Received: 9 June 2020 | Revised: 28 September 2020 | Accepted: 8 October 2020

DOI: 10.1002/wics.1538

ADVANCED REVIEW

WIREs COMPUTATIONAL STATISTICS

WILEY

On semiparametric regression in functional data analysis

Nengxiang Ling[1] | **Philippe Vieu**[2]

[1]School of Mathematics, Hefei University of Technology, Hefei, China

[2]Institut de Mathématiques, Université Paul Sabatier, Toulouse, France

Correspondence
Philippe Vieu, Institut de Mathématiques, Université Paul Sabatier, Toulouse, France.
Email: philippe.vieu@math.univ-toulouse.fr

Funding information
NNSF, Grant/Award Number: 72071068

Abstract

The aim of this paper is to provide a selected advanced review on semiparametric regression which is an emergent promising field of researches in functional data analysis. As a deliberate strategy, we decided to focus our discussion on the single functional index regression (SFIR) model in order to fix the ideas about the stakes linked with infinite dimensional problems and about the methodological challenges that one has to solve when building statistical procedure: one of the most challenging issue being the question of dimensionality effects reduction. This will be the first (and the main) part of this discussion and a complete survey of the literature on SFIR model will be presented. In a second attempt, other semiparametric models (and more generally, other dimension reduction models) will be shortly discussed with the double goal of presenting the state of art and of defining challenging tracks for the future. At the end, we will discuss how additive modeling is an appealing idea for more complicated models involving multifunctional predictors and some tracks for the future will be pointed in this setting.

This article is categorized under:

Statistical Models > Semiparametric Models

Data: Types and Structure > Time Series, Stochastic Processes, and Functional Data

Statistical Learning and Exploratory Methods of the Data Sciences > Modeling Methods

KEYWORDS

dimensionality reduction, functional data analysis, review, semiparametric modeling

1 | INTRODUCTION

In the last decades, in any field of applied sciences the technologies for recording and storing information had significantly improved leading to new challenges for the mathematicians and the statisticians for modeling and analyzing these new kinds of data. In particular, when the data are coming from continuous processes (i.e., when the data are of functional nature), this field of research is known as functional data analysis (FDA). FDA was 20 ago a very confidential area of researches and its popularization started really with Ramsay and Silverman's books (see Ramsay & Silverman, 1996, 2002, 2005). Twenty years later, many among standard multivariate statistical procedures have been adapted to functional data and there is now a wide literature on FDA as attested by the books (Aneiros, Bongiorno, Cao, & Vieu, 2017; Bongiorno, Goia, Salinelli, & Vieu, 2014; Bosq, 2000; Bosq & Blanke, 2007; Ferraty & Vieu, 2006; Horváth & Kokoszka, 2012; Hsing & Eubank, 2015; Shi & Choi, 2011; Zhang, 2013a), and by various bibliographical discussions such as Aneiros, Cao, and Vieu (2019), Aneiros, Cao, Fraiman, Genest, and Vieu (2019), Cuevas (2014), Geenens (2011), Goia and Vieu (2016), Horváth and Rice (2015), Jacques and Preda (2014), Ling and Vieu (2018), Müller and Hall (2016), Nagy (2017), Shang (2014a), and Vieu (2018).

A typical question arising in Statistics is the regression one, when the goal is to analyze the links between two variables, one is an explanatory variable and the other is a responses variable. When one (or both) among these variables are infinite dimensional, the problem is known as functional regression analysis and it can be modeled for instance as:

$$Y_i = m(\Delta_i) + \epsilon_i, \quad i = 1, 2, \ldots, n, \tag{1.1}$$

where $\{(\Delta_i, Y_i), 1 \le i \le n\}$ is a sample and Δ_i $(1 \le i \le n)$ takes values in some infinite dimensional space E. The recent literature provides a wide scope of applied scientific fields having to face with continuous data and for which such kind of functional regression models is useful. This contribution is deliberately based on a discussion on methodological advances and, concerning many possible general examples, we refer to Ullah and Finch (2013), Ferraty and Vieu (2002), or Aneiros, Cao, and Vieu (2019) for details. Even if in any practical problem the functional variables are only observed on a discrete grid, this grid becomes in reality more and more fine and this is the aim of the model (1.1) to deal with these data in a continuous way as they are in reality.

The analysis of this type of data leads to the statistical question of the estimation of the nonparametric functional operator $m(.)$.[1] To fix the idea let us stay with the simplest case of scalar response (that is when $(Y_1, ..., Y_n)$ are real variables) and when the statistical sample is composed of independent pairs $\{(\Delta_i, Y_i), 1 \le i \le n\}$. The literature is mainly composed of two kinds of approaches based on parametric/linear or nonparametric modeling ideas.

- *Linear approach*. The main idea of parametric modeling is to change the problem of estimating the functional operator $m(.)$ into a simpler problem of estimating some unknown element of E. For instance, the functional linear regression model consists in assuming that E is an Hilbert space endowed with some inner product $< ., . >$ and to assume that the operator $m(.)$ is linear and continuous. By this way, because of Riesz representation theorem, there exists some element (let say θ_1) in E such that $m(.) = < ., \theta_1 >$, in such a way that the regression model becomes

$$Y_i = \langle \Delta_i, \theta_1 \rangle + \epsilon_i, \quad i = 1, 2, \ldots, n. \tag{1.2}$$

From one hand this kind of model presents two nice features: from an exploratory point of view it provides an interpretable output θ_1 and from an explanatory point of view the target θ_1 is easier to estimate. However, in the other hand this kind of model suffers from the important drawback of being too much flexible and hence not so much trustable in many situations. This model has been widely studied in last years and it is out of the scope of this paper to discuss this wide literature (the reader may find a recent theoretically oriented overview in Chapter 11 of Hsing & Eubank, 2015, while practical aspects are discussed in the recent paper by Febrero, Galeano, & Gonzalez Manteiga, 2017).

- *Nonparametric approach*. To face this lack of flexibility of parametric approaches, a natural approach consists in relaxing the linear hypothesis to simple smooth condition. One of the most common models consists in assume that the functional operator $m(.)$ satisfies one kind of Hölder condition of the form:

$$\exists \beta > 0, \forall (x, y) \in E \times E, \ | m(x) - m(y) | \le C d(x, y)^{\beta}, \tag{1.3}$$

$d(., .)$ being some measure of proximity acting on $E \times E$. Undoubtedly, this kind of model is much more trustable in practice than the linear model (1.2) and is an appealing way for solving the lack of flexibility depicted just before. However in counterpart this model loses the two nice features exhibited by parametric approaches: from an exploratory point of view the nonlinear operator $m(.)$ is hardly representable and interpretable, and from an explanatory point of view the target $m(.)$ is much more difficult to estimate with poor rates of convergence (see for instance Mas, 2012 and references therein). It is once again out of scope of our paper to discuss the literature on such nonparametric FDA approaches (the reader may find that in the main ideas in the precursor book; Ferraty & Vieu, 2006 and the most recent results are discussed in the surveys papers; Geenens, 2011; Ling & Vieu, 2018).

There is therefore an important challenge for developing intermediary models able to combine both the flexibility of nonparametric ones and the easiness of estimation and/or interpretation of the parametric ones. Such a necessary balance was already pointed before in multivariate regression leading to a wide literature (Härdle, Müller, Sperlich, & Werwatz, 2004; Horowitz, 2009; Sperlich, Härdle, & Aydinli, 2006), and a few recent advances have been developed for functional variable (see (Goia & Vieu, 2014 for discussion). The common feature of the all the works in this direction is to build models including unknown operators acting on spaces being of lower dimension than the functional space E. The aim of our paper is to present, through some necessarily selected bibliography, some discussion about the state of the art in this field in order to highlight a few open questions and tracks for future.

Our discussion is organized as follows. In a first attempt we consider the simple model involving only one functional predictor. Our purpose will be mainly centered around semiparametric models, and more specifically around the single functional index modeling (see Section 2) which is the most commonly studied and which allows to present in a simple and easily understandable all the ins and outs of the problem, while other semiparametric functional regression models will be discussed in Section 3. Then we extend our purpose to the general situation when more than one predictor is used in the regression relationship, and we will present in Section 4 other dimensionality reduction models that will be mainly based on additive modeling. Section 5 aims to promote further advances in the field by presenting our points of view on what will be the main open questions and the main challenges along the next future in FDA.

2 | SINGLE FUNCTIONAL INDEX REGRESSION

2.1 | The SFIR model

The single index regression model is a kind of important semiparametric model, and it assumes that the explanatory variable acts on the responses only through its projection onto some one-dimensional subspace. This idea was presented by Härdle et al. (2004) in the standard unfunctional multivariate case. The same procedure can be followed in the functional setting, leading to the single functional index regression (SFIR) model introduced in Ferraty, Peuch, and Vieu (2003), which is a typical example of how semiparametric can be used in FDA for balancing the trade-off between too few flexibility (linear models) and dimensionality sensitivity (nonparametric models). We can also refer to Ferraty and Romain (2011) for more details on this model. Here, let us assume that E is an Hilbert space endowed with some inner product $\langle ., . \rangle$, and suppose that the regression model can be written as

$$Y_i = m(\Delta_i) + \epsilon_i = g(\langle \Delta_i, \theta_1 \rangle) + \epsilon_i, \quad i = 1, 2, \ldots, n, \tag{2.1}$$

while g and θ_1 are unknown, and θ_1 is simply such that $\theta_1 \in \mathscr{E}$ and $\langle \theta_1, e_1 \rangle = 1$, where e_1 is the first element of an orthonormal basis of $\mathscr{E}$. From one hand, this model can be seen as an extension on the linear model (1.2) but with increased flexibility because of the smooth component g. From an other hand, it can also be seen as a dimensionality reduced model with nonparametric component g acting on the one dimensional space $\mathbb{R}$ and hence being less sensitive to dimensionality effects as the nonparametric model (1.1)–(1.3) can be. For instance, if the space E is a standard curves space like $E = L^2([0, 1])$ then the model (2.1) involves only 2 one-dimensional functions g and θ_1, in opposition with the pure nonparametric model (1.1) which involves one infinite dimensional (nonlinear) operator $m(.)$.

2.2 | Some comments about identifiability

Clearly the model SFIR is not identifiable in the sense that there may have various pairs (g, θ) fulfilling the condition (2.1). Earlier study about identifiability was provided in Ferraty et al. (2003), and in particular the directional parameter θ_1 is defined up to a constant term. See also the early work by Amato, Antoniadis, and De Feis (2006) for a slightly similar presentation of the model. A nice way allowing both for checking identifiability and for interpreting the outputs of the model has been provided in Ferraty, Park, and Vieu (2011), is based on average derivatives ideas such as developed in multivariate analysis in Härdle and Stoker (1989) and can be summarized as follows. Assume that for $x \in E$ the functional operator $m(.)$ admits some derivative m_x:

$$m(x+\alpha u)\tilde{m}(x)+\alpha m_x(u) \quad \text{as } \alpha\to 0.$$

From one hand, because the functional derivative $m_x(.)$ is a continuous and linear operator, the standard Riesz representation theorem insures that there exists some element $\tilde{m}_x \in E$ such that

$$m_x(u)=\langle u,\tilde{m}_x\rangle.$$

In an other hand, under the SFIR assumption (2.1) one has that $m(x) = g(< x, \theta_1 >)$ and so one has

$$m_x(u)=\langle u,\theta g'(\langle x,\theta_1\rangle)\rangle.$$

By applying both previous results to a random element Δ and then taking the expectancy, one arrives at

$$E(\tilde{m}_\Delta)=\theta E(g'(\langle \Delta,\theta_1\rangle)),$$

which can be written again as

$$E(\tilde{m}_\Delta)=C\theta_1, \quad \text{for some } C\in\mathbb{R}.$$

At the end, because the model (2.1) is identifiable only up to a constant, one can decide to take $C = 1$, and finally the SFIR model is defined by using the following identification assumption[2]:

$$\theta_1=E(\tilde{m}_\Delta). \tag{2.2}$$

2.3 | Some basic results

This short section is somewhat technical. It aims to recall how most of statistical techniques usually developed for estimating both the link function g and the functional directional parameter θ_1 is constructed. Roughly speaking, the link function is estimated first, either by direct approach (see (ii)) or by some minimization equation (see (iii)) and then the smooth link estimation is estimated by standard nonparametric techniques (see (iv)).

i. *Preliminary step*. To estimate g and θ in model (2.1) an usual starting point is to construct, for each $\theta \in E$, a nonparametric estimate of the function $g_\theta(x) = g(< x, \theta >)$ and the most natural way is to use kernel smoothers:

$$\hat{g}_\theta(x)=\frac{\sum_{i=1}^{n} Y_i W\left(h^{-1}\langle x-\Delta_i,\theta\rangle\right)}{\sum_{i=1}^{n} W\left(h^{-1}\langle x-\Delta_i,\theta\rangle\right)}, \tag{2.3}$$

where $W(.)$ is a kernel weight function and $h = h(n) > 0$ is a smoothing parameter. This approach has been developed in Ferraty et al. (2003) and the rate of convergence has been stated to be the same as if Δ was a one-dimensional real variable, showing that SFIR model achieves its goal of being insensitive to dimensionality effects.

ii. *Estimation of the functional index*. Once the smooth component is estimated, it remains to deal with the directional parameter θ. Two approaches have been developed for that purpose.

- *Direct approach*. The idea is to use the expression (2.2) and to propose an estimator having the form

$$\hat{\theta}_1 = \frac{1}{n}\sum_{i=1}^{n} \hat{\tilde{m}}_{\Delta_i}. \tag{2.4}$$

For estimating the derivative $\tilde{m}$, one decomposes it into an orthonormal basis Ψ_j of the functional space E:

$$\tilde{m}_x = \sum_{j=0}^{\infty} a_{x,j}\Psi_j$$

Estimate of the coefficients $a_{x,j}$ is an easier task (see for instance Hall, Müller, & Yao, 2009 for the construction of some kernel estimate $\hat{a}_{x,j}$) and finally one can estimate the derivative $\tilde{m}$ by defining (for some integer $k = k_n > 0$):

$$\hat{\tilde{m}}_x = \sum_{j=0}^{k} \hat{a}_{x,j}\Psi_j. \tag{2.5}$$

This approach was developed in Ferraty et al. (2011) and the directional estimate defined by Equations (2.4) and (2.5) is consistent.

- *Cross-validation approach*. The cross-validation idea consists in not estimating directly the parameter θ_1 but in choosing the one leading to the best estimation of the target g_θ. The errors can be for instance measured by

$$\mathrm{ASE}(\theta) = \sum_{i=1}^{n}\left(g_{\theta_1}(\Delta_i) - \hat{g}_\theta(\Delta_i)\right)^2,$$

  but the optimal direction $\theta_0 = \mathrm{argmin}\ \mathrm{ASE}(\theta)$ is uncomputable in practice. Cross-validation intents to overpass this difficulty by proposing a data-driven criterion

$$CV(\theta) = \sum_{i=1}^{n}\left(g_{\theta_1}(\Delta_i) - \hat{g}_\theta^{-i}(\Delta_i)\right)^2,$$

  where $\hat{g}_\theta^{-i}$ is the same as $\hat{g}_\theta$ but based on the leave-one-out sample $\{(\Delta_j, Y_j), j \neq i\}$. This approach was proposed in Ait-Saïdi, Ferraty, Kassa, and Vieu (2008) and it is shown how the data-driven selected direction

$$\tilde{\theta}_1 = \mathrm{argmin} CV(\theta)$$

  is asymptotically equivalent to the optimal one θ_0, being therefore a consistent estimate of the true directional parameter θ_1 of the model.

- *Estimation of the smooth component*. Once the directional parameter is estimated, it suffices to plug this estimate $\hat{\theta}$ ($\hat{\theta}$ being either $\hat{\theta}_1$ or $\tilde{\theta}_1$) into the definition (2.3) to construct a fully automatic estimate $\hat{m} = \hat{g}_{\hat{\theta}}$ of the regression operator in the model (2.1). Consistency properties of these estimate have been provided in Ferraty et al. (2011) when the estimated direction $\hat{\theta}_1$ is used and in Ferraty, Goia, Salinelli, and Vieu (2013) when the estimated direction $\tilde{\theta}_1$ is used. In both cases it is shown that the rates of convergence are the same as if Δ was a standard one-dimensional real random variable, showing that the functional single index model reaches its main goal of being insensitive to the dimensionality of the problem.

2.4 | Survey on SFIR model

The results described just before have been extended in many directions, with main objective to show in each case that the one-dimensional rate can be obtained. Here is a fast review of the literature in this sense.

- *Alternative estimates*. While most of the literature on the SFIR model is concerning kernel smoothers (see discussion before) some alternative techniques have been developed. For instance B-splines ideas have been used in Ma (2016), and a mixed procedure using both kernel and splines has been proposed in Jiang, Ma, and Wang (2015). Also, k nearest neighbors smoothers have been studied in Nagy (2017).
- *On testing SFIR model*. The problem of developing statistical test procedures for assessing the validity of the SFIR model has been essentially investigated in the paper by Delsol and Goia (2020).
- *Case of non-independent samples*. The SFIR model has been also studied for dependent data with direct applications in time series analysis. In this framework, Ait-Saïdi, Ferraty, and Kassa (2005) obtained the almost complete convergence rates of regression operator in SFIR model under the case of some strong mixing structure on the sample.
- *Estimation of other operators than regression*. The SFIR model has been also used for estimating alternative functional operators such as the conditional density (see Attaoui, 2014; Ling, Li, & Yang, 2014; Ling & Xu, 2012), the conditional quantiles (see Bouchentouf, Mekkaoui, & Rabhi, 2015; Kadiri, Mekkaoui, & Rabhi, 2017), the conditional cumulative distribution (see Ait-Saïdi & Kheira, 2016; Attaoui & Ling, 2016) or the conditional hazard function (see Rabhi, Belkhir, & Soltani, 2016).
- *Case of functional response*. Versions of the SFIR model have been adapted to the situation when the response Y is also functional (see Jiang & Wang, 2011; Li, Huang, & Zhu, 2017; Luo, Zhu, & Zhu, 2016).
- *Miscellaneous*. A dynamic version of the SFIR model was introduced in Ma, Bai, and Zhu (2016), while a partitioned version of the model has been studied in Goia and Vieu (2015).
- *About applications*. Even if the scope of this discussion is restricted to theoretical issues, it is worth being stressed that most of papers presented before include real data analysis. One of the main applied appealing feature of the SFIR model is the fact that the outputs of the model (namely the link function g and the directional parameter θ_1) are simple mathematical objects (while the regression operator $m(.)$ does not) which are easy to represent and to interpret. The reader may find, for instance, a nice real data analysis highlighting this phenomenon in Novo, Aneiros, and Vieu (2019).

3 | OTHER SEMIPARAMETRIC MODELS FOR DIMENSION REDUCTION

A few other semiparametric ideas have been passed recently from the multivariate setting to FDA. This is what we will discuss now, by starting along Section 3.1 with those among the functional semiparametric models which can be seen as more or less direct extensions of the above discussed SFIR model and then by presenting a general way for semiparametric dimension reduction modeling (see Section 3.2).

3.1 | Extending the SFIR model

The SFIR model has been extended in several directions and this is what we will shortly discuss below.

- *Multiple index functional regression*. The first extension consists in assuming that more than one functional direction can be of interest for the response Y. This idea has been developed by Bouraine, Aït-Saidi, Ferraty, and Vieu (2010) through the following multiple index functional regression model:

$$Y = g\left(\langle\theta_1, \Delta\rangle, \ldots, \langle\theta_p, \Delta\rangle\right) + \varepsilon. \tag{3.1}$$

This model is more flexible than the SFIR one but it is more sensitive to dimensionality effects because the link function g is now a p-dimensional one.

- *Functional projection pursuit.* A nice compromise between models (2.1) and (3.1) is the following projection pursuit model introduced simultaneously in Chen, Hall, and Müller (2011) or Ferraty et al. (2013):

$$Y = g_1(\langle\theta_1,\Delta\rangle) + \cdots + g_p(\langle\theta_p,\Delta\rangle) + \varepsilon. \tag{3.2}$$

The papers by Chen et al. (2011) or Ferraty et al. (2013) show that, each link function g_j ($j = 1, 2, ..., p$) can be estimated to the one-dimensional rate, making this model fully insensitive to dimensionality effects.

3.2 | A general formulation for dimension reduction model

As discussed for instance in Wang, Zhou, Wu, and Chen (2017), a general way for building a dimension reduction model in FDA consists in assuming that the distribution of Y given Δ depends only on some linear combinations of Δ. In the regression framework this turns to assume the following model

$$Y = g(\langle\theta_1,\Delta\rangle, ..., \langle\theta_p,\Delta\rangle, \varepsilon). \tag{3.3}$$

All the models discussed before, namely models (2.1), (3.1), and (3.2) are special cases of model (3.3).

There exists a general way for estimating the parameters in model (3.3) which consists in introducing some specific conditions on the probability distribution of (Y, Δ) in such a way that $E(\Delta|Y)$ can be expressed by a finite number of elements included in a subspace. The main point is that, because Y is a real random variable, the form $E(\Delta|Y)$ is a one-dimensional function that can therefore be estimated at the usual one-dimensional rate. This idea is called sliced inverse functional regression and was first developed by Ferré and Yao (2003, 2005), and revisited in Cook, Forzani, and Yao (2010), Forzani and Cook (2007), Li and Hsing (2010), and Wang et al. (2017). Let us conclude this short discussion by mentioning that nonlinear versions of the model (3.3) have been recently developed (see Li & Hsing, 2010; Li & Song, 2017; Lian & Li, 2014).

4 | DIMENSION REDUCTION FOR MULTIFUNCTIONAL COVARIATE

In a natural way, the dimensional reduction ideas have be developed in the more complex situation when the functional covariate Δ^i is in fact composed by more than one variable

$$\boldsymbol{\Delta} = (\Delta^1, ..., \Delta^p).$$

Saying that, keep in mind that for some k some among the Δ^j are functional but some other ones may possibly be standard real random variables. Let $k \le p$ and assume that

$$\Delta^j \text{ are functional for } j \le k,$$

$$\Delta^j \text{ are scalar for } j > k.$$

Basically, additive modeling assumes that the covariate acts on the response Y in some additive way. We will review below some models constructed along these lines (let us mention that additive ideas have also been developed following different lines but still for regression analysis in multifunctional purpose in Han, Müller, and Park (2018).

- *Nonparametric additive functional regression*. The additive ideas can be also used in a fully nonparametric way, for instance by means of the model

$$Y = m_1\left(\Delta^1\right) + \cdots + m_p\left(\Delta^p\right) + \varepsilon, \tag{4.1}$$

in which the unknown parts $m_j(.)$ (j = 1, 2, ..., p) are still nonlinear operators but acting a lower dimensional space than the non-additive one $m(.)$ defined in model (1.1). As illustrated by the asymptotic provided in Ferraty and Vieu (2009), this model is less sensitive to dimensionality effects than the nonparametric model (1.1), but it cannot achieve the one-dimensional rate of convergence since the targets $m_j(.)$ are not one-dimensional objects as could be the $g_j(.)$ in model (4.2). Further advances on this model can be found in Fan, James, and Radchenko (2015).

- *Multifunctional linear regression*. At the opposite of the pure additive multi-functional nonparametric model (4.1) is the naive additive linear model which is constructed by assuming that each component in (4.1) the same linear form as in (1.2). Most recent advances for this model can be found in Aneiros and Vieu (2016b), including asymptotic normality and confidence prediction intervals for the case when Y can also be functional. Of course this linear approach suffers from the same drawbacks as for the case of a single functional covariate (see discussion before), and there is again the need for developing new models being intermediary between linear and nonparametric ones.
- *A multiple index type model*. As being directly inspired from the single index and projection pursuit models discussed before, the following model

$$Y = g_1\left(\left\langle\theta_1, \Delta^1\right\rangle\right) + \cdots + g_p\left(\left\langle\theta_p, \Delta^p\right\rangle\right) + \varepsilon, \tag{4.2}$$

has been studied in Aneiros and Vieu (2016b). Statistical inference developed in this paper shows that, without surprise, any component $g_j(.)$ can be estimated with the one-dimensional rate of convergence.

- *Functional partial linear regression*. Another way for building semiparametric functional regression models consists in assuming that in the additive decomposition (4.1), some components are linear and some other ones are not. There are many models of this kind, differing by the fact that the linear part of the model acts either on the scalar variables, or on the functional ones.
- When the scalar variables Δ^j, $j > k$ are linearly modeled and the functional ones Δ^j, $j \leq k$ are nonparametrically modeled, the model was proposed in Aneiros and Vieu (2006) in which asymptotic properties are proved when kernel estimates are used for the nonparametric functional element. These results for kernel estimates have been extended in various directions, including time series framework (Aneiros & Vieu, 2008), smoothing parameter selection (Aneiros & Vieu, 2011), testing procedure construction (Aneiros & Vieu, 2013), variance error estimation (Aneiros, Ling, & Vieu, 2015), and so on. The ideas in Aneiros and Vieu (2006) have been also extended to other kind of estimates, including local linear approaches (Feng & Xue, 2016) and robust procedures (Boente & Vahnovan, 2017). See also Zhang (2013b).
- When the scalar variables Δ^j, $j > k$ are nonparametrically modeled and the functional ones Δ^j, $j \leq k$ are linearly modeled, the paper by Zhou and Chen (2012) is stating asymptotics for spline based estimates while the one by Huang, Wang, Cui, and Wang (2015) develops similar study for M-estimates.
- Even in the situation when only functional variables are observed (that is when $k = p$) one can also use partial linear ideas by assuming that some among the component $m_j(.)$ in the model (4.1) are linear and some other are nonparametric. Asymptotics for this model are given in Lian (2011).
- Partial linear ideas have been adapted along a few other lines in FDA. For instance, a general formulation of the partial linear functional model allowing to treat as well regression as median or quantile is proposed in Qingguo (2015). Also worth being mentioned is the contribution by Maity and Huang (2012) in which a similar model is studied with the specificity of using the functional covariates for stratifying. Finally, a mixture of partial linear and single index

ideas is developed in Ding, Liu, Xu, and Zhang (2017), Wang, Feng, and Chen (2016), Yang, Lin, and Lian (2019), and Yu, Du, and Zhang (2020).

- *By-product: partial linear model for single functional covariate*. As discussed before, partial linear models are introduced for multifunctional problem, but they may have direct impact on single functional problem. For instance, if one has a simple regression problem as in (1.1) with one functional predictor Δ, one can decide to put:

$$k=1,\ \Delta^1=\Delta \text{ and } \Delta^j=\Delta\left(t^j\right) \text{ for } j>1,$$

where the t^j are the grid on which the functional Δ is observed. In this situation, both the continuous feature of the predictor Δ and its (scalar) discretized values Δ^j can be used in the model for predicting the response Y. The specific question to be emphasized here concerns the fact that, in general, the grid t^j is rather fine leading to a problem with a very large number p of predictors. To overpass these difficulties, sparse[3] versions of the additive models can be developed (see Aneiros & Vieu, 2015 for partial linear type models and Aneiros & Vieu, 2016a for nonparametric additive model).

- *Case of functional predictor*. For sake of simplicity our discussion focused on situation when the response variable is scalar. Note that some advances exist when both the response and the covariate are functional, such as the functional additive model by Müller and Yao (2008) in which ideas similar to the single functional index model are developed with an estimation of directional parameters based on the Eigen decomposition of the covariance operator.

5 | CONCLUDINGS AND TRACKS FOR FUTURE

The first conclusion that one can draw from this short selected survey discussion is that semiparametric FDA is a rather young area of research but having already received increasing attention in the last 10 years. Clearly, the state of art is still underdeveloped (compared for instance with what it is in nonparametric FDA, see Ling & Vieu, 2018) but the intensive recent activities may expect that this will become in the next future a major track in the field. In particular, the nice properties of being few sensitive to dimensionality effects (as discussed along this survey) make semiparametrics appealing ideas for FDA, and our wish is that this short survey can promote further advances in the area. To help in this sense, we would like to conclude that contribution by presenting some among the most important open questions.

Among the specific questions remaining to be emphasized in the short future, we wish to stress on the three following challenging points.

- First of all, this survey shows that almost all the literature on semiparametric FDA is based on kernel estimates while one can reasonably think in developing similar ideas for other statistical techniques. What about, for instance, the behavior of location adaptive techniques (such as kNN) whose efficiency is highly acknowledged in nonparametric FDA (see Kara-Zaitri, Laksaci, Rachdi, & Vieu, 2017 and references therein). Undoubtedly this is a wide scope field, since kNN ideas may be expected to be of interest for any among the operatorial problems discussed before (regression, conditional distribution, etc.) and for any among the semiparametric models presented before (single index, projection pursuit, etc.).
- A second question concerns the needs for developing fully automatically data-driven semiparametric methods. While in nonparametrics there are now at hand various approaches to this question, this is far to be the case in semi-parametric FDA (see however first advances in Novo et al., 2019). What about the optimality of data-driven parameter selection rules such as cross-validation methods (Rachdi & Vieu, 2007) or Bayesian ones (Shang, 2014b), or adaptative ones (Chagny & Roche, 2014, 2016). Once again, this is a wide scope field being of interest for many operator problem (regression, conditional distribution, etc.) and for many semiparametric models index, projection pursuit, and so on.
- A further natural question when developing various statistical approaches is the one of having at hand some statistical procedure for choosing the most accurate model. Testing procedures in this sense would be of great help but are still underdeveloped. Ideas as in Delsol (2013), Delsol, Ferraty, and Vieu (2011), or Zhu, Zhang, Yu, Lian, and Liu (2019) could be of interest for that purpose.

- Finally it is worth being mentioned that there is also a rich literature on Bayesian FDA which has received more attention in the last decade as well on parametric, nonparametric or semiparametric ways (see Scarpa & Dunson, 2009, 2014; Jeong & Park, 2016; Lee et al., 2019 for many details and the review paper in the general survey by Morris, 2015 including some additional examples).

In addition to these specific short-future views, it should be stressed that semiparametrics are really crossing two fields of modern statistics, namely FDA and big data analysis. While these two areas followed in the last 20 years rather separated ways, the developments of bridges between them have been pointed as in important necessity as well by the FDA community (see the discussion in Goia & Vieu, 2016) as by the big data community (see for instance Ahmed, 2017; Sangalli, 2018). Crossing these two fields will probably become at the center of many further advances in FDA, and semiparametrics is one among these bridges (as could be sparse functional modeling as discussed in Aneiros & Vieu, 2015, 2016a) and references therein).

ACKNOWLEDGMENTS

The authors would like to thank both the anonymous referees and the Editor in Chief for their constructive suggestions that have led to improve the presentation of paper substantially. This work is supported by the NNSF of China (72071068).

CONFLICT OF INTEREST

The authors have declared no conflicts of interest for this article.

AUTHOR CONTRIBUTIONS

Philippe Vieu: Investigation. **Nengxiang Ling:** Investigation.

ORCID

Philippe Vieu https://orcid.org/0000-0002-1379-0588

ENDNOTES

[1] At this stage, as pointed for instance in Ferraty and Vieu (2006) and Hsing and Eubank (2015), it is worth being emphasized on the fact that in the FDA community the statisticians are using the wording “functional operator” but this does not mean necessarily that the operator is “linear,” as it could be in other mathematical areas such as Functional Analysis.

[2] In the special case when $E = \mathbb{R}^p$ner product, then (2.2) is exactly what was stated in Härdle and Stoker (1989).

[3] See Aneiros and Vieu (2016b) for a wide discussion on sparsity in functional data setting.

RELATED WIREs ARTICLE

High-dimensional covariance estimation for Gaussian directed acyclic graph models with given order

FURTHER READING

Aneiros, G., Raña, P., Vieu, P., & Vilar, J. (2018). Bootstrap in semi-functional partial linear regression under dependence. *Test*, *27*(3), 659–679.

REFERENCES

Ahmed, S. E. (2017). Big and complex data analysis. Methodologies and applications. In *Contributions to statistics*. New York: Springer.

Ait-Saïdi, A., Ferraty, F., & Kassa, R. (2005). Single functional index model for a time series. *Revue Roumaine de Mathématiques Pures et Appliquées*, *50*(4), 321–330.

Ait-Saïdi, A., Ferraty, F., Kassa, R., & Vieu, P. (2008). Cross-validated estimations in the single-functional index model. *Statistics*, *42*(6), 475–494.

Ait-Saïdi, A., & Kheira, M. (2016). The conditional cumulative distribution function in single functional index model. *Communications in Statistics: Theory and Methods*, *45*(16), 4896–4911.

Amato, U., Antoniadis, A., & De Feis, A. (2006). Dimension reduction in functional regression with applications. *Computational Statistics & Data Analysis*, *50*, 2422–2446.

Aneiros, G., Bongiorno, E., Cao, R., & Vieu, P. (2017). An introduction to the 4th edition of the international workshop on functional and operatorial statistics. In *Functional statistics and related fields* Contributions to statistics (pp. 1–5). New York: Springer.

Aneiros, G., Cao, R., Fraiman, R., Genest, C., & Vieu, P. (2019). Recent advances in functional data analysis and high-dimensional statistics. *Journal of Multivariate Analysis*, *170*, 3–9.

Aneiros, G., Cao, R., & Vieu, P. (2019). Editorial on the special issue on functional data analysis and related topics. *Computational Statistics*, *34*(2), 447–450.

Aneiros, G., Ling, N., & Vieu, P. (2015). Error variance estimation in semi-functional partially linear regression models. *Journal of Nonparametric Statistics*, *27*(3), 316–330.

Aneiros, G., & Vieu, P. (2006). Semi-functional partial linear regression. *Statistics & Probability Letters*, *76*(11), 1102–1110.

Aneiros, G., & Vieu, P. (2008). Nonparametric time series prediction: A semi-functional partial linear modeling. *Journal of Multivariate Analysis*, *99*(5), 834–857.

Aneiros, G., & Vieu, P. (2011). Automatic estimation procedure in partial linear model with functional data. *Statistical Papers*, *52*(4), 751–771.

Aneiros, G., & Vieu, P. (2013). Testing linearity in semi-parametric functional data analysis. *Computational Statistics*, *28*(2), 413–434.

Aneiros, G., & Vieu, P. (2015). Partial linear modelling with multi-functional covariates. *Computational Statistics*, *30*(3), 647–671.

Aneiros, G., & Vieu, P. (2016a). Sparse nonparametric model for regression with functional covariate. *Journal of Nonparametric Statistics*, *28*(4), 839–859.

Aneiros, G., & Vieu, P. (2016b). Comments on: Probability enhanced effective dimension reduction for classifying sparse functional data. *Test*, *25*(1), 27–32.

Attaoui, S. (2014). On the nonparametric conditional density and mode estimates in the single functional index model with strongly mixing data. *Sankhya A*, *76*(2), 356–378.

Attaoui, S., & Ling, N. (2016). Asymptotic results of a nonparametric conditional cumulative distribution estimator in the single functional index modeling for time series data with applications. *Metrika*, *79*(5), 485–511.

Boente, G., & Vahnovan, A. (2017). Robust estimators in semi-functional partial linear regression models. *Journal of Multivariate Analysis*, *154*, 59–84.

Bongiorno, E., Goia, A., Salinelli, E., & Vieu, P. (2014). An overview of IWFOS'2014. In *Contributions in infinite-dimensional statistics and related topics* (pp. 1–5). Bologna: Società Editrice Esculapio.

Bosq, D. (2000). Linear processes in function spaces. Theory and applications. In *Lecture notes in statistics* (Vol. 149). New York: Springer-Verlag.

Bosq, D., & Blanke, D. (2007). Inference and prediction in large dimension. In *Wiley series in probability and statistics*. Chichester: John Wiley & Sons.

Bouchentouf, A., Mekkaoui, S., & Rabhi, A. (2015). Strong uniform consistency rates of conditional quantiles for time series data in the single functional index model. *New Trends in Mathematical Sciences*, *3*(2), 181–198.

Bouraine, M., Aït-Saidi, A., Ferraty, F., & Vieu, P. (2010). Choix optimal de l'indice multi-fonctionnel: Méthode de validation croisée (in French). *Revue Roumaine des Mathematiques Pures et Appliquees*, *5*(5), 355–367.

Chagny, G., & Roche, A. (2014). Adaptive and minimax estimation of the cumulative distribution function given a functional covariate. *Electronic Journal of Statistics*, *8*(2), 2352–2404.

Chagny, G., & Roche, A. (2016). Adaptive estimation in the functional nonparametric regression model. *Journal of Multivariate Analysis*, *146*, 105–118.

Chen, D., Hall, P., & Müller, H. (2011). Single and multiple index functional regression models with nonparametric link. *Annals of Statistics*, *39*, 1720–1747.

Cook, R., Forzani, L., & Yao, A.-F. (2010). Necessary and sufficient conditions for consistency of a method for smoothed functional inverse regression. *Statistica Sinica*, *20*(1), 235–238.

Cuevas, A. (2014). A partial overview of the theory of statistics with functional data. *Journal of Statistical Planning and Inference*, *147*, 1–23.

Delsol, L. (2013). No effect tests in regression on functional variable and some applications to spectrometric studies. *Computational Statistics*, *28*(4), 1775–1811.

Delsol, L., Ferraty, F., & Vieu, P. (2011). Structural test in regression on functional variables. *Journal of Multivariate Analysis*, *102*(3), 422–447.

Delsol, L., & Goia, A. (2020). Testing a specification form in single functional index model. In G. Aneiros, I. Horová, M. Hušková, & P. Vieu (Eds.), *Functional and high-dimensional statistics and related fields* (pp. 61–67). New York: Springer.

Ding, H., Liu, Y., Xu, W., & Zhang, R. (2017). A class of functional partially linear single-index models. *Journal of Multivariate Analysis*, *161*, 68–82.

Fan, Y., James, G., & Radchenko, P. (2015). Functional additive regression. *Annals of Statistics*, *43*(5), 2296–2325.

Febrero, M., Galeano, P., & Gonzalez Manteiga, W. (2017). Functional principal component regression and functional partial least-squares regression: An overview and a comparative study. *International Statistical Review*, *85*, 61–83.

Feng, S., & Xue, L. (2016). Partially functional linear varying coefficient model. *Statistics*, *50*(4), 717–732.

Ferraty, F., Goia, A., Salinelli, E., & Vieu, P. (2013). Functional projection pursuit regression. *Test*, *22*(2), 293–320.

Ferraty, F., Park, J., & Vieu, P. (2011). Estimation of a functional single index model. In *Recent advances in functional data analysis and related topics* Contribution to statistics (pp. 11–116). Heidelberg: Physica-Verlag/Springer, Heidelberg.

Ferraty, F., Peuch, A., & Vieu, P. (2003). Modèle à indice fonctionnel simple. (French). *Comptes Rendus de l'Académie des Sciences*, *336*(12), 1025–1028.

Ferraty, F., & Romain, Y. (2011). *The oxford handbook of functional data analysis*, Oxford: Oxford University Press.

Ferraty, F., & Vieu, P. (2002). The functional nonparametric model and application to spectrometric data. *Computational Statistics*, *17*, 545–564.

Ferraty, F., & Vieu, P. (2006). *Nonparametric functional data analysis. Theory and practice*. New York: Springer-Verlag.

Ferraty, F., & Vieu, P. (2009). Additive prediction and boosting for functional data. *Computational Statistics & Data Analysis*, *53*(4), 1400–1413.

Ferré, L., & Yao, A.-F. (2003). Functional sliced inverse regression analysis. *Statistics*, *37*(6), 475–488.

Ferré, L., & Yao, A.-F. (2005). Smoothed functional inverse regression. *Statistica Sinica*, *15*(3), 665–683.

Forzani, L., & Cook, R. (2007). A note on smoothed functional inverse regression (with discussions). *Statistica Sinica*, *17*(4), 1677–1681.

Geenens, G. (2011). Curse of dimensionality and related issues in nonparametric functional regression. *Statistics Surveys*, *5*, 30–43.

Goia, A., & Vieu, P. (2014). Some advances on semi-parametric functional data modelling. In *Contributions in infinite-dimensional statistics and related topics* (pp. 135–140). Bologna: Esculapio.

Goia, A., & Vieu, P. (2015). A partitioned single functional index model. *Computational statistics*, *30*(3), 673–692.

Goia, A., & Vieu, P. (2016). An introduction to recent advances in high/infinite dimensional statistics. *Journal of Multivariate Analysis*, *146*, 1–6.

Hall, P., Müller, H., & Yao, F. (2009). Estimation of functional derivatives. *The Annals of Statistics*, *37*, 3307–3329.

Han, K., Müller, H., & Park, B. (2018). Smooth backfitting for additive modeling with small errors-in-variables, with an application to additive functional regression for multiple predictor functions. *Bernoulli*, *24*(2), 1233–1265.

Härdle, W., Müller, M., Sperlich, S., & Werwatz, A. (2004). Nonparametric and semiparametric models. In *Springer series in statistics*. New York: Springer-Verlag.

Härdle, W., & Stoker, T. (1989). Investigating smooth multiple regression by the method of average derivatives. *Journal of the American Statistical Association*, *84*(408), 986–995.

Horowitz, J. (2009). Semiparametric and nonparametric methods in econometrics. In *Springer series in statistics*. New York: Springer.

Horváth, L., & Kokoszka, P. (2012). Inference for functional data with applications. In *Springer series in statistics*. New-York: Springer.

Horváth, L., & Rice, G. (2015). An introduction to functional data analysis and a principal component approach for testing the equality of mean curves. *Revista Matemática Complutense*, *28*(3), 505–548.

Hsing, T., & Eubank, R. (2015). Theoretical foundations to functional data analysis with an introduction to linear operators. In *Wiley series in probability and statistics*. Chichester: John Wiley & Sons.

Huang, L., Wang, H., Cui, H., & Wang, S. (2015). Sieve M-estimator for a semi-functional linear model. *Science China. Mathematics*, *58*(11), 2421–2434.

Jacques, J., & Preda, C. (2014). Functional data clustering: A survey. *Advances in Data Analysis and Classification*, *8*(3), 231–255.

Jeong, S., & Park, T. (2016). Bayesian Semiparametric inference on functional relationships in linear mixed models. *Bayesian Analysis*, *11*(4), 1137–1163.

Jiang, C. R., & Wang, J. L. (2011). Functional single index models for longitudinal data. *Annals of Statistics*, *39*(1), 362–388.

Jiang, F., Ma, Y., & Wang, Y. (2015). Fused kernel-spline smoothing for repeatedly measured outcomes in a generalized partially linear model with functional single index. *Annals of Statistics*, *43*(5), 1929–1958.

Kadiri, N., Mekkaoui, S., & Rabhi, A. (2017). Rate of uniform consistency for nonparametric of the conditional quantile estimate with functional variables in the single functional index model. *International Journal of Mathematics and Statistics*, *18*(2), 1–29.

Kara-Zaitri, L., Laksaci, A., Rachdi, M., & Vieu, P. (2017). Data-driven kNN estimation in nonparametric functional data-analysis. *Journal of Multivariate Analysis*, *153*, 176–188.

Lee, W., Miranda, M. F., Rausch, P., Fazio, M., Downs, J. G., & Morris, J. (2019). Bayesian semiparametric functional mixed models for serially correlated functional data, with application to glaucoma data. *Journal of the American Statistical Association*, *114*(526), 495–513. https://doi.org/10.1080/01621459.2018.1476242

Li, B., & Song, J. (2017). Nonlinear sufficient dimension reduction for functional data. *Annals of Statistics*, *45*(3), 1059–1095.

Li, J., Huang, C., & Zhu, H. (2017). A functional varying-coefficient single-index model for functional response data. *Journal of the American Statistical Association*, *112*(519), 1169–1181.

Li, Y., & Hsing, T. (2010). Deciding the dimension of effective dimension reduction space for functional and high-dimensional data. *Annals of Statistics*, *38*(5), 3028–3062.

Lian, H. (2011). Functional partial linear model. *Journal of Nonparametric Statistics*, *23*(1), 115–128.

Lian, H., & Li, G. (2014). Series expansion for functional sufficient dimension reduction. *Journal of Multivariate Analysis*, *124*, 150–165.

Ling, N., Li, Z., & Yang, W. (2014). Conditional density estimation in the single functional index model for α-mixing functional data. *Communications in Statistics: Theory and Methods*, *43*(3), 441–454.

Ling, N., & Vieu, P. (2018). Nonparametric modelling for functional data: Selected survey and tracks for future. *Statistics*, *52*(4), 934–949.

Ling, N., & Xu, Q. (2012). Asymptotic normality of conditional density estimation in the single index model for functional time series data. *Statistics & Probability Letters*, *82*(12), 2235–2243.

Luo, X., Zhu, L., & Zhu, H. (2016). Single-index varying coefficient model for functional responses. *Biometrics*, *72*(4), 1275–1284.

Ma, H., Bai, Y., & Zhu, Z. (2016). Dynamic single-index model for functional data. *Science China. Mathematics*, *59*(12), 2561–2584.

Ma, S. (2016). Estimation and inference in functional single-index models. *Annals of the Institute of Statistical Mathematics*, *68*(1), 181–208.

Maity, A., & Huang, J. (2012). Partially linear varying coefficient models stratified by a functional covariate. *Statistics & Probability Letters*, *82*(10), 1807–1814.

Mas, A. (2012). Lower bound in regression for functional data by representation of small ball probabilities. *Electronic Journal of Statistics*, *6*, 1745–1778.
Morris, J. (2015). Functional regression. *Annual Review of Statistics and its Application*, *2*, 321–359.
Müller, H. G., & Hall, P. (2016). Functional data analysis and random objects. *Annals of Statistics*, *44*(5), 1867–1887.
Müller, H. G., & Yao, F. (2008). Functional additive models. *Journal of the American Statistical Association*, *103*(484), 1534–1544.
Nagy, S. (2017). An overview of consistency results for depth functionals. In *Functional statistics and related topics* Springer contributions to statistics. New York: Springer.
Novo, S., Aneiros, G., & Vieu, P. (2019). Automatic and location-adaptive estimation in functional single-index regression. *Journal of Non-parametric Statistics*, *31*(2), 364–392.
Qingguo, T. (2015). Estimation for semi-functional linear regression. *Statistics*, *49*(6), 1262–1278.
Rabhi, A., Belkhir, N., & Soltani, S. (2016). On strong uniform consistency of conditional hazard function in the functional single-index model. *International Journal of Mathematics and Statistics*, *17*(2), 85–109.
Rachdi, M., & Vieu, P. (2007). Nonparametric regression for functional data: Automatic smoothing parameter selection. *Journal of Statistical Planning and Inference*, *137*(9), 2784–2801.
Ramsay, J., & Silverman, B. (1996). Functional data analysis. In *Springer series in statistics*. New York: Springer.
Ramsay, J., & Silverman, B. (2005). Functional data analysis. In *Springer series in statistics* (2nd ed.). New York: Springer.
Ramsay, J. O., & Silverman, B. W. (2002). *Applied functional data analysis*, Springer Series in Statistics. New-York: Springer-Verlag.
Sangalli, L. (2018). The role of statistics in the era of big data (editorial). *Statistics & Probability Letters*, *136*, 1–3.
Scarpa, B., & Dunson, D. B. (2009). Bayesian hierarchical functional data analysis via contaminated informative priors. *Biometrics*, *65*, 772–780.
Scarpa, B., & Dunson, D. B. (2014). Enriched stick-breaking processes for functional data. *Journal of the American Statistical Association*, *109*(506), 647–660.
Shang, H. (2014b). Bayesian bandwidth estimation for a semi-functional partial linear regression model with unknown error density. *Computational Statistics*, *29*(3–4), 829–848.
Shang, H. L. (2014a). A survey of functional principal component analysis. *AStA Advances in Statistical Analysis*, *98*(2), 121–142.
Shi, J. Q., & Choi, T. (2011). *Gaussian process regression analysis for functional data* (Vol. 2011). Boca Raton, FL: CRC Press.
Sperlich, S., Härdle, W., & Aydinli, G. (2006). The art of semiparametrics. Selected papers from the conference held in Berlin, 2003. In *Contributions to statistics*. Heidelberg: Physica-Verlag/Springer.
Ullah, S., & Finch, C. (2013). Applications of functional data analysis: A systematic review. *BMC Medical Research Methodology*, *13*, 43. https://doi.org/10.1186/1471-2288-13-43
Vieu, P. (2018). On dimension reduction models for functional data. *Statistics & Probability Letters*, *136*, 134–138.
Wang, G., Feng, X., & Chen, M. (2016). Functional partial linear single-index model. *Scandinavian Journal of Statistics*, *43*(1), 261–274.
Wang, G., Zhou, J., Wu, W., & Chen, M. M. (2017). Robust functional sliced inverse regression. *Statistical Papers*, *58*(1), 227–245.
Yang, G., Lin, H., & Lian, H. (2019). On double-index dimension reduction for partially functional data. *Journal of Nonparametric Statistics*, *31*(3), 761–768.
Yu, P., Du, J., & Zhang, Z. (2020). Single-index partially functional linear regression model. *Statistical Papers*, *61*(3), 1107–1123.
Zhang, J. (2013a). Analysis of variance for functional data. In *Monographs on statistics & applied probability*, (Vol. 127). Boca Raton, FL: CRC Press.
Zhang, T. (2013b). Asymptotic properties in semiparametric partially linear regression models for functional data. *Acta Mathematicae Applicatae Sinica (English Series)*, *29*(3), 631–644.
Zhou, J., & Chen, M. (2012). Spline estimators for semi-functional linear model. *Statistics & Probability Letters*, *82*(3), 505–513.
Zhu, H., Zhang, R., Yu, L., Lian, H., & Liu, Y. (2019). Estimation and testing for partially functional linear errors-in-variables models. *Journal of Multivariate Analysis*, *170*, 296–314.

How to cite this article: Ling N, Vieu P. On semiparametric regression in functional data analysis. *WIREs Comput Stat*. 2020;e1538. https://doi.org/10.1002/wics.1538