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Data Reduction Methods for Reverse Engineering

K. H. Lee, H. Woo and T. Suk
Kwangju Institute of Science and Technology (K-JIST), 1 Oryong-dong, Puk-gu, Kwangju, 500–712, Korea

Reverse engineering is an emerging technology that promises to play a role in reducing product development time. Reverse engineering in this paper refers to the process of creating engineering design data from existing parts. It creates or clones an existing part by acquiring its surface data using a scanning or measurement device. Recently, laser scanning technology has improved significantly and has become a viable option in capturing geometries of complicated design models. 3D laser scanners have become more accurate and the speed of data acquisition has increased dramatically. However, they generate up to thousands of points per second, and handling the huge amount of point data is a major problem. It becomes quite important, therefore, to reduce the amount of acquired point data and convert it into formats required by the manufacturing processes while maintaining the accuracy. This paper presents methods that can reduce efficiently the amount of point data.

Keywords: Point cloud data; Point data reduction; Reverse engineering; 3D laser scanner; Uniform and non-uniform grid methods

1. Introduction

Products are manufactured from 2D or 3D CAD models, but these CAD models are not always available in the production environment, and thus it is often necessary to generate a model from an existing physical part. Reverse engineering technology makes it possible to recreate an existing part by reconstructing its surface geometry in a 3D digital file using a scanning or measurement device. This technology enables us to create a CAD model of a part that has no design data or has gone through many design changes. For certain industries such as in the automotive industry, the part geometry is obtained from a mock-up model using this technology.

Correspondence and offprint requests to: Dr K. H. Lee, Department of Mechatronics, Kwangju Institute of Science and Technology, 1 Oryong-dong, Puk-gu, Kwanglu, 500–712, Korea. E-mail: lee@kyebek.kjist.ac.kr

In capturing surface information, either contact or non-contact measuring devices can be used. Coordinate measuring machines have usually been used in the past for capturing surface information. These machines typically use touch probes and the data acquisition process is very slow when they are used for measuring parts having complex freeform surfaces. In contrast, non-contact devices such as 3D-laser scanners are extremely fast in data acquisition, but they are rarely used in production lines because of their poor accuracy compared to that of contact devices [1]. Among these devices, those using laser-scanning technology have been improved significantly in recent years, and are now being used in the production mode [2]. However, there are problems in using these scanning devices since they produce extremely dense point data at a great rate. Not all these point data are necessary for generating a surface model, moreover, bottlenecks are created owing to inefficiencies in storing and manipulating them. Therefore, it takes a long time to generate a surface model from these scanned data.

To avoid these problems, it is important to reduce the amount of scanned data while maintaining the required accuracy. This paper presents data reduction methods that can reduce the amount of point data acquired during laser scanning. The proposed methods are applied to different types of surfaces and the results are discussed.

1.1 Previous Research

Contact devices were usually used to acquire surface geometry in the past, and research issues were mainly on point-sampling strategies for different types of surface. The amount of point data did not need to be reduced for contact devices. However, in recent years, some workers have shown interest in managing the large amount of point data acquired by laser scanners. Some research related to point data reduction for reverse engineering is described below.

Martin et al. [3] proposed a data reduction method using a uniform grid in their EU Copernicus project. Their method uses a “median filtering” approach, which has been widely used in image processing. The procedure starts by building a grid structure, and the input data points are assigned to the corresponding grid. From all of the points assigned to a given grid, a median point is selected to represent data points belonging to that cell. This approach overcomes the limitation of averaging or simple sampling methods. Their method, however, has a drawback due to the use of uniform size grids that can be insensitive in capturing a part shape.

Fujimoto and Kariya [4] also indicated that the amount of data should be reduced since a large amount of point data causes problems in downstream manufacturing operations. They suggested an improved sequential data reduction method for 2D digitised point data. Their method guarantees that an error range of the reduced data remains within the given angle and distance tolerances.

Chen et al. [5] suggested a method to reduce the point data by reducing the number of triangles required in a polyhedral model. They generated the STL file of a part directly from the point data acquired by a coordinate measuring machine, and reduced the amount of data by decreasing the number of triangles in the STL file. Triangles in planar or near planar regions are combined by comparing the normal vectors of neighbouring triangles. As a result, larger triangles are formed for flat areas and smaller ones for highly curved areas. They demonstrated their algorithm by reducing the number of triangles in an STL file of a human face digitised by a CMM machine.

Hamann [6] also presented a method of data reduction for triangulation files based on an iterative triangle removal principle. As a measure of the reduction of file size, each triangulation is weighted according to the principal curvature estimates at its vertices and interior angles.

Véron and Léon [7] introduced an approach to reduce the number of nodes of a polyhedral model using error zones assigned to each point of the initial polyhedron so that the simplified polyhedron intersects with each error zone.

Hamann and Chen [8] proposed a method to reduce the point data in making various planar curves, compressing 2D images, and visualising volumes. In their method, points are selected with respect to local absolute curvature estimates for piecewise linear curve approximation. The degree of reduction is controlled either by the number of points to be selected or by the error tolerance level.

Major research efforts in the existing data reduction methods lie in manipulating polyhedral models. Various schemes are also used to reduce the amount of point data from the initial point clouds; however, none of the existing methods has considered the characteristics of the scanning device.

2. Reverse Engineering by Laser Scanning

The reverse engineering process consists of the following steps, as shown in Fig. 1: point data acquisition, noise filtering, data reduction/arrangement/registration, segmentation, curve/surface fitting, and 3D surface model generation.

The data acquisition step creates raw point data to work with and it is considered as a crucial step in reverse engineering since in many cases, the quality of raw point data determines the quality of the resulting surfaces. Once raw point data are acquired, noisy data points, so called outliers or spikes, generated during a scanning process, must be eliminated. The quality of point data can be further improved by performing preprocess operations such as spike removal, smoothing and merging. For spike removal, several methods can be used:

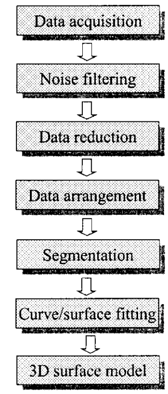


Fig. 1. General procedure for reverse engineering.

1. The angles between two consecutive points are considered, and a point that makes an angle with the previous point larger than the given value is eliminated.
2. The points can be moved to a median value.
3. The points can be moved either upward or downward close to the given level along the specified axis within the allowable distance [9].

The data reduction methods are applied after the removal of spikes and they should be performed with consideration of the characteristics of the scanning devices.

2.1 3D Laser Scanning

A 3D laser-scanning device acquires the surface information of a part, by sending laser beams which are radiated from the surface and received by CCD cameras. In general, the probe of a laser scanner consists of a beam projector radiating the laser beam and a CCD camera sensing the reflected beam from the surface. The laser beam can typically be categorised as a point type, or a stripe type. A stripe-type scanner radiates a line of laser beams, called a stripe, onto the surface so that several points can be acquired at once, whereas a point type laser scanner obtains only one point at a time. Laser scanning devices can also be classified based on the configuration of the machine, as shown in Fig. 2.

There are other configurations, but these three are the most popular ones. Each one has advantages and disadvantages; the choice of a machine depends on the characteristics of the part to be scanned, as well as its application. The first type shown in Fig. 2(*a*) uses three linear axes that are perpendicular to each other. This one follows the same structure as for the traditional fabrication machines and shows better accuracy compared to the others owing to its structural stability. A coordinate measuring machine can be retrofitted to form this type by mounting a scanning probe. Extra degrees of freedom can be obtained by adding a rotary table with tilt and other motions. In the second type of device, the scanning is perfor-

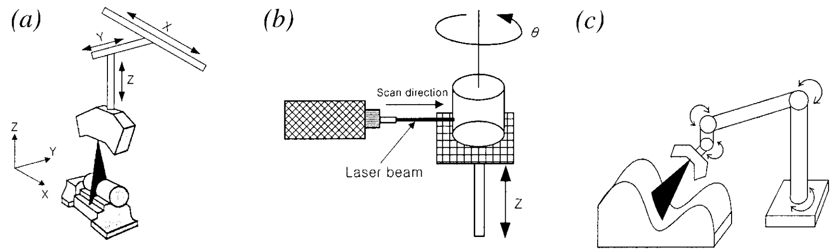


Fig. 2. Types of laser scanning device.

med by rotating the platform on which the object is mounted, as depicted in Fig. 2(*b*). In this configuration, at times, the object is stationary and the laser probe rotates around it. Scanners of this type are operated by obtaining contours of an object from the bottom to the top by lowering the platform in a stepwise manner. This operation is similar to obtaining magnetic resonance images in medical applications. It can be advantageous for acquiring the entire surface data of a part, compared to the first type. However, it may collect less accurate point data since the *z*-increment generally is much larger than the stripe intervals used in the machines of the first type. The third type uses an articulated robot arm with a laser probe attached, as the end of arm tooling. This type usually requires a skilled operator to move the laser head around the part while maintaining certain spacing between the heads and the part. This type has a greater flexibility in scanning since it allows us to scan any shape of part by moving the robot arm, as long as the probe motion is within the robot work envelope. The surface data of a scaled model of a car, for example, can be collected efficiently using this type of scanner, but, this type also has poor accuracy compared to the first type.

As the laser probe scans an object, the rays sensed by the CCD camera are stored as intensity data in pixels; this information is then converted to 3D coordinates for scanned points through image processing and triangulation methods. Figure 3(*a*) shows the principle of the triangulation method.

The laser scanner considered in this research belongs to the first type. It uses a probe that radiates a series of laser stripes while moving along the scan path illustrated by Fig. 3(*b*). Among the coordinates of 3D points, *x*- and *y*-values are obtained according to the position of the probe that is directly controlled by the transport mechanism. The *x*- and *y*-values are mostly reliable, but *z*-values generally show poor accuracy since they are determined by processing the sensed rays at the CCD camera. The error range in the *z*-axis depends on the quality of the scanning devices, but most current state-of-the-art devices can accommodate ranges from 0.01 mm to over 0.10 mm with a good calibrated accuracy. In managing the point cloud data generated from laser scanners, therefore, the z-axis errors should be controlled in order to create accurate surfaces.

2.2 Point Data Reduction Using Uniform Grids

The amount of point data can be reduced by dividing it into grids and by sampling a representative point from each grid. Martin et al. [3] used a uniform grid method as described below. An array of grids perpendicular to the scanning direction (*z*-direction) is used for extracting points from the point cloud data. Since *z*-values, as discussed in the previous section, are more prone to errors due to the characteristics of laser scanners, median filtering is used with the grids. First, a grid plane is created consisting of same-sized grids perpendicular to the scanning direction. The data reduction ratio is determined by the size of a grid which is user defined. The smaller the grid size, the more points are sampled from the entire point cloud. After creating a uniform grid plane, all the points are projected on the grid plane and each grid is assigned with the corresponding points. Then, one point from each grid is selected based on the median-filtering rule [10]. The points within each grid are sorted with respect to the distances from the grid plane, and a point, which is located in the middle, is chosen, as shown in Fig. 4. When the number of points within a grid is n, the $(n+1)/2$th point is selected if n is odd and the $n/2$th point or the $(n+2)/2$th point is selected if n is even.

Using median filtering with uniform grids, those points that are regarded as noise are likely to be discarded. This method shows better performance if the scanned surface is perpendicular to the scanning direction. Furthermore, this method is good for maintaining the original point data, as

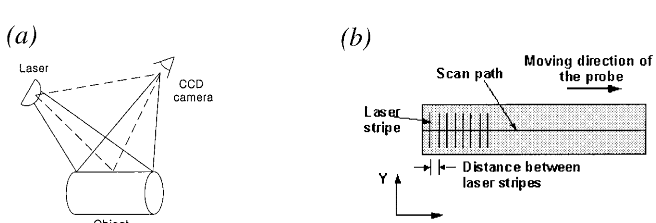


Fig. 3. (*a*) Triangulation methods. (*b*) Scanning operation.

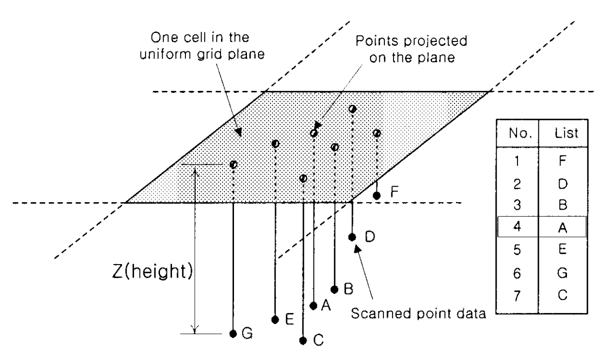


Fig. 4. Uniform grid method.

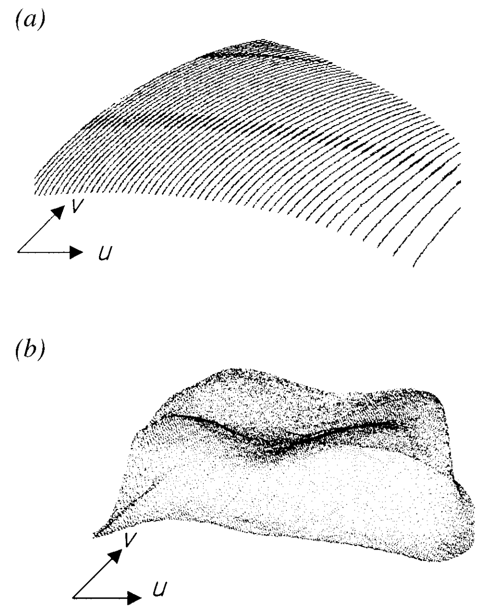


Fig. 5. Scanned data using a laser scanner.

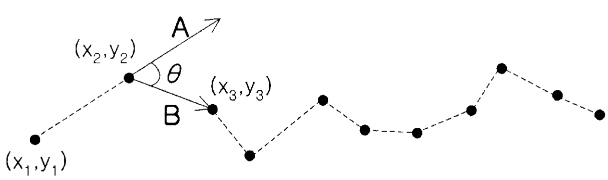


Fig. 6. Angular deviation method.

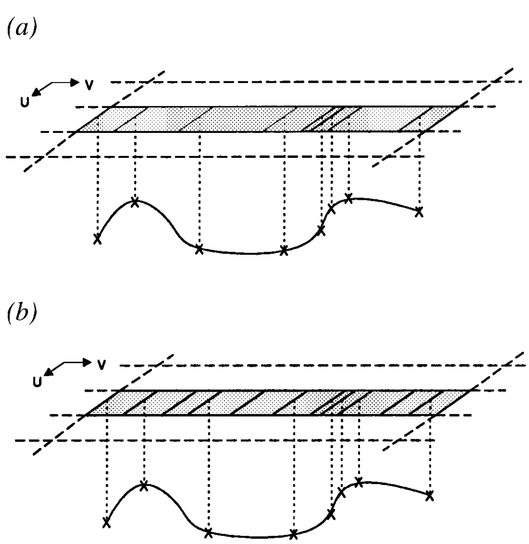


Fig. 7. One-directional non-uniform grid generation.

it selects points rather than changes the positions of points. The uniform grid method is especially useful in cases where data reduction must be done very quickly for parts having relatively simple surfaces.

3. Improved Data Reduction Methods Using Non-Uniform Grids

When applying the uniform grid method, some points for which the part shape drastically changes, such as edges, can be lost because no consideration of part shape is provided. In reverse engineering, it is critical to recreate part shape accurately, and the uniform grid method has limitations in this regard. In this work, non-uniform grid methods are proposed in which the size of grids can be varied based on the part shape. Two levels of non-uniform grid methods are proposed: one-directional and bi-directional. They can be applied considering the characteristics of the measured data.

When measuring a part with a stripe-type laser scanner, the scan path and the interval between laser stripes are usually specified by the user. The moving direction of the laser probe follows the scan path, and the interval between laser stripes controls the density of the scanned points. When a part having simple surfaces is measured, it does not have to be scanned densely in all directions. The point data shown in Fig. 5(*a*) have more points along one direction (v-direction) than in the other direction (u-direction). In this case, the one-directional non-uniform grid method is appropriate for capturing the shape of the surface, and is, therefore, recommended. On the other hand, when a part to be measured has complex and freeform surfaces, the point data should be dense along both the u- and v-directions, as shown in Fig. 5(*b*). In this case, the bi-directional non-uniform grid method is more appropriate than the one-directional method.

For applying both one-directional and bi-directional methods, the scan data are obtained from one view and therefore can be projected on a 2D plane. If the user requires a complete 3D model of an object, the object must be scanned several times by changing the set-up. Upon completion of scanning of all surfaces, the point clouds from different views should be registered together to construct a complete 3D model. However, the single-view scan data is sufficient in many cases, for meeting various purposes required by industrial parts.

3.1 One-Directional Non-Uniform Grid Method

In the one-directional non-uniform grid method, points are sampled from the point cloud acquired from the surfaces of a part using an angular deviation method. As shown in Fig. 6, the angular deviation method selects the points based on the angle which is calculated by the vectors created from three consecutive points, for example, (x_1,y_1), (x_2,y_2) and (x_3,y_3). The angles represent the curvature information; the curvature is small when the angle is small and vice versa. Using these angles, the points with high curvature can be extracted. The size of grids along the u-direction is fixed by the interval of the laser stripes, which is determined by the user. In the v-direction, the grid size is determined depending on the geometric information about the part shape. The points extracted by angular deviation represent high curvature areas, and they must be preserved during data reduction in order to express the part shape accurately. Thus, after extracting points by using the angular deviation method, the grid along the v-direction is

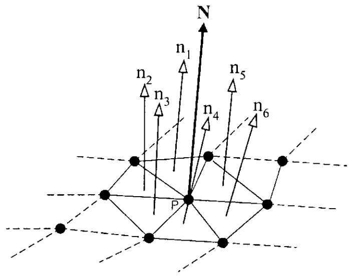


Fig. 8. Normal value calculation.

divided based on the extracted points, as shown in Fig. 7(*a*). When dividing the grids, if a grid is larger than the maximum size, which is predetermined by the user, it is further divided so as not to exceed the maximum grid size as shown in Fig. 7(*b*). Then, median filtering is applied to the points in each grid. This will result in a representative point for each grid, as in the case of the uniform grid method. The final retained points in this method include the points selected by using median filtering for each grid and the points extracted by angular deviation. Using these points, this method reduces the point data more effectively while maintaining the accuracy of the part shape, when compared to the uniform grid method.

3.2 Bi-Directional Non-Uniform Grid Method

In the bi-directional non-uniform grid method, the normal vectors of individual points are calculated and data reduction is performed based on this information. First, the point data are polygonised using triangles. In determining the normal vector of a point, the normal vectors of neighbouring triangles are used. As shown in Fig. 8, for the point, *p*, six neighbouring triangles exist and the normal value of the point, *N*, can be calculated using Eq. (1).

$$N = \frac{n_1 + n_2 + n_3 + n_4 + n_5 + n_6}{|n_1 + n_2 + n_3 + n_4 + n_5 + n_6|} \tag{1}$$

Upon calculating the normal vectors of all points, a grid plane is generated. The grid size is defined by the user, and

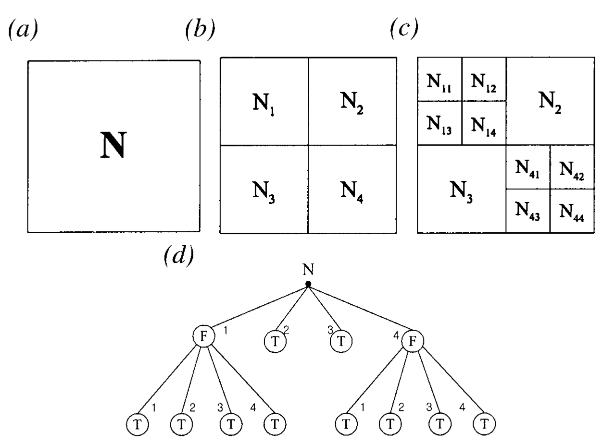


Fig. 9. Bi-directional non-uniform grid method. (*a*) Initial cell. (*b*) First split. (*c*) Second split. (*d*) QuadTree structure.

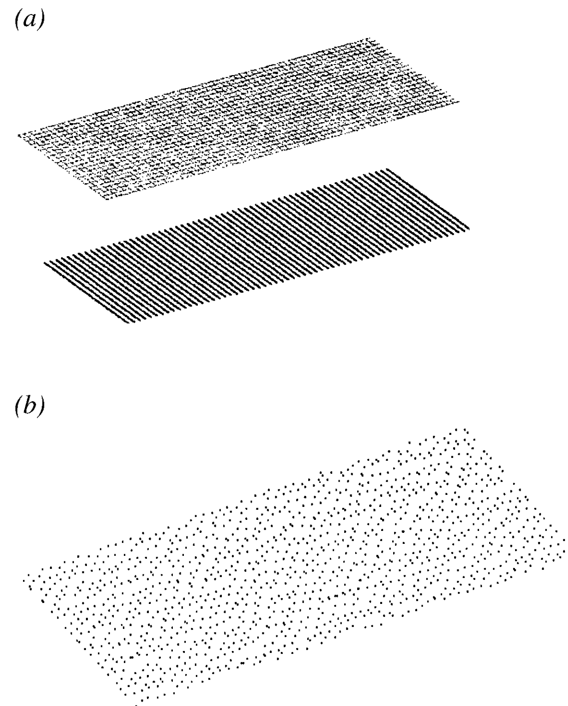


Fig. 10. (*a*) Original data with uniform grids, and (*b*) reduced data.

Table 1. Comparison of flatness.

Plane	CMM	Original point data	Uniform grid	Uniform sampling
Flatness	0.0050	0.0953	0.0653	0.0826
Number of points	50	14611	1221	1219

Table 2. Comparison between uniform grid method and uniform sampling for a shpere (unit: mm).

Reduction ratio	Number of points	Positive deviation	Negative deviation	Radius difference
Original	16857			
Uniform grid method				
0.1	1669	0.04685	−0.05589	0.0267
0.01	166	0.03901	−0.05907	0.0232
0.001	17	0.02323	−0.02716	0.0111
Uniform sampling				
0.1	*1700*	*0.05606*	*−0.07412*	*0.02736*
0.01	*179*	*0.0377*	*−0.07918*	*0.03492*
0.001	*19*	*0.03438*	*−0.03536*	*0.04822*

it depends on the intended data reduction ratio for the given part shape. If it is necessary to reduce the point data greatly, the size of a grid will be increased; whereas, if less reduction of point data is necessary, the grid size will be decreased. By projecting the points on the grid plane, the points corresponding

Table 3. Summary of deviation for sample model 1 (unit: mm).

Reduction ratio	Number of points	Positive deviation	Negative deviation
Original	27861		
One-directional non-uniform grid method			
1/5	6276	0.00289	–0.00425
1/10	2967	0.0057	–0.00512
1/20	1556	0.00859	–0.006
1/50	555	0.0288	–0.0159
1/100	267	0.0411	–0.0405
Uniform sampling			
1/5	6563	0.00489	–0.00496
1/10	2875	0.00722	–0.00785
1/20	1564	0.0157	–0.01265
1/50	558	0.10318	–0.08857
1/100	266	0.1825	–0.17825
Uniform grid			
1/5	6421	0.00324	–0.00382
1/10	2890	0.00675	–0.00728
1/20	1600	0.01178	–0.00988
1/50	552	0.0923	–0.06181
1/100	272	0.16214	–0.16783
Chordal deviation sampling			
1/5	6500	0.00311	–0.00422
1/10	2927	0.00628	–0.00735
1/20	1586	0.0105	–0.00815
1/50	562	0.0509	–0.0412
1/100	268	0.0615	–0.0866

to each grid are grouped and the normal values of these points are averaged. As a criterion for subdivision of grids, the standard deviation of the point normal values is used. The level of standard deviation is predetermined considering both the part shape and the desired point data reduction ratio. If the standard deviation in a grid is, for instance, large, it indicates that the part geometry corresponding to the grid is complicated, and therefore, further subdivision of the grid is required in order to sample more points. The process of subdivision is shown in Fig. 9. This is called quadtree subdivision and it has been widely used for image processing and computer graphics from the early 1970s [11,12]. If the standard deviation of a grid is larger than the given value, the grid is subdivided into four cells. This process repeats until the standard deviation of a grid is smaller than the given value, or the grid size reaches the minimum limit specified by the user. The minimum size of a grid varies depending on the complexity of the part shape. Upon completion of grid making, a representative point is selected from the points belonging to each grid, using median filtering. This bi-directional method extracts more points compared to the one-directional method, and thereby represents the part shape more accurately.

4. Experimental Results

The proposed non-uniform grid based point data reduction methods are tested using different types of part surfaces, and the results are discussed. In implementing the data reduction methods, a commercial laser scanner, Surveyor Model 1200 from Laser Design Inc., was used to acquire the point data. The algorithms were developed using the open architecture of a reverse engineering software package, Surfacer version 7.1 from Imageware Inc., and were run on a Silicon Graphics Indigo2 engineering workstation. Before applying the reduction algorithms, outliers or spikes were removed from the initial point cloud.

4.1 Uniform Grids to Simple Surfaces

Before we discuss the experimental results of the proposed data reduction methods, the uniform grid method is performed to show the effectiveness of median filtering for a point cloud obtained by a laser scanner. Figure 10 shows the result of applying the uniform grid method with median filtering to the point cloud of a planar surface. A comparison is made between the uniformly sampled data [13] and the reduced data using the uniform grid method. Figure 11 shows the difference by displaying points using polylines, projected on the *Y,Z*-plane. The point cloud reduced by the uniform grid method expresses the measured plane with less fluctuation compared to that by the uniform sampling method. The flatness and the number of points in the original point cloud, in the point cloud by uniform grids, and in the point cloud by uniform sampling are summarised in Table 1. The result shows that the error range of the point cloud in the *z*-axis is minimised using median filtering. Median filtering is also used in the non-uniform sampling method proposed in this work and this result shows that median filtering can generate reliable point data for a point cloud created by a laser scanning device. The uniform grid method with median filtering also gives a good performance for a surface having constant curvature such as a sphere

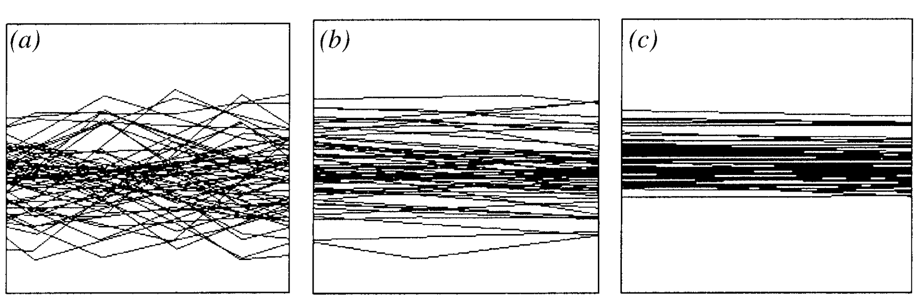
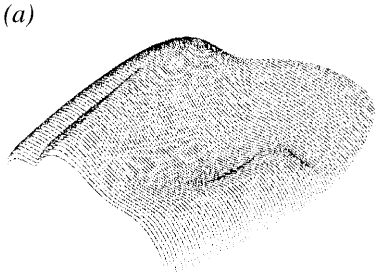


Fig. 11. Point data deviations in the *Z*-direction. (*a*) Original data. (*b*) Reduced data by uniform sampling. (*c*) Reduced data by uniform grids.

Table 4. Summary of deviation for sample model 2 (unit: mm).

Reduction ratio	Number of points	Positive deviation	Negative deviation
Original	36000		
Bi-directional non-uniform grid method			
1/5	4681	0.00100	−0.00098
1/10	3553	0.00125	−0.00206
1/20	1534	0.01552	−0.01927
1/50	702	0.01682	−0.02614
1/100	349	0.09671	−0.09322
Uniform sampling			
1/5	4680	0.00187	−0.00155
1/10	3720	0.00248	−0.00314
1/20	1564	0.0157	−0.01265
1/50	720	0.78819	−0.68422
1/100	360	2.14127	−1.92231
Uniform grid			
1/5	4690	0.00200	−0.00150
1/10	3468	0.00280	−0.00258
1/20	1584	0.02402	−0.02039
1/50	720	0.08142	−0.08845
1/100	363	0.11683	−0.13511
Chordal deviation sampling			
1/5	4692	0.00370	−0.05291
1/10	3576	0.00861	−0.00417
1/20	1534	0.08364	−0.00243
1/50	720	0.31240	−0.26035
1/100	360	2.14100	−1.92221

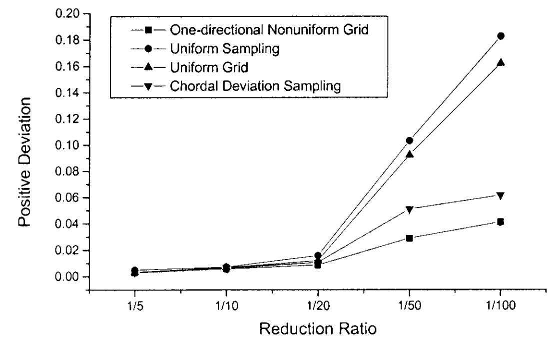
(a)



(b)

Fig. 12. Sample model 1 for the one-directional non-uniform grid method. (*a*) Initial data. (*b*) Reduced data.

(a)



(b)

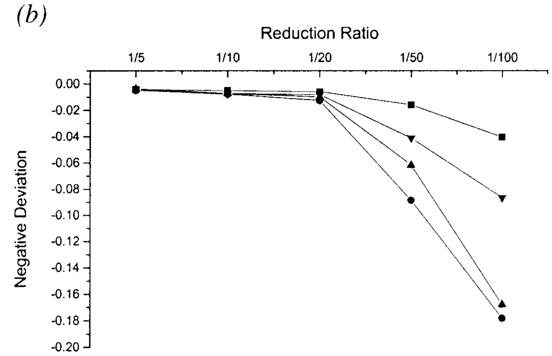


Fig. 13. Errors at different reduction ratio (for sample model 1). (*a*) Positive deviation. (*b*) Negative deviation.

or a cylinder. The uniform grid method is applied to a master ball whose diameter is 12.7 mm, and Table 2 shows the result.

4.2 Non-Uniform Grids to Freeform Surfaces

In the case of a freeform shaped object, the uniform grid method may not be the best choice. However, it is still worthwhile in cases where the data reduction must be performed very quickly without considering accuracy. The non-uniform grid methods are more effective in dealing with freeform shaped objects owing to the flexibility of the grid size: for example, smaller grids are used for highly curved and detailed areas and larger grids are used for planar and near planar areas.

Two clay models having freeform surfaces are made to demonstrate the performance of both the one-directional and bi-directional non-uniform grid methods. The measured point data for the first model is shown in Fig. 12(*a*); Fig. 12(*b*) shows the reduced point data after using the one-directional non-uniform grid method. Table 3 summarises the results of the one-directional non-uniform grid method and other methods such as uniform sampling, chordal deviation sampling, and the uniform grid method. Figure 13 shows the results for different levels of data reduction. The size of deviation increases as the reduction ratio increases from 1/5 to 1/100; however, the deviation values of the one-directional non-uniform grid method are smaller than those of other methods at each level.

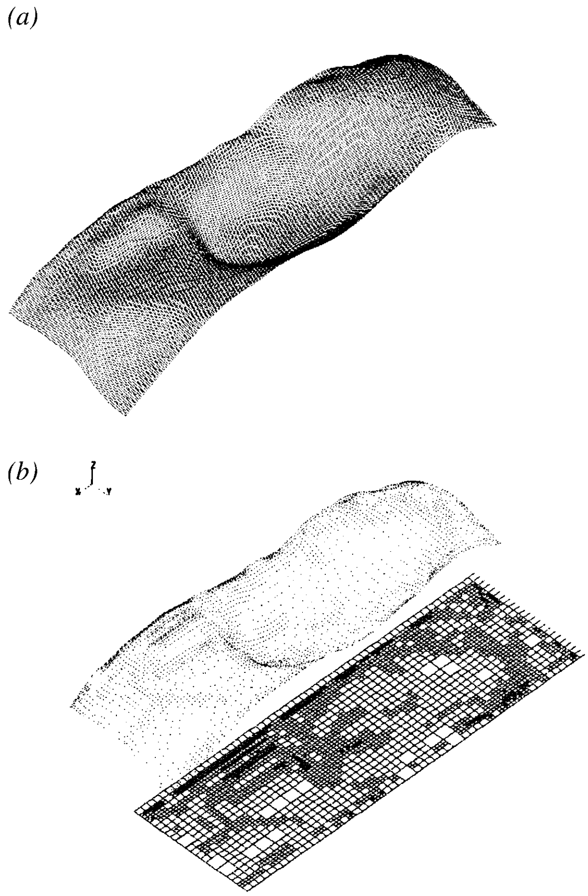


Fig. 14. Sample model 2 for the bi-directional non-uniform grid method (*a*) Measured point data. (*b*) Reduced data with non-uniform grid.

The second clay model shown in Fig. 14(*a*) was used to observe the performance of the bi-directional non-uniform grid method. Figure 14(*b*) shows the point cloud reduced with the bi-directional non-uniform grids generated using point normal vectors. For comparison purposes, other data reduction methods are also applied to the same model, and the results are summarised in Table 4.

As the reduction ratio increases, the reduced point data set shows more deviation from the original model; however, for all levels of reduction, the bi-directional non-uniform grid method shows less deviation than the other methods. Furthermore, even for an increase in the reduction, e.g. up to 1/100, the amount of deviation from the proposed method is kept constant whereas that of the conventional methods increases quickly as shown in Figs 15(*a*) and 15(*b*). Determining the optimal data reduction ratio considering both the accuracy and the number of points depends upon the part shape. Although only one freeform model was tested for each method, this study demonstrated the effectiveness of the non-uniform grid method in reducing the point cloud data for freeform shaped parts.

5. Conclusion

In this paper, a procedure for handling point cloud data acquired by laser scanners has been studied. As the technology

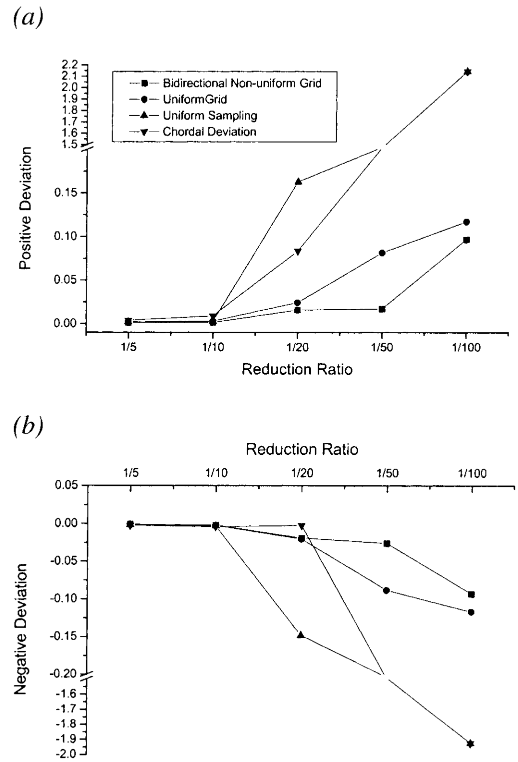


Fig. 15. Errors at different reduction ratios (for sample model 2). (*a*) Positive deviation. (*b*) Negative deviation.

of scanning devices improves, it is likely that the speed of data acquisition will increase, and this will automatically increase the amount of point data that must be handled. From the point of view of part shape and *z*-axis error occurring in laser scanning, the data reduction methods using non-uniform grids are proposed and implemented. These non-uniform grid methods are implemented either by using one-directional or bi-directional non-uniform grids. The proposed methods are applied to sample models having freeform shapes, and the results are analysed in comparison with the other conventional methods. The proposed methods demonstrated that data reduction can be performed effectively while maintaining the quality of the point data.

When a complete 3D model of a part needs to be processed, these 2D grid methods may not be suitable, since they are projection-based techniques. However, these methods are still useful if the registration of point clouds is performed after reducing the data size for each point cloud. The non-projection based data reduction method that deals with 3D scanned data will be the subject of our further research.

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