysical phenomena of time, in that the subjective experience of time is not equivalent to the objective measurement of time (Takahashi, 2005; Zauberman, Kim, Malkoc, & Bettman, 2009), this nonlinear scaling results in an imbalance in the relative contribution of each indifference point to the total AUC estimate of discounting; successive delays disproportionately contribute to the total AUC. This imbalance is in contrast to the theoretical assumptions underlying delay-discounting models of intertemporal choice that depend on equivalent contributions from multiple time points to accurately estimate delay discounting. Figure 1a shows how this might be manifested in a procedure with the delays noted above; the shading indicates the relative contribution of each potential trapezoid to the total AUC (darker = more), with a clear indication that indifference points at longer delays have a disproportionate impact on this model-free index of delay

[1]Kirby (1997) is one example of an exception, where delay intervals were always 2 days.

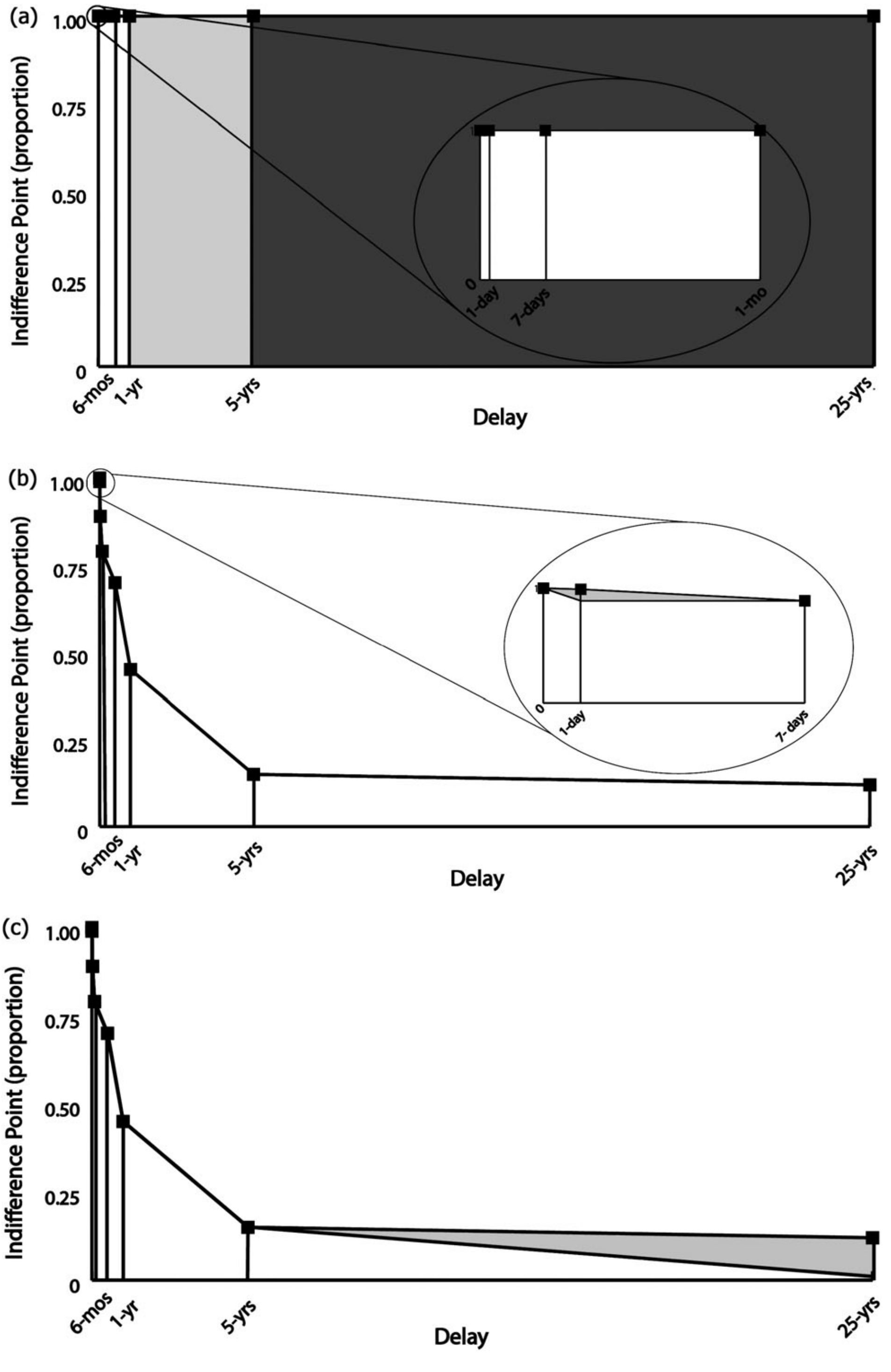

Fig. 1. Panel 1a demonstrates the contribution of each indifference point to AUC when delays are: 1 day, 1 week, 1 month, 6 months, 1 year, 5 years, and 25 years, and when all of the indifference points are equal to 1.0. The percent contribution of each area is listed in Table 1. Shading reflects the relative contribution of each division. Panel 1b illustrates the change in AUC (shaded area) following a 0.1 reduction in the indifference point at the 1-day delay for a real-data example. Panel 1c illustrates the change in AUC (shaded area) following a 0.1 reduction in the indifference point at the 25-year delay for the same participant.

Table 1

The percent contribution of the indifference point associated with each delay to the total AUC, $AUC_{\log d}$ and $AUC_{\mathrm{or}d}$ values.

Common Delay Scaling	Trapezoid	Percent Contribution via AUC *(Figure 1)*	Percent Contribution via $AUC_{\log d}$ *(Figure 2)*	Percent Contribution via $AUC_{\mathrm{or}d}$ *(Figure 3)*
1 day	1	0.011%	7.60%	14.29%
7 days	2	0.07%	15.20%	14.29%
1 month	3	0.26%	15.01%	14.29%
6 months	4	1.67%	19.35%	14.29%
1 year	5	1.99%	7.56%	14.29%
5 years	6	16.00%	17.63%	14.29%
25 years	7	80.00%	17.64%	14.29%
All Delays	1-7	100%	100%	100%

discounting[2]. More specifically, the delay interval between no delay and the first obtained indifference point (1 day) serves as the height measure of the first trapezoid (i.e., x_2-x_1), which corresponds to a very small proportion (0.00011) of the total delay. In contrast, the delay interval between the second-to-last (5 years) and last (25 years) indifference points corresponds to a very large proportion (0.80) of the total delay (see Table 1).

We consider hypothetical data from a typical participant in a delay discounting task using the above-referenced delays (solid lines in Figs. 1b and 1c). For demonstration purposes, we consider the impact on AUC of a 0.1 reduction of one indifference point. When this minimal reduction was applied to the first obtained indifference point (at delay = 1 day), there was only a 0.02% change in total AUC (Fig. 1b). In contrast, when the same 0.1 reduction was applied to the last indifference point (25 years), there was a 22.14% change in total AUC (Fig. 1c). This is a relative impact that is 1107 times greater than the same reduction at the 1-day delay. We believe this demonstration highlights the unjustifiably disproportionate impact that indifference points from longer delays have on total AUC. By inflating or deflating the total AUC depending on the delays where differences might be observed, this disproportionateness has important implications for assessing within-individual changes or between-individual differences in delay discounting.

[2]Model-based approaches to calculating rate of delay discounting typically use nonlinear regression methods, where the least-squares solution ensures that each indifference point is equally weighted.

Proposed Methods for Calculating Alternatives to Conventional AUC

To address the theoretical and empirical limitations of the conventional method of calculating AUC, we propose two minor but significant modifications, the choice of which depends on the theoretical orientation of the experimenter. The first method, $AUC_{\log d}$, accounts for the subjective experience of time that may be implicitly incorporated into the conventional scaling of delays. This method reflects this scaling while also partially correcting for the unequal contributions of each indifference point in the original model. The second method, $AUC_{\mathrm{or}d}$, remains theoretically neutral in regards to the original delay scaling and corrects for the pseudoexponential scaling of delays via an ordinal transformation. Each method will be discussed in turn.

The first modification, $AUC_{\log d}$, retains some of the original delay scaling for experimenters who seek to retain this scaling while still adjusting the relative contribution of each delay point. Although there remains debate regarding the psychophysical law governing time perception (e.g., Steven's Power Law, Fechner's Law), for this transformation we settle on the base-10 logarithmic scale of subjective time given the procedural details relevant for most delay discounting procedures with humans (i.e., prospective time that is months and years into the future; Takahashi, Oono, & Radford, 2008). A small constant (e.g., 1.00) can be added to all of the delay values (Bartlett, 1947) before logarithmic transformation to address the problem of transforming a value of zero. To calculate a standardized AUC measure, proportions of the maximum delay can still be used for calculating the total AUC by dividing each logged delay by the longest logged delay, resulting in proportions along the abscissa. The values of the resulting scale will range from 0 to 1 as in the original standardized AUC method, yet allow relative equivalence of the impact of each indifference point. Overall, the $AUC_{log\mathit{d}}$ method retains the strength of an atheoretical index of delay discounting without overweighting the contribution of specific indifference points or individual trapezoids.

For the second method, $AUC_{or\mathit{d}}$, we propose an ordinal transformation of delays. In this method, the original scaling for the delay discounting measure (e.g., 1 day, 7 days, 30 days, etc.) can be replaced with integers from 1 through 7. Modifications can be made depending on the original number of delay points within a discounting measure, such that the number of integers may be greater than or less than seven. Replacing each delay with an integer results in equal spacing between delays (i.e., one unit). The remaining steps of the conventional AUC method can be completed following this transformation. Delays can be transformed into proportions (i.e., 1/7, 2/7, 3/7 etc.) and the area of each polygon can be calculated using these proportions and the original indifference point values. Finally, these areas can be summed to form the total $AUC_{or\mathit{d}}$ estimate for discounting as in the original AUC method.

Figure 2a and Figure 3a (and Table 1) show the effect of this time scaling on $AUC_{log\mathit{d}}$ and $AUC_{or\mathit{d}}$. Noting that the shading indicates the relative contribution of each potential trapezoid to the total $AUC_{log\mathit{d}}$ and $AUC_{or\mathit{d}}$ (like Fig. 1a), the equivalence of the relative contributions of each potential trapezoid to the total $AUC_{log\mathit{d}}$ and $AUC_{or\mathit{d}}$ is apparent, with the exception of the first potential trapezoid (between 0 and 1 day delay) for $AUC_{log\mathit{d}}$. In other words, the proposed log transformation and ordinal transformation of delay minimizes the systematically uneven contribution of indifference points to calculation of the conventional AUC.

When considering the same hypothetical data from a typical participant (Figs. 1b and 1c) in a delay discounting task using this modified approach, a 0.1 reduction of the first obtained indifference point results in a 1.84% change in total $AUC_{log\mathit{d}}$ (Fig. 2b), and the same reduction of the last indifference point results in a 1.42% change in total $AUC_{log\mathit{d}}$ (Fig. 2c). For $AUC_{or\mathit{d}}$, a 0.1 reduction of the first obtained indifference point results in a 2.17% change in total $AUC_{or\mathit{d}}$ (Fig. 3b), and the same reduction of the last indifference point results in a 1.09% change in total $AUC_{or\mathit{d}}$ (Fig. 3c). Not only was the imbalance of the effect on the calculation of delay discount rate eliminated, it was slightly reversed. We believe this reversal is appropriate, given that the reduction of the 1-day delay indifference point impacts two trapezoids while the reduction of the 25-year indifference point only impacts one trapezoid.

Utility of Alternative AUC Methods

To examine the convergence of $AUC_{log\mathit{d}}$ and $AUC_{or\mathit{d}}$ with established approaches to assessing rate of delay discounting, we contrasted these alternative AUC measures with data previously reported (hypothetical $50 reward in Matusiewicz, Carter, Landes, & Yi, 2013) with two conventional approaches: AUC and hyperbolic discount rate k. Discount rate k is calculated in hyperbolic equation $V_d = V / (1 + kD)$, where V_d is the discounted value of an outcome, V is the undiscounted value, D is delay in days, and k is the index of discount rate (Mazur, 1987). Bivariate correlations indicate stronger relations between $AUC_{log\mathit{d}}$ and hyperbolic k ($r = 0.98$, $p < .001$) and $AUC_{or\mathit{d}}$ and hyperbolic k ($r = 0.98$, $p < .001$) than between conventional AUC and hyperbolic k ($r = 0.76$, $p < .001$). Likewise, the relation between $AUC_{log\mathit{d}}$ and AUC ($r = 0.76$, $p < .001$) as well as $AUC_{or\mathit{d}}$ and AUC ($r = 0.76$, $p < .001$) are not as strong, with $AUC_{log\mathit{d}}$ and $AUC_{or\mathit{d}}$ highly correlated ($r = 0.99$, $p < .001$). Given that the calculation of hyperbolic k via nonlinear regression ensures equal contribution of all indifference points to determining rate of delay discounting, this analysis indicates that $AUC_{log\mathit{d}}$ and $AUC_{or\mathit{d}}$ appear to better equate the contribution of each indifference point, in a manner consistent with the calculation of hyperbolic k or likely any other index determined from a mathematical model of delay discounting.

The conventional AUC, by inflating the contributions of some indifference points relative to others, could be argued as enhancing effect sizes by magnifying differences that

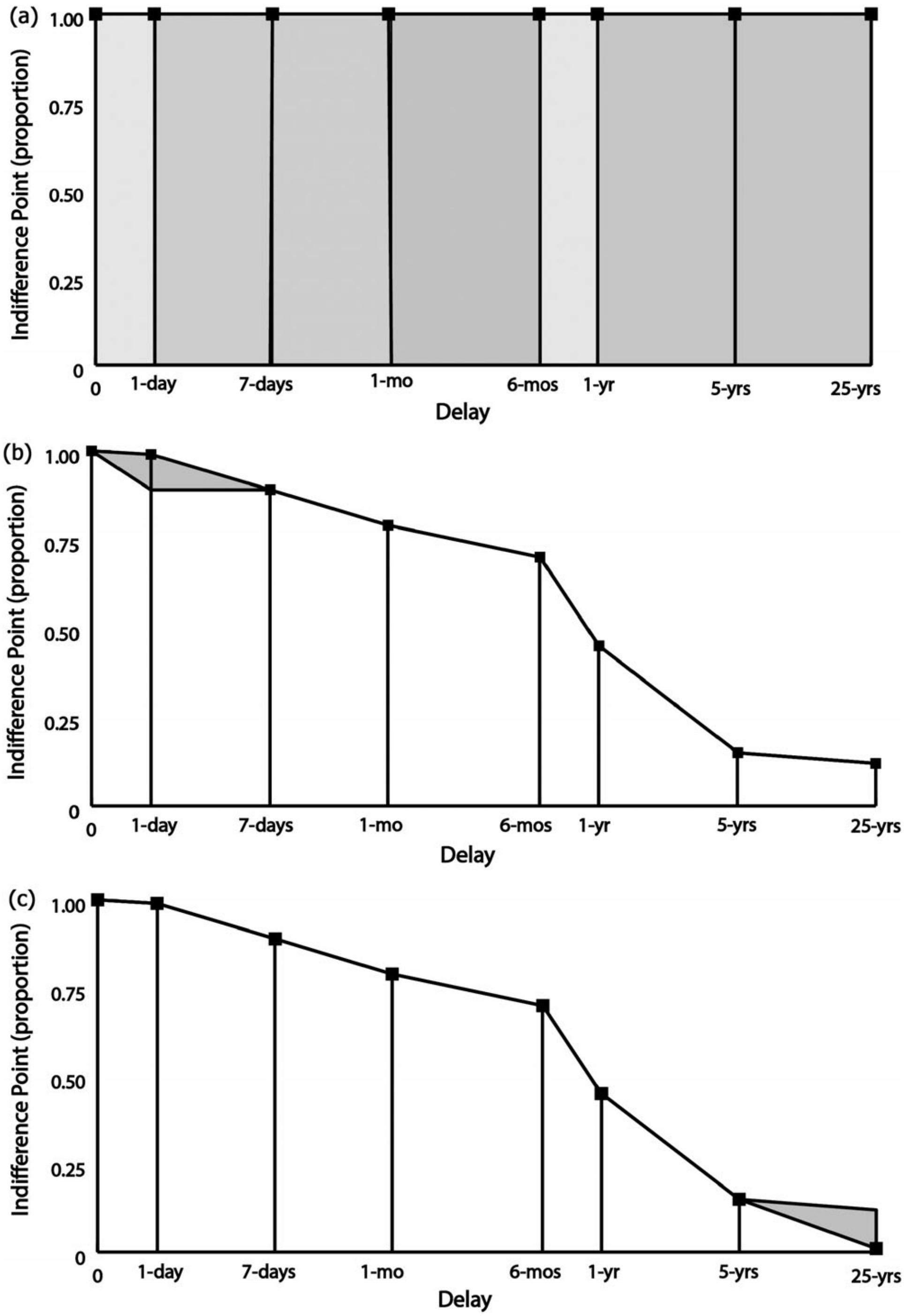

Fig. 2. Panel 2a demonstrates the contribution of each indifference point to $AUC_{\log d}$, with shading reflecting the relative contribution of each division (Table 1). Panel 2b illustrates the change in $AUC_{\log d}$ (shaded area) following a 0.1 reduction in the indifference point at the 1-day delay. Panel 2c illustrates the change in $AUC_{\log d}$ (shaded area) following a 0.1 reduction in the indifference point at the 25-year delay.

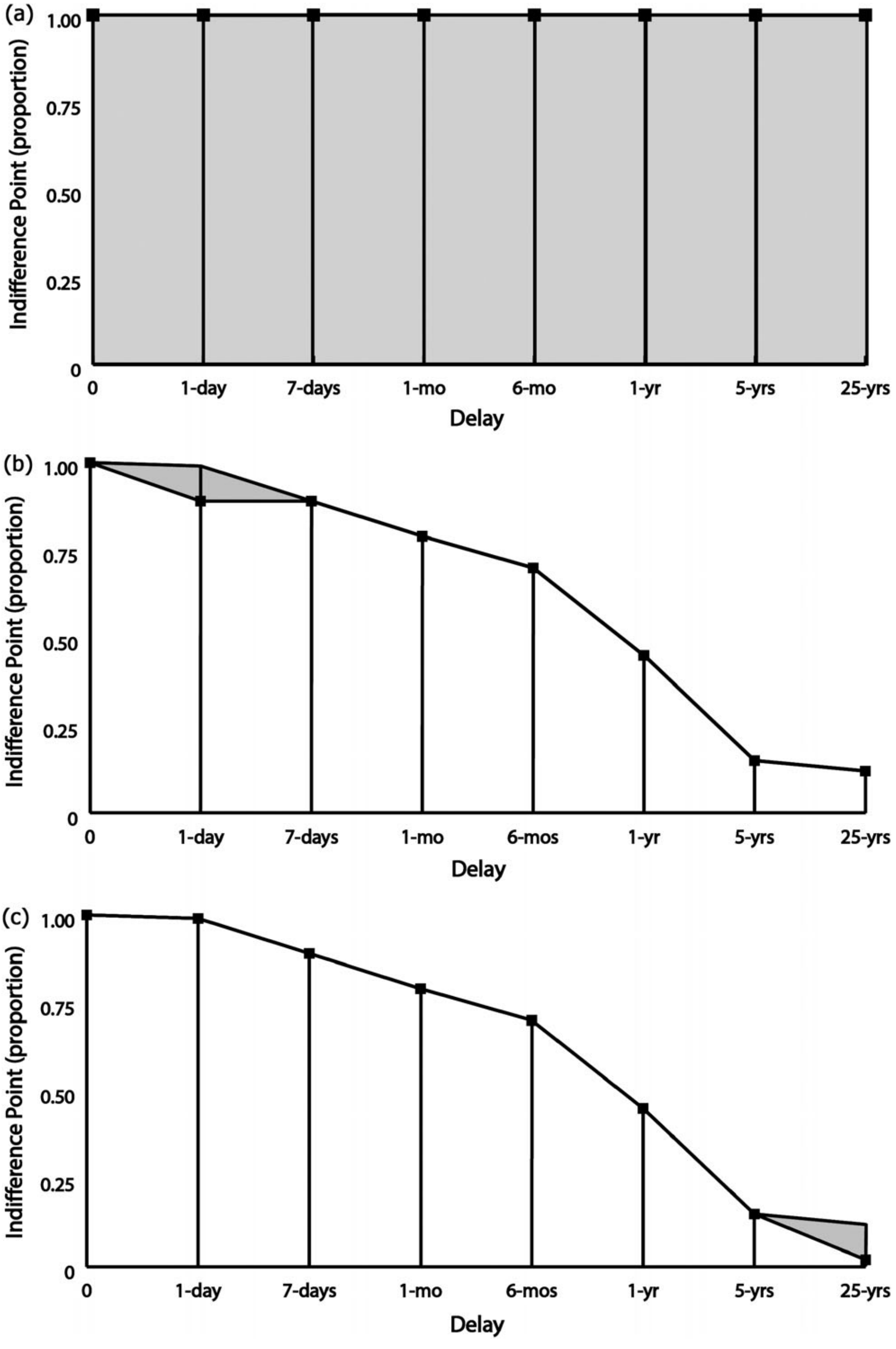

Fig. 3. Panel 3a demonstrates the contribution of each indifference point to AUC_{ord}, with shading reflecting the relative contribution of each division (Table 1). Panel 3b illustrates the change in AUC_{ord} (shaded area) following a 0.1 reduction in the indifference point at the 1-day delay. Panel 3c illustrates the change in AUC_{ord} (shaded area) following a 0.1 reduction in the indifference point at the 25-year delay.

may occur at long delays. We provide one counterexample of that here, examining the magnitude effect in a previously published dataset. Replicated in many studies of delay discounting, the well-established magnitude effect (Green, Myerson, & McFadden, 1997) is observed when small-magnitude rewards are discounted more (i.e., have higher discount rates / lower AUC) than large-magnitude rewards. To examine whether the proposed $\mathrm{AUC}_{\log d}$ and $\mathrm{AUC}_{\mathrm{ord}}$ also demonstrate the magnitude effect, both were calculated for two hypothetical money reward conditions ($50 and $1,000) using indifference points from the same previously published study (Matusiewicz et al., 2013). $\mathrm{AUC}_{\log d}$ for $50 and $1000 were 0.58 ($SD$ = 0.20) and 0.65 (SD = 0.23), respectively, and this difference was statistically significant ($t(27) = -3.12$, $p = .004$, $d = -0.624$). $\mathrm{AUC}_{\mathrm{ord}}$ for $50 and $1000 were 0.62 ($SD$ = 0.18) and 0.68 (SD = 0.22), respectively, and this difference was also statistically significant ($t(27) = -3.13$, $p = .004$, $d = -0.565$). This analysis is consistent with the examination of the magnitude effect in the original publication using hyperbolic k ($t(27) = 3.72$, $p = .001$, $d = 0.79$). In contrast, when conventional AUC was calculated using the same data, the mean difference between $50 ($M = 0.33$, $SD = 0.23$) and $1,000 ($M = 0.37$, $SD = 0.29$) magnitude conditions failed to reach statistical significance ($t(27) = -1.84$, $p = .077$, $d = -0.33$). The nonsignificant finding using AUC may again be the result of the nonequivalent contributions of each indifference point to the overall discounting estimate.

Replicability Using Alternative AUC Methods

To examine the performance of the newly proposed methods, $\mathrm{AUC}_{\log d}$ and $\mathrm{AUC}_{\mathrm{ord}}$, we attempted to replicate published results that initially employed model-based estimates of delay discounting. The first replication that was conducted involved data initially reported in Yi and Landes (2012). This study examined the effects of abstinence on discounting of hypothetical monetary gains for cigarette smokers. Delay discounting was calculated using the exponential-power model, and the analysis plan contrasted the effects of magnitude ($50 vs. $1000) and smoking abstinence (abstinent vs. nonabstinent). In the published results, there was a significant main effect of magnitude on discounting, such that individuals discounted small-magnitude ($50) gains more than large-magnitude ($1000) gains. There was also a significant main effect for abstinence on discounting, such that smokers discounted gains more when they were abstinent for 24 hours than when they were not abstinent.

Within-subject ANOVAs were conducted to examine the effects of abstinence status and magnitude on conventional and alternative AUC indices. Using the conventional AUC method, the significant difference as a function of abstinence was replicated (F[1, 26] =12.84, $p = .001$), but no significant difference was observed between magnitude conditions (F[1, 26] = 0.092, $p = .76$). In contrast, when using $\mathrm{AUC}_{\log d}$, both effects of abstinence (F[1, 26] = 13.70, $p = .001$) and magnitude (F[1, 26] = 8.66, $p = .007$) were replicated. Both effects of abstinence (F[1, 26] = 14.74, $p = .001$) and magnitude (F[1, 26] = 8.57, $p = .007$) were also replicated when using $\mathrm{AUC}_{\mathrm{ord}}$.

A second replication was conducted using data previously reported in Yi, Carter, and Landes (2012). This study examined differences in delay discounting of hypothetical monetary gains between methamphetamine using and non-using controls. Delay discounting of gains was calculated using the exponential-power model, and analysis examined the effect of magnitude ($50 vs. 1000) and group status (using vs. non-using). Statistically higher rates of delay discounting were reported for the $50 (compared to $1000) and using (compared to non-using) conditions. Similar analyses were conducted to examine these results using discounting estimates calculated via conventional AUC, $\mathrm{AUC}_{\log d}$, and $\mathrm{AUC}_{\mathrm{ord}}$, and results were fully replicated using all three AUC indices.

In summary, although the majority of conclusions resulting from hypothesis testing using the various indices of delay discounting were consistent, results using $\mathrm{AUC}_{\log d}$ and $\mathrm{AUC}_{\mathrm{ord}}$ were fully concordant with model-based (using regression) indices which equally weight all indifference points. In contrast, results using conventional AUC failed to replicate the common magnitude effect in two studies reanalyzed here (Matusiewicz et al., 2013; Yi & Landes, 2012).

Discussion

The Area-Under-the-Curve method was proposed by Myerson and colleagues (2001) as an alternative to model-based indices of delay discounting due to the lack of consensus on the form of the discounting function. Although AUC has substantial utility as an atheoretical index of delay discounting, the conventional approach to calculating AUC is problematic for the majority of human studies of delay discounting; specifically, indifference points from longer delays disproportionately contribute to the total AUC. To address this limitation, we propose two modifications to the conventional AUC method, $AUC_{\log d}$ and $AUC_{\mathrm{or}d}$, as small but significant variations to determining a model-free index of delay discounting.

In instances where intervals between delays increase as delays get longer (i.e. a pseudoexponential scaling of delays), we proposed a logarithmic transformation of delay in order to calculate $AUC_{\log d}$ and an ordinal transformation of delay in order to calculate $AUC_{\mathrm{or}d}$. For both of these methods, equality is enhanced relative to conventional AUC and well-aligned with indices of delay discounting where contributions of each indifference point are equal (e.g., hyperbolic k). Indeed, the proposed $AUC_{\log d}$ and $AUC_{\mathrm{or}d}$ maintain some of the preferred features of model-based indices of delay discounting that are calculated via nonlinear regression while remaining neutral in support of a specific model. In model-based indices of delay discounting, each delay point equally contributes to the estimated discounting rate. The proposed methods, as opposed to conventional AUC, retain this feature when estimating discounting by equally weighting each delay point and corresponding indifference point. Our reanalyses of published datasets indicate that the proposed modifications to the AUC method successfully replicated original results, while the conventional AUC method did not reliably replicate results. In three separate instances, the proposed methods of $AUC_{\log d}$ and $AUC_{\mathrm{or}d}$ yielded similar conclusions as model-based estimates of discounting (Matusiewicz et al., 2013; Yi, Carter, & Landes, 2012; Yi & Landes, 2012). In contrast, the conventional AUC method fully replicated the results of one of the empirical studies, but did not fully replicate results reported by Matusiewicz and colleagues (2013) nor Yi and Landes (2012). The sensitivity of the proposed methods may be the result of improved measurement of discounting from the equal weighting of indifference points. These results suggest that the proposed modifications to conventional AUC may improve the performance of this index of delay discounting, and that these modifications should be seriously considered when utilizing this method in place of a model-based estimate.

Although the results of the replication analyses indicate that the proposed methods are a better approximation of results obtained via a model-based method, one limitation should be noted. As previously discussed, the logarithmic transformation of delays in the $AUC_{\log d}$ method does not fully correct for nonequivalent contribution of delay points. Following the transformation there appears to be a difference in the relative contribution of delay points between the 1-year delay and the other delays, such that the indifference point corresponding to the 1-year delay contributes less to the total AUC than the 6-month delay point. This may be an important consideration for experimenters when determining which modification to employ.

The thesis of this paper is not to suggest that estimation of rate of delay discounting using conventional AUC methods is incorrect. Rather, we believe that $AUC_{\log d}$ and $AUC_{\mathrm{or}d}$ are alternative discounting indices that should be preferred in many human studies of delay discounting where procedures are scaled in a pseudoexponential manner or implicitly incorporate the subjective scaling of time, because (a) these analytic approaches are more consistent with the parameters of the delay discounting task, (b) better equate the relative impact of each indifference point to the overall index of delay discounting, (c), require a very simple scaling transformation, and (d) do not appear to result in a loss of statistical power.

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Received: February 11, 2016
Final Acceptance: July 20, 2016