IDS 702: Module 8.1

Bootstrap

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Introduction

- When building statistical models, we often need to quantify the uncertainty around the estimated parameters we are interested in.
- So far in this class, we have been doing so using standard errors and confidence intervals.
- Computing standard errors is often straightforward when we have closed forms.
- For example, the standard error for $\bar{X}$ is $\sigma/\sqrt{n}$.
- When σ is unknown, replace with $s = \hat{\sigma}$.
- What to do when we do not have closed forms?

Introduction

- Setting confidence intervals and conducting hypotheses testing often requires us to know the distribution of the parameter of interest.
- A key tool for doing this is the central limit theorem.
- Recall that according to CLT, for large samples, averages and sums are approximately normally distributed.
- With some work, the CLT allows confidence intervals and hypotheses testing on means, proportions, sums, intercepts, slopes, and so on.
- But...what if we want to set confidence intervals on a correlation or an sd or a ratio?

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- Once neat solution is to approximate whatever distribution you have in mind via re-sampling from the true population.
- For example, suppose I would like to estimate the average income of Durham residents and quantify uncertainty around my estimate.
- First I need a sample (of course!).
- Suppose I sample 1000 residents and record their income as $X_1, \ldots, X_{1000}$. Then, my estimate of average income is $\bar{X}$.
- Next, I should quantify my uncertainty around that number. I can do so using the standard error $\sigma/\sqrt{n}$ mentioned earlier, which relies on the CLT.

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- Alternatively, I could approximate the entire distribution of average income myself as follows:

  1. Generate $B = 100$ different samples of 1000 Durham residents.

  2. For each set $b = 1, \ldots, B$ of 1000 residents, compute $\bar{X}^b$.

  3. Make a histogram of all $\bar{X}^1, \ldots, \bar{X}^{100}$ values. This approximates the distribution of average income of Durham residents.

- Point estimate of average income is thus the mean of $\bar{X}^1, \ldots, \bar{X}^{100}$.

- To quantify uncertainty, can use the standard deviation of $\bar{X}^1, \ldots, \bar{X}^{100}$.

- For confidence intervals, use the quantiles of the histogram.

- In practice, however, the procedure above cannot be applied, because we usually cannot generate many samples from the original population.

- What to do then? Bootstrap!

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- Bootstrap is a very powerful statistical tool.
- It can be used to "approximate" the distribution of almost any parameter of interest.
- Bootstrap allows us to mimic the process of obtaining new sample sets by repeatedly sampling observations from the original data set.
- That is, replace step 1 of the previously outlined approach with

   1. Generate $B = 100$ different samples of 1000 Durham residents by re-sampling from the original observed sample with replacement.

- Can then follow the remaining steps to approximate the distribution of the parameter of interest.
- Ideally, the sample you start with should be representative of the entire population. Bootstrap relies on the original sample!

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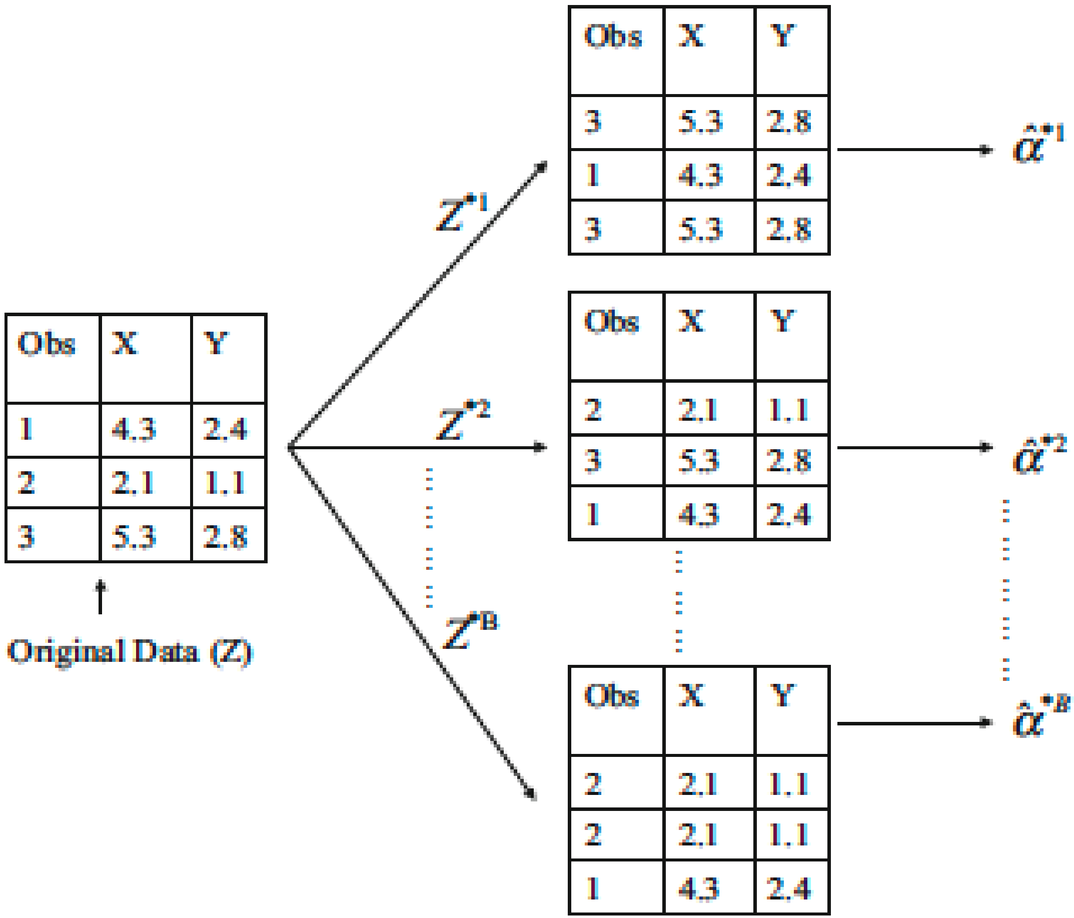
Here's a figure from the ISL book illustrating the approach.



What's next?

Move on to the readings for the next module!

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