

# The architecture of complex weighted networks

A. Barrat\*, M. Barthélemy†, R. Pastor-Satorras‡, and A. Vespignani\*§

\*Laboratoire de Physique Théorique (Unité Mixte de Recherche du Centre National de la Recherche Scientifique 8627), Bâtiment 210, Université de Paris-Sud, 91405 Orsay Cedex, France; †Commissariat à l'Energie Atomique-Département de Physique Théorique et Appliquée, 91191 Bruyères-Le-Chatel, France; and ‡Departament de Física i Enginyeria Nuclear, Universitat Politècnica de Catalunya, Campus Nord, Mòdul B4, 08034 Barcelona, Spain

Communicated by Giorgio Parisi, University of Rome, Rome, Italy, January 8, 2004 (received for review October 29, 2003)

Networked structures arise in a wide array of different contexts such as technological and transportation infrastructures, social phenomena, and biological systems. These highly interconnected systems have recently been the focus of a great deal of attention that has uncovered and characterized their topological complexity. Along with a complex topological structure, real networks display a large heterogeneity in the capacity and intensity of the connections. These features, however, have mainly not been considered in past studies where links are usually represented as binary states, i.e., either present or absent. Here, we study the scientific collaboration network and the world-wide air-transportation network, which are representative examples of social and large infrastructure systems, respectively. In both cases it is possible to assign to each edge of the graph a weight proportional to the intensity or capacity of the connections among the various elements of the network. We define appropriate metrics combining weighted and topological observables that enable us to characterize the complex statistical properties and heterogeneity of the actual strength of edges and vertices. This information allows us to investigate the correlations among weighted quantities and the underlying topological structure of the network. These results provide a better description of the hierarchies and organizational principles at the basis of the architecture of weighted networks.

A large number of natural and man-made systems are structured in the form of networks. Typical examples include large communication systems (the Internet, the telephone network, the World Wide Web), transportation infrastructures (railroad and airline routes), biological systems (gene and/or protein interaction networks), and a variety of social interaction structures (1–3). The macroscopic properties of these networks have been the subject of intense scientific activity that has highlighted the emergence of a number of significant topological features. Specifically, many of these networks show the small-world property (4), which implies that the network has an average topological distance between the various nodes increasing very slowly with the number of nodes (logarithmically or even slower), despite showing a large degree of local interconnectivity typical of more ordered lattices. Additionally, several of these networks are characterized by a statistical abundance of “hubs” with a very large number of connections  $k$  compared with the average degree value  $\langle k \rangle$ . The empirical evidence collected from real data indicates that this distinctive feature finds its statistical characterization in the presence of scale-free degree distributions  $P(k)$ , i.e., showing a power-law behavior  $P(k) \sim k^{-\gamma}$  for a significant range of values of  $k$  (5). These topological features turn out to be extremely relevant because they have a strong impact in assessing such networks’ physical properties as their robustness or vulnerability (6–9).

While these findings alone might provide insight for threat analysis and policy decisions, networks are specified not only by their topology but also by the dynamics of information or traffic flow taking place on the structure. In particular, the heterogeneity in the intensity of connections may be very important in the understanding of social systems. Analogously, the amount of traffic characterizing the connections in communication systems and large transport infrastructures is fundamental for a full description of these networks.

Motivated by these observations, we undertake in this paper the statistical analysis of complex networks whose edges have been assigned a given weight (the flow or the intensity) and thus can be generally described in terms of *weighted graphs* (10, 11). Working with two typical examples of this kind of network, we introduce some metrics that combine in a natural way both the topology of the connections and the weight assigned to them. These quantities provide a general characterization of the heterogeneous statistical properties of weights and identify alternative definitions of centrality, local cohesiveness, and affinity. By appropriate measurements it is also possible to exploit the correlation between the weights and the topological structure of the graph, unveiling the complex architecture shown by real weighted networks.

## Weighted Networks Data

To proceed to the general analysis of complex weighted networks we consider two specific examples for which it is possible to have a full characterization of the links among the elements of the systems, the world-wide airport network (WAN) and the scientist collaboration network (SCN).

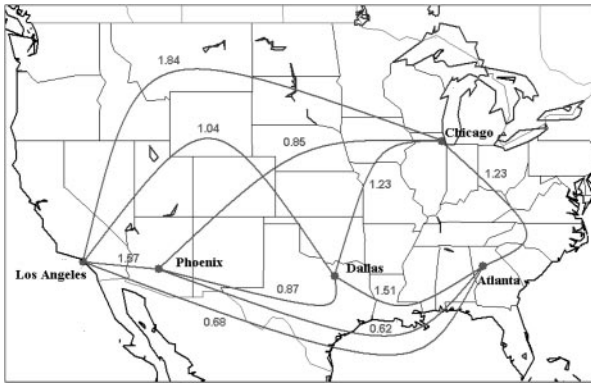
**WAN.** We analyze the International Air Transportation Association ([www.iata.org](http://www.iata.org)) database containing the world list of airports pairs connected by direct flights and the number of available seats on any given connection for the year 2002. The resulting air-transportation graph comprises  $N = 3,880$  vertices denoting airports and  $E = 18,810$  edges accounting for the presence of a direct flight connection. The average degree of the network is  $\langle k \rangle = 2E/N = 9.70$ , while the maximal degree is 318. The topology of the graph exhibits both small-world and scale-free properties as already observed in different dataset analyses (12, 13). In particular, the average shortest path length, measured as the average number of edges separating any two nodes in the network, shows the value  $\langle \ell \rangle = 4.37$ , very small compared with the network size  $N$ . The degree distribution takes the form  $P(k) = k^{-\gamma} f(k/k_c)$ , where  $\gamma \cong 2.0$  and  $f(k/k_c)$  is an exponential cut-off function that finds its origin in physical constraints on the maximum number of connections that a single airport can handle (3, 13). The airport connection graph is therefore a clear example of a network with an heterogeneous degree distribution, showing scale-free properties on a wide range of degree values.

**SCN.** We consider the network of scientists who have authored manuscripts submitted to the e-Print Archive relative to condensed matter physics (<http://xxx.lanl.gov/archive/cond-mat>) between 1995 and 1998. Scientists are identified with nodes, and an edge exists between two scientists if they have coauthored at least one paper. The resulting connected network has  $N = 12,722$  nodes, with an average degree (i.e., average number of collaborators)  $\langle k \rangle = 6.28$  and maximal degree 97. The topological properties of this network and other similar networks of scientific collaborations have been studied in refs. 14–16.

Abbreviations: WAN, world-wide airport network; SCN, scientist collaboration network.

§To whom correspondence should be addressed. E-mail: alexv@th.u-psud.fr.

© 2004 by The National Academy of Sciences of the USA



**Fig. 1.** Pictorial representation of the weighted graph obtained from the airport network data. Major U.S. airports are connected by edges denoting the presence of a nonstop flight in both directions whose weights represent the number of available seats (million/year).

The properties of a graph can be expressed by its adjacency matrix  $a_{ij}$ , whose elements take the value 1 if an edge connects the vertex  $i$  to the vertex  $j$  and 0 otherwise. The data contained in the previous datasets permit one to go beyond this topological representation by defining a weighted graph (10) that assigns a weight or value characterizing each connecting link. In the case of the WAN the weight  $w_{ij}$  of an edge linking airports  $i$  and  $j$  represents the number of available seats in flights between these two airports. The inspection of the weights shows that the average numbers of seats in both directions are identical  $w_{ij} = w_{ji}$  for an overwhelming majority of edges. In the following we will thus work with the symmetric undirected graph and avoid the complication deriving from flow imbalances. We show an example of the resulting weighted graph in Fig. 1. Noticeably, the above definition of weights is a straightforward and objective measure of the traffic flow on top of the network.

For the SCN we follow the definition of weight introduced in refs. 14 and 15: The intensity  $w_{ij}$  of the interaction between two collaborators  $i$  and  $j$  is defined as

$$w_{ij} = \sum_p \frac{\delta_i^p \delta_j^p}{n_p - 1} \quad \text{if } i \neq j, \quad w_{ii} = 0, \quad [1]$$

where the index  $p$  runs over all papers,  $n_p$  is the number of authors of paper  $p$ , and  $\delta_i^p$  is 1 if author  $i$  has contributed to paper  $p$  and 0 otherwise. While any definition of the intensity of a connection in social networks depends on the particular elements chosen to be relevant, the above definition seems to be rather objective and representative of the scientific interaction: It is large for collaborators having many papers in common but the contribution to the weight introduced by any given paper is inversely proportional to the number of authors.

### Centrality and Weights

To take into account the information provided by the weighted graph, we shall identify the appropriate quantities characterizing its structure and organization at the statistical level. The statistical analysis of weights  $w_{ij}$  between pairs of vertices indicates the presence of right-skewed distributions, already signaling a high level of heterogeneity in the system for both the WAN and the SCN as also reported in refs. 12, 14, and 15. It has been observed, however, that the individual edge weights do not provide a general picture of the network's complexity (11). A more significant measure of the network properties in terms of the actual weights is obtained by extending the definition of vertex degree  $k_i = \sum_j a_{ij}$  in terms of the vertex *strength*  $s_i$ , defined as

$$s_i = \sum_{j=1}^N a_{ij} w_{ij}. \quad [2]$$

This quantity measures the strength of vertices in terms of the total weight of their connections. In the case of the WAN the vertex strength simply accounts for the total traffic handled by each airport. For the SCN, on the other hand, the strength is a measure of scientific productivity because it is equal to the total number of publications of any given scientist, excluding single-author publications. This quantity is a natural measure of the importance or centrality of a vertex  $i$  in the network.

The identification of the most central nodes in the system is a major issue in network characterization (17). The most intuitive topological measure of centrality is given by the degree: more connected nodes are more central. However, more is not necessarily better. Indeed, by considering solely the degree of a node we overlook that nodes with small degree may be crucial for connecting different regions of the network by acting as bridges. To quantitatively account for the role of such nodes, betweenness centrality (14, 15, 17, 18) has been defined as the number of shortest paths between pairs of vertices that pass through a given vertex.<sup>†</sup> Central nodes are therefore part of more shortest paths within the network than peripheral nodes. Moreover, the betweenness centrality is often used in transport networks to provide an estimate of the traffic handled by the vertices, assuming that the number of shortest paths is a zeroth-order approximation to the frequency of use of a given node.<sup>‡</sup> The above definition of centrality relies only on topological elements. It is therefore intuitive to consider the alternative definition of centrality constructed by looking at the strength  $s_i$  of the vertices as a more appropriate definition of the importance of a vertex in weighted networks. For instance, in the case of the WAN this quantity provides the actual traffic going through the vertex  $i$ , and it is natural to study how it compares and correlates with other topological measures of centrality.

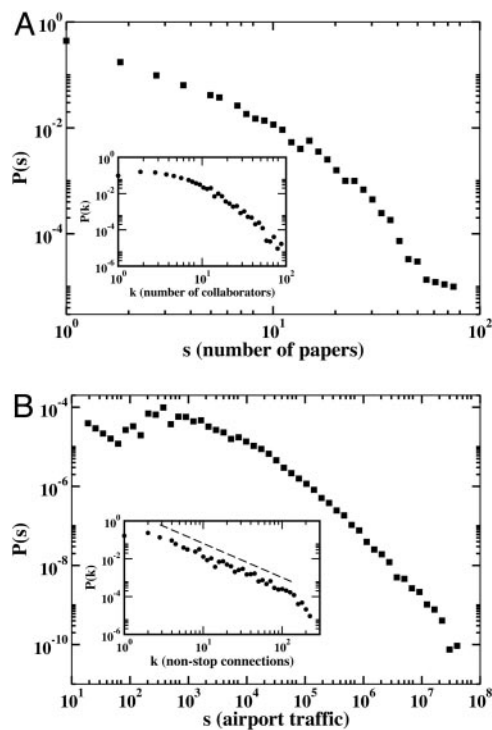
The probability distribution  $P(s)$  that a vertex has strength  $s$  is heavy tailed in both networks, and the functional behavior exhibits similarities with the degree distribution  $P(k)$  (see Fig. 2). A precise functional description of the heavy-tailed distributions may be very important in understanding the network evolution and will be deferred to future analysis. This behavior is not unexpected because it is plausible that the strength  $s_i$  increases with the vertex degree  $k_i$ , and thus the slow decaying tail of  $P(s)$  stems directly from the very slow decay of the degree distribution. To shed more light on the relationship between the vertices' strength and degree, we investigate the dependence of  $s_i$  on  $k_i$ . We find that the average strength  $s(k)$  of vertices with degree  $k$  increases with the degree as

$$s(k) \sim k^\beta. \quad [3]$$

In the absence of correlations between the weight of edges and the degree of vertices, the weights  $w_{ij}$  are on average independent of  $i$  and  $j$ , and therefore we can approximate  $w_{ij} = \langle w \rangle = (2E)^{-1} \sum_{i,j} a_{ij} w_{ij}$ , where  $\langle w \rangle$  is the average weight in the network. From Eq. 2 we then have  $s_i = \langle w \rangle \sum_j a_{ij} = \langle w \rangle k_i$ . That is, the strength of a vertex is simply proportional to its degree, yielding an exponent  $\beta = 1$ , and the two quantities provide therefore the same information on the system. In Fig. 3 we report the behavior

<sup>†</sup>More precisely, if  $D_{hj}$  is the total number of shortest paths from  $h$  to  $j$  and  $D_{hi}(i)$  is the number of these shortest paths that pass through the vertex  $i$ , the betweenness of the vertex  $i$  is defined as  $b_i = \sum_{h,j} D_{hi}(i) D_{hj} / D_{hj}$ , where the sum runs over all  $h, j$  pairs with  $j \neq h \neq i$ . An efficient algorithm to compute betweenness centrality is reported in ref. 19.

<sup>‡</sup>For the airport network, the analysis of the betweenness centrality and its correlation with the degree has been discussed in ref. 13.



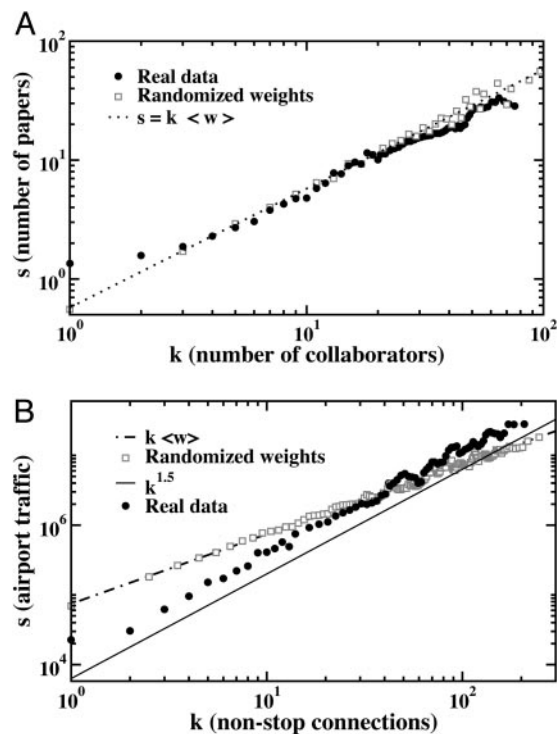
**Fig. 2.** (A) Degree (*Inset*) and strength distribution in the SCN. The degree  $k$  corresponds to the number of coauthors of each scientist, and the strength  $s$  represents the scientist's total number of publications. The distributions are heavy-tailed even if it is not possible to distinguish a definite functional form. (B) The same distributions for the WAN. The degree  $k$  (*Inset*) is the number of nonstop connections to other airports, and the strength  $s$  is the total number of passengers handled by any given airport. In this case, the degree distribution can be approximated by the power-law behavior  $P(k) \sim k^{-\gamma}$  with  $\gamma = 1.8 \pm 0.2$ . The strength distribution has a heavy tail extending over more than four orders of magnitude.

obtained for both the real weighted networks and their randomized versions, generated by a random redistribution of the actual weights on the existing topology of the network. For the SCN the curves are very similar and well fitted by the uncorrelated approximation  $s(k) = \langle w \rangle k$ . Interestingly, this is not the case of the WAN. Fig. 3B clearly shows a very different behavior for the real data set and its randomized version. In particular, the power-law fit for the real data gives an “anomalous” exponent  $\beta_{\text{WAN}} = 1.5 \pm 0.1$ . This value implies that the strength of vertices grows faster than their degree, i.e., the weight of edges belonging to highly connected vertices tends to have a value higher than the one corresponding to a random assignment of weights. This tendency denotes a strong correlation between the weight and the topological properties in the WAN, where the larger is an airport, the more traffic it can handle.

The fingerprint of these correlations is also observed in the dependence of the weight  $w_{ij}$  on the degrees of the end-point nodes  $k_i$  and  $k_j$ . As we can see in Fig. 4, for the WAN the behavior of the average weight as a function of the end-point degrees can be well approximated by a power-law dependence

$$\langle w_{ij} \rangle \sim (k_i k_j)^\theta \quad [4]$$

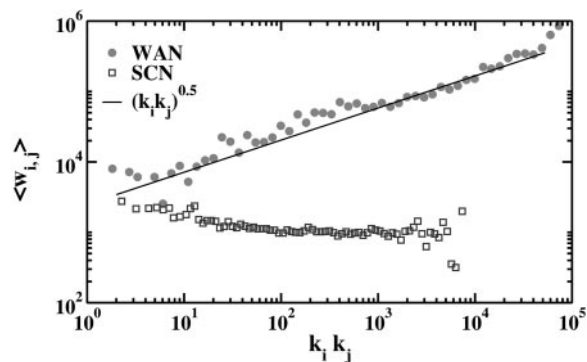
with an exponent  $\theta = 0.5 \pm 0.1$ . This exponent can be related to the  $\beta$  exponent by noticing that  $s_i \sim k_i \langle w_{ij} \rangle \sim k_i^{1+\theta} k_j^\theta$ , resulting in  $\beta = 1 + \theta$ , if the topological correlations between the degrees of connected vertices can be neglected. This is indeed the case of the WAN, where the above scaling relation is well satisfied by the numerical values provided by the independent measure-



**Fig. 3.** Average strength  $s(k)$  as function of the degree  $k$  of nodes. (A) In the SCN the real data are very similar to those obtained in a randomized weighted network. Only at very large  $k$  values is it possible to observe a slight departure from the expected linear behavior. (B) In the WAN real data follow a power-law behavior with exponent  $\beta = 1.5 \pm 0.1$ . This value denotes anomalous correlations between the traffic handled by an airport and the number of its connections.

ments of the exponents. In the SCN, instead,  $\langle w_{ij} \rangle$  is almost constant for more than two decades, confirming a general lack of correlations between the weights and the vertex degrees.

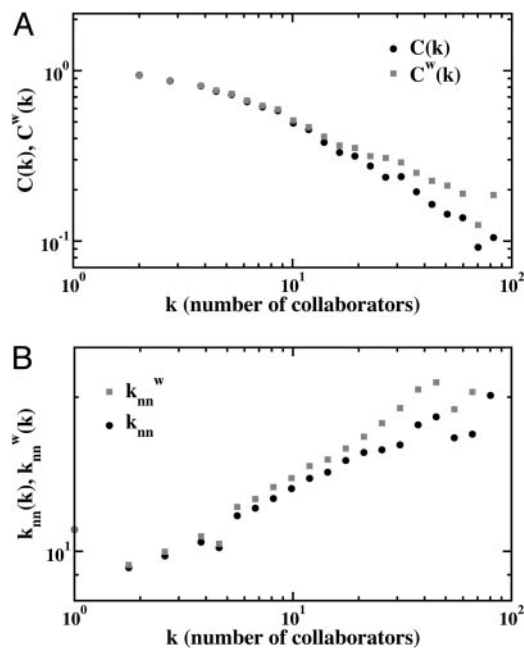
Analogously, a study of the average value  $s(b)$  of the strength for vertices with betweenness  $b$  shows that the functional behavior can be approximated by a scaling form  $s(b) \sim b^\delta$  with  $\delta_{\text{SCN}} \cong 0.5$  and  $\delta_{\text{WAN}} \cong 0.8$  for the SCN and the WAN, respectively. As before, the comparison between the behavior of the real data and the randomized case shows more pronounced differences in the case of the WAN. In both networks, the strength grows with the betweenness faster than in the randomized case, especially in the WAN. This behavior is another clear signature of the



**Fig. 4.** Average weight as a function of the end-point degree. The solid line corresponds to a power-law behavior  $\langle w_{ij} \rangle \sim (k_i k_j)^\theta$ , with exponent  $\theta = 0.5 \pm 0.1$ . In the case of the SCN it is possible to observe an almost flat behavior for roughly two orders of magnitude.



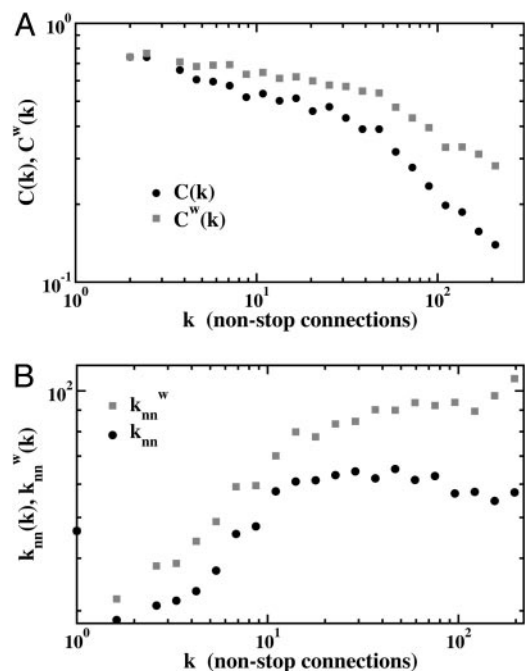




**Fig. 6.** Topological and weighted quantities for the SCN. (A) The weighted clustering separates from the topological one around  $k \geq 10$ . This value marks a difference for authors with larger number of collaborators. (B) The assortative behavior is enhanced in the weighted definition of the average nearest-neighbors degree.

The  $k_{nn,i}^w$  thus measures the effective *affinity* to connect with high- or low-degree neighbors according to the magnitude of the actual interactions. As well, the behavior of the function  $k_{nn}^w(k)$  marks the weighted assortative or disassortative properties considering the actual interactions among the system's elements.

As a general test, we inspect the results obtained for both the SCN and the WAN by comparing the regular topological quantities with those obtained with the weighted definition introduced here. The topological measurements tell us that the SCN has a continuously decaying spectrum  $C(k)$  (see Fig. 6A). This implies that hubs present a much lower clustered neighborhood than low-degree vertices. This effect can be interpreted as the evidence that authors with few collaborators usually work within a well defined research group in which all of the scientists collaborate (high clustering). Authors with a large degree, however, collaborate with different groups and communities, which in their turn do not often have collaborations, thus creating a lower clustering coefficient. Furthermore, the SCN exhibits an assortative behavior in agreement with the general evidence that social networks are usually denoted by a strong assortative character (24) (see Fig. 6B). The analysis of weighted quantities confirms this topological picture, providing further information on the network architecture. The weighted clustering coefficient is very close to the topological one ( $C^w/C \approx 1$ ). This fact states in a quantitative way that group collaborations tend on average to be stable and determine the average intensity of the interactions in the network. In addition, the inspection of  $C^w(k)$  (see Fig. 6A) shows generally that for  $k \geq 10$  the weighted clustering coefficient is larger than the topological one. This difference implies that high-degree authors (i.e., with many collaborators) tend to publish more papers with interconnected groups of coauthors. This finding suggests that influential scientists form stable research groups where the largest part of their production is obtained. Finally, the assortative properties find



**Fig. 7.** Topological and weighted quantities for the WAN. (A) The weighted clustering coefficient is larger than the topological one in the whole degree spectrum. (B)  $k_{nn}(k)$  reaches a plateau for  $k > 10$  denoting the absence of marked topological correlations. In contrast,  $k_{nn}^w(k)$  exhibits a more definite assortative behavior.

a clearcut confirmation in the weighted analysis with a  $k_{nn}^w(k)$  growing as a power of  $k$ .

A different picture is found in the WAN, where the weighted analysis provides a richer and somehow different scenario (Fig. 7). This network also shows a decaying  $C(k)$ , a consequence of the role of large airports that provide nonstop connections to very far destinations on an international and intercontinental scale. These destinations are usually not interconnected among them, giving rise to a low clustering coefficient for the hubs. We find, however, that  $C^w/C \approx 1.1$ , indicating an accumulation of traffic on interconnected groups of vertices. The weighted clustering coefficient  $C^w(k)$  also has a different behavior in that its variation is much more limited in the whole spectrum of  $k$ . This observation implies that high-degree airports have a progressive tendency to form interconnected groups with high-traffic links, thus balancing the reduced topological clustering. Because high traffic is associated to hubs, we have a network in which high-degree nodes tend to form cliques with nodes with equal or higher degree, the so-called *rich-club phenomenon* (25). Interesting evidence emerges also from the comparison of  $k_{nn}(k)$  and  $k_{nn}^w(k)$ . The topological  $k_{nn}(k)$  does show an assortative behavior only at small degrees. For  $k > 10$ ,  $k_{nn}(k)$  approaches a constant value, a fact revealing an uncorrelated structure in which vertices with very different degrees have a very similar neighborhood. The analysis of the weighted  $k_{nn}^w(k)$ , however, exhibits a pronounced assortative behavior in the whole  $k$  spectrum, providing a different picture in which high-degree airports have a larger affinity for other large airports where the major part of the traffic is directed.

## Conclusions

We have shown that a more complete view of complex networks is provided by the study of the interactions defining the links of these systems. The weights characterizing the various connec-

tions exhibit complex statistical features with highly varying distributions and power-law behavior. In particular we have considered the specific examples of SCN and WAN where it is possible to appreciate the importance of the correlations between weights and topology in the characterization of real network properties. Indeed, the analysis of the weighted quantities and the study of the correlations between weights and topology provide a complementary perspective on the structural organization of the network that might be undetected by quantities based only on topological information. Our study thus offers a quantitative and general approach to understand the complex architecture of real weighted networks.

We thank the International Air Transportation Association for making the airline commercial flight database available to us. We also thank M. E. J. Newman for giving us the possibility of using the SCN data (see [www-personal.umich.edu/~mejn/collaboration](http://www-personal.umich.edu/~mejn/collaboration)). We are grateful to L. A. N. Amaral and R. Guimerà for many discussions and sharing of results during the various stages of this work, and to L. Brualla for help with Fig. 1. A.B., R.P.-S., and A.V. are partially funded by the European Commission, Future and Emerging Technologies Open Project COSIN (Coevolution and Self-Organisation in Dynamical Networks) IST-2001-33555. R.P.-S. acknowledges financial support from the Ministerio de Ciencia y Tecnología (Spain) and from the Departament d'Universitats, Recerca i Societat de la Informació, Generalitat de Catalunya (Spain).

1. Albert, R. & Barabási, A.-L. (2002) *Rev. Mod. Phys.* **74**, 47–97.
2. Dorogovtsev, S. N. & Mendes, J. F. F. (2003) *Evolution of Networks: From Biological Nets to the Internet and WWW* (Oxford Univ. Press, Oxford).
3. Amaral, L. A. N., Scala, A., Barthélemy, M. & Stanley, H. E. (2000) *Proc. Natl. Acad. Sci. USA* **97**, 11149–11152.
4. Watts, D. J. & Strogatz, S. H. (1998) *Nature* **393**, 440–442.
5. Barabási, A.-L. & Albert, R. (1999) *Science* **286**, 509–512.
6. Cohen, R., Erez, K., ben Avraham, D. & Havlin, S. (2000) *Phys. Rev. Lett.* **85**, 4626–4628.
7. Callaway, D. S., Newman, M. E. J., Strogatz, S. H. & Watts, D. J. (2000) *Phys. Rev. Lett.* **85**, 5468–5471.
8. Albert, R., Jeong, H. & Barabási, A.-L. (2000) *Nature* **406**, 378–382.
9. Pastor-Satorras, R. & Vespignani, A. (2001) *Phys. Rev. Lett.* **86**, 3200–3203.
10. Clark, J. & Holton, D. A. (1998) *A First Look at Graph Theory* (World Scientific, Singapore).
11. Yook, S. H., Jeong, H., Barabási, A.-L. & Tu, Y. (2001) *Phys. Rev. Lett.* **86**, 5835–5838.
12. Li, W. & Cai, X. (2003) e-Print Archive, <http://xxx.lanl.gov/abs/cond-mat/0309236>.
13. Guimerà, R., Mossa, S., Turtleschi, A. & Amaral, L. A. N. (2003) e-Print Archive, <http://xxx.lanl.gov/abs/cond-mat/0312535>.
14. Newman, M. E. J. (2001) *Phys. Rev. E* **64**, 016131.
15. Newman, M. E. J. (2001) *Phys. Rev. E* **64**, 016132.
16. Barabási, A. L., Jeong, H., Neda, Z., Ravasz, E., Schubert, A. & Vicsek, T. (2002) *Physica A* **311**, 590–614.
17. Freeman, L. C. (1977) *Sociometry* **40**, 35–41.
18. Goh, K.-I., Kahng, B. & Kim, D. (2001) *Phys. Rev. Lett.* **87**, 278701.
19. Brandes, U. (2001) *J. Math. Soc.* **25**, 163–177.
20. Vázquez, A., Pastor-Satorras, R. & Vespignani, A. (2002) *Phys. Rev. E* **65**, 066130.
21. Ravasz, E. & Barabási, A.-L. (2003) *Phys. Rev. E* **67**, 026112.
22. Pastor-Satorras, R., Vázquez, A. & Vespignani, A. (2001) *Phys. Rev. Lett.* **87**, 258701.
23. Maslov, S. & Sneppen, K. (2001) *Science* **296**, 910–913.
24. Newman, M. E. J. (2002) *Phys. Rev. Lett.* **89**, 208701.
25. Zhou, S. & Mondragon, R. J. (2003) e-Print Archive, <http://xxx.lanl.gov/abs/cs.NI/0303028>.