$$X \mid Y=k \sim N(\mu_k, \sigma^2), \quad k=1, \cdots K$$

(LDA)

$f_k(x) \to$ pdf of $N(\mu_k, \sigma^2)$

logistic regression:

$$P(Y=1 \mid X=x) = \frac{e^{\beta_0+\beta_1 X}}{1+e^{\beta_0+\beta_1 X}}$$

$$P(Y=k \mid X=x) \overset{\text{Bayes' Theorem}}{=\!=\!=} \frac{P(Y=k, X=x)}{P(X=x)} \begin{matrix} \leftarrow 1 \\ \leftarrow 2 \end{matrix}$$

$$\underset{\substack{\uparrow \\ 1}}{P(Y=k, X=x)} = \underset{\substack{\uparrow \\ \text{p density f}}}{P(X=x \mid Y=k)} \underset{\substack{\uparrow \\ \text{pmf}}}{P(Y=k)}$$

$$= f_k(x)\, \pi_k$$

$$\underbrace{P(X=x) = \sum_{k=1}^{K} P(Y=k, X=x)}_{\substack{\uparrow \\ 2}} = \sum_{\ell=1}^{K} f_\ell(x)\, \pi_\ell$$

$$P(Y=k \mid X=x) = \frac{\pi_k f_k(x)}{\sum_{\ell=1}^{K} \pi_\ell f_\ell(x)}$$

k for the largest $P(Y=k \mid X=x)$

$\Longleftrightarrow$ k for the largest $\pi_k f_k(x)$

$$\pi_k f_k(x) = \pi_k \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x-\mu_k)^2}{2\sigma^2}\right]$$

$$\log(\pi_k f_k(x)) = \log \pi_k - \frac{(x-\mu_k)^2}{2\sigma^2} - \log(\sqrt{2\pi}\sigma)$$

Find k such that $\log(\pi_k f_k(x))$ is largest

$$= \log \pi_k - \frac{x^2 - 2\mu_k x + \mu_k^2}{2\sigma^2} - \log(\sqrt{2\pi}\sigma)$$

$$= \log \pi_k + \frac{\mu_k x}{2\sigma^2} - \frac{\mu_k^2}{2\sigma^2} \boxed{- \frac{x^2}{2\sigma^2} - \log(\sqrt{2\pi}\sigma)}$$

does not depend on k !

LDA: choose k such that

$$\log \pi_k + \frac{\mu_k \cancel{x}}{2\sigma^2} - \frac{\mu_k^2}{2\sigma^2}$$ is the largest among $k = 1, \cdots K$